This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

L. Irving Neale
1008 Western Ave.,
Allegheny City
Pa.

Educ T 219.00.275

Harvard College Library
FROM
Laurance I. Neale

1.2.1

3 2044 097 017 420

L. Irving Neale.
Desk 184.
Shady Side Academy,
Ellsworth Ave.,
Pittsburgh,
Pa.

A

TEXT-BOOK OF PHYSICS

BY

G. A. WENTWORTH, A.M.
AUTHOR OF A SERIES OF TEXT-BOOKS IN MATHEMATICS
FORMERLY PROFESSOR OF MATHEMATICS IN
PHILLIPS EXETER ACADEMY

AND

G. A. HILL, A.M.
AUTHOR OF A GEOMETRY FOR BEGINNERS, FORMERLY
ASSISTANT PROFESSOR OF PHYSICS IN
HARVARD COLLEGE

BOSTON, U.S.A.
GINN & COMPANY, PUBLISHERS
The Athenæum Press
1900

EducT 219.00.875

HA... Oct 19, 1937 LIBRARY

Laurance I. Neale

Copyright, 1898
By George A. Wentworth and George A. Hill

ALL RIGHTS RESERVED

PREFACE.

THIS book aims to give a rational explanation of the more important physical phenomena, and to prepare the way for further investigation and study of physical science.

The progress of Physics during the nineteenth century has been so rapid, and the useful applications of physical knowledge have become so numerous and important that there is danger of underrating the importance of the fundamental principles established by the great founders of the science. These principles will always remain the necessary basis of all systematic knowledge of the subject. Prominent among them are the parallelogram law, the law of moments, the laws of Pascal, Archimedes, and Boyle, the laws of uniformly accelerated motion, Newton's laws of motion, and the laws of energy. In this book much space has been devoted to the statement, proof, and application of these laws. The experimental methods by which the laws are established have been given; and, when possible, the line of thought of the discoverer has been indicated.

Modern Physics assumes that bodies consist of molecules, and modern Chemistry assumes that molecules consist of atoms. The student cannot too early become familiar with these assumptions and the grounds on which they rest. In Chapter IV some of the evidence for the theory of the molecular structure of bodies is brought together, and the nature of a chemical change is carefully explained. The reader is thus prepared to understand the references to chemical changes found in later chapters.

Any one who masters the first six chapters of this book will have laid a good foundation for the further study of Physics. The last two chapters are independent of each other; if found necessary, either one of them may be omitted.

Laboratory experiments, requiring simple apparatus only, are described for the purpose of *verifying* laws previously stated, *not for discovering* laws; and many numerical exercises are introduced into each chapter for practice in applying the principles of Physics to the common problems of life.

We are indebted to Prof. John Trowbridge, of Harvard University, and to Mr. Walter O. Pennell, of Philadelphia, Electrical Engineer, for their kindness in reading the proof sheets of the chapter on Magnetism and Electricity, and for many valuable suggestions. We are also indebted to Mr. William A. Stone, Instructor of Physics in Phillips Exeter Academy, who has read the proof sheets of the whole book, and made many valuable criticisms.

A pamphlet containing solutions of the numerical problems and full directions for performing the laboratory experiments will be published for teachers only. The pamphlet can be had on applying to the Publishers by any teacher who is using the text-book in his school.

G. A. WENTWORTH,
G. A. HILL.

JUNE, 1898.

CONTENTS.

PAGE

Table of Fundamental Units viii

CHAPTER I.

BALANCED FORCES.

Elementary Ideas about Matter 1
Fundamental Units 6
Elementary Ideas about Force 13
Elasticity 18
Friction 25
The Parallelogram Law 29
Law of Moments 36
Gravity Acting on Solids 43
Review Questions 50

CHAPTER II.

FLUID PRESSURE.

Gravity Acting on Liquids 51
Law of Archimedes 62
Atmospheric Pressure 71
Boyle's Law 78
Pneumatic Instruments 86
Review Questions 93

CHAPTER III.

HEAT.

PAGE
Change of Volume 95
Change of State 106
Calorimetry 115
Transmission of Heat 125
Heat in the Atmosphere 136
Review Questions 141

CHAPTER IV.

MATTER.

Molecular Phenomena 143
Molecular Forces 153
Chemical Changes 161
Review Questions 172

CHAPTER V.

MOTION.

Elementary Ideas about Motion 173
Falling Bodies and Projectiles 179
First Law of Motion 188
Second Law of Motion 192
Third Law of Motion 199
The Pendulum 205
Circular Motion 210
Universal Gravitation 214
Review Questions 216

CHAPTER VI.

ENERGY.

Mechanical Work 217
Machines 223

PAGE
The Two Types of Energy 234
Heat as a Form of Energy 242
The Steam Engine 250
Energy Considered as Subject to Change 254
Review Questions 261

CHAPTER VII.

MAGNETISM AND ELECTRICITY.

Magnetism 262
Electrostatics 271
The Electric Current 286
Magnetic Effects of a Current 294
Ohm's Law 304
Thermal Effects of a Current 318
Chemical Effects of a Current 327
Induced Currents 334
Review Questions 349

CHAPTER VIII.

SOUND AND LIGHT.

Sound 351
Transmission of Light 375
Reflection of Light 383
Refraction of Light 393
Dispersion of Light 403
Optical Instruments 411
Physical Nature of Light and Color 420
Review Questions 427

Appendix 429
Index 431

TABLES OF FUNDAMENTAL UNITS.

	English.	**Metric.**
Length.	inch (in.)	millimeter (mm.)
	foot (ft.) = 12 in.	centimeter (cm.) = 10 mm.
	yard (yd.) = 3 ft.	decimeter (dm.) = 10 cm.
	rod = 16½ ft.	meter (m.) = 10 dm.
	mile = 5280 ft.	kilometer (km.) = 1000 m.
Surface.	square inch (sq. in.)	square centimeter (qcm.)
	square foot (sq. ft.) = 144 sq. in.	square decimeter (qdm.) = 100 qcm.
	square yard (sq. yd.) = 9 sq. ft.	square meter (qm.) = 100 qdm.
Volume.	cubic inch (cu. in.)	cubic centimeter (ccm.)
	cubic foot (cu. ft.) = 1728 cu. in.	cubic decimeter (cdm.) = 1000
	cubic yard (cu. yd.) = 27 cu. ft.	also called a liter ccm.
	liquid gallon = 231 cu. in.	cubic meter (cbm.) = 1000 cdm.
Mass and Weight.	grain (gr.)	milligram (mg.)
	ounce (oz.) = 437½ gr.	gram (g.) = 1000 mg.
	pound (lb.) = 16 oz.	kilogram (kg.) = 1000 g.
	ton = 2000 lb.	tonne = 1000 kg.
	1 inch = 2.54 cm.	1 centimeter = 0.3937 in.
	1 mile = 1.609 km.	1 meter = 3.28 ft.
	1 square inch = 6.45 qcm.	1 square centimeter = 0.155 sq. in.
	1 cubic inch = 16.387 ccm.	1 cubic centimeter = 0.061 cu. in.
	1 grain = 0.0648 g.	1 milligram = 0.01543 gr.
	1 ounce = 28.35 g.	1 gram = 0.03527 oz.
	1 pound = 0.4536 kg.	1 kilogram = 2.2046 lb.

1 cubic foot of water weighs 1000 ounces, very nearly.

A TEXT-BOOK OF PHYSICS.

CHAPTER I.

BALANCED FORCES.

Elementary Ideas about Matter.

1. Body. That which fills space and acts on our senses is called *matter.* A limited portion of matter is called a *body;* for example, a table, a raindrop, the earth, the sun.

If two bodies have the *same properties* we say that they are composed of the *same substance,* and give that substance a name, as glass, wood, charcoal, iron, sulphur, water.

2. Compressibility. If we squeeze a sponge with the hand, the sponge is reduced in size, or *compressed.* If we push an air-tight piston down a cylinder, the lower end of which is closed (Fig. 1), the air within can be very much compressed.

Air and all gaseous bodies are very compressible. There is good reason to believe that every body can be compressed if sufficient force is used. Even iron is compressed by rolling and hammering, and in the process of making compressed steel.

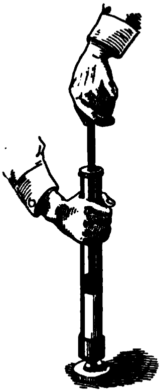


FIG. 1.

Compressibility is, therefore, a general property of matter.

3. Porosity. A body is said to be *porous* when it contains small cavities or *pores* not filled with the substance of which the body consists. Sponge and charcoal, for example, contain pores which can be easily seen (sensible pores).

In many cases we *infer* that a body is porous from its action upon other bodies or the action of other bodies upon it.

Thus a liquid must be porous if it will dissolve a certain quantity of a solid without sensibly increasing in bulk, and a solid must be porous if it will allow a liquid or a gas to pass through it. The following experiments are illustrations.

1. Dissolve fine sugar in a measuring glass containing water up to a certain mark. The water will dissolve a large quantity of sugar without sensibly increasing in bulk. The only rational explanation is that the water receives the sugar into its pores, somewhat as a hod full of coal will receive a large number of bullets and then in addition a large quantity of sand.

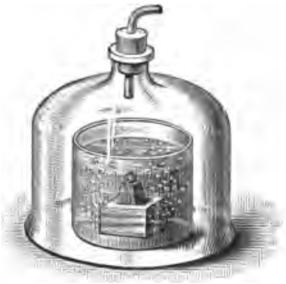


FIG. 2.

2. When a piece of wood is placed under water in an air-tight vessel (Fig. 2), and the air is then removed, large numbers of air bubbles are seen to form on the surface of the wood and also in the water, and to rise to the surface of the water. What do you infer from this experiment?

Two general facts lead us to believe that even the densest body contains very minute pores, called *physical* pores, too small to be seen even under the most powerful microscope.

First, bodies are compressible; and secondly, bodies in general expand when heated and contract when cooled.

Porosity is, therefore, a general property of matter.

Glass apparently has no pores, and no solid or liquid or gas has ever been made to pass through it. But glass expands when heated and contracts when cooled; therefore it must contain physical pores.

4. Changes of Matter. When an apple falls to the ground, it suffers a change of place, but no change of substance; the apple after falling has the same properties as before.

If the apple is left on the ground it gradually decays; it suffers a change in color, in taste, in weight, and in various other ways. In this case a change of substance occurs. The decayed apple has properties quite unlike those possessed by the sound apple.

Every *change of substance* is called a *chemical* change.

Every change which *does not involve a change of substance* is called a *physical* change.

Every change perceived by us, whether physical or chemical, is called a *natural phenomenon.*

5. States of Matter. Matter exists in three different states, the *solid* state, the *liquid* state, and the *gaseous* state.

Bodies that offer resistance to change of shape or to division into parts are called *solids.*

Bodies that offer practically no resistance to change of shape or to division into parts are called *liquids.*

Bodies that offer no resistance to change of shape or to division into parts, and *show a constant tendency to expand* or occupy more space are called *gases.*

Gases are easily compressed, liquids are almost incompressible.

Liquids and gases, however, have certain properties in common; as, for example, that of being able to *flow.* For this reason they are often put into one class and called *fluids.*

By means of heat, solids may be changed to liquids and liquids to gases. Conversely, by cooling (or cooling and pressure combined), gases can be converted to liquids and liquids to solids. These changes are called *changes of state.*

Water at ordinary temperatures is a liquid, but in winter it often changes to ice (solid water), and when it boils it changes to steam (gaseous water).

6. Indestructibility. There is one change that matter cannot undergo, namely, destruction or annihilation. When the oil in a lamp burns, it disappears from view, and at first seems to have been destroyed. But it is now well known that the oil in burning unites with oxygen from the air and forms an invisible gas which mixes with the air. So it is in all cases. Whenever matter appears to be destroyed, it merely suffers some change in form or properties which renders it incapable of affecting our senses as it did before. There is no truth in science more firmly established than that *matter is indestructible.*

7. Molecules and Atoms. The properties of compressibility and porosity, and the behavior of matter in the liquid and gaseous states, suggest strongly the idea that every body is composed of a great multitude of small distinct parts which have fixed relative positions in a solid, but are capable of moving freely from place to place in a liquid, and still more freely in a gas. Accordingly this hypothesis is accepted as true by men of science. The smallest portion of a body capable of retaining the properties of the body is called a *molecule.* All physical changes are believed to be due to the motions either of bodies or of their molecules (molecular motion). When a chemical change occurs, each molecule is supposed to break up into two or more smaller parts called *atoms,* which recombine in such a way as to form *new molecules* possessing properties unlike the original ones.

Molecules and atoms are too small to be seen. Their existence is inferred from the changes which matter undergoes and which can be explained only by supposing that they exist. Some of the grounds for believing that molecules and atoms exist will be given in Chap. IV.

8. Physics Defined. *Physics* is the systematic study of physical changes, their laws, and their causes. *Chemistry* is the study of chemical changes, their laws, and their causes.

LABORATORY EXERCISES.

1. Show that air is compressible (as in Fig. 1, or otherwise).

2. Test rock salt or alum for porosity by putting a lump of the substance into water and slowly heating the water.

3. Weigh a piece of chalk. Put it into water. Observe what takes place. After the chalk has been in the water some time, take it out and weigh it again. Compare the two weights. What inference as to the nature of chalk do you draw? Write in a note-book what you have done, what you have observed, and what you infer.

4. Place a sponge over the hole on the bottom of a flower pot, and above the sponge some layers of sand and powdered charcoal. Then pour dirty water into the pot. What happens to the water? Write out an account of the experiment, as in Exercise 3, and explain the effect on the water. What property of matter is illustrated?

CLASS-ROOM EXERCISES.

1. What is matter? How do you know that air consists of matter?

2. What property of matter is illustrated when you put a lump of sugar into a cup of tea and stir it around?

3. In Fig. 3 we see a beam bent by a heavy load. What effect has the bending on the length of the upper side? What effect has the bending on the lower side? How could you prove that these are the effects?

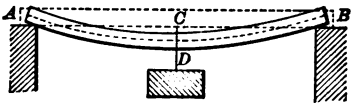


FIG. 3.

4. A hollow globe of lead was once filled with water by Lord Bacon, and the hole securely closed. Then the globe was hammered so as to flatten it out and diminish the space inside. The outside was then found to be covered with a fine dew. What did this prove?

5. Water and alcohol when mixed contract in bulk. What do you infer from this fact?

6. Is the falling of a stone a physical or a chemical change? the bending of a bow? the melting of ice? the boiling of water? the burning of coal? the rusting of iron? the ringing of a bell? the sprouting of a seed?

7. When phosphorus is burned in a closed vessel, it is found that the vessel and its contents weigh exactly the same after the burning as before. What truth does this experiment illustrate? What kind of a change does the phosphorus undergo?

Fundamental Units.

9. Units of Extension. To *measure* a magnitude of any kind is to find the number of times a magnitude of the same kind chosen as a *unit* is contained in the magnitude to be measured.

The *standard units* of length are the *yard* and the *meter*. Both are defined by law with reference to material standards, the yard with reference to a bronze bar kept at London, the meter with reference to a platinum bar kept at Paris.

From these standards other units of length, and units of surface and volume, are derived. Those derived from the meter form the so-called *metric system*.

Angles are expressed in *degrees*, *minutes*, and *seconds*.

If the radius of a circle is made to turn through one revolution it describes an angle of 360 degrees.

A degree $(^\circ) = 60$ minutes $(')$; a minute $= 60$ seconds $('')$.

10. Measurement of Extension. Short lengths (such as lines drawn on paper) are measured by applying to them an *inch scale* or a *millimeter scale*. For longer lines *yardsticks*, *meter sticks*, etc., are used.

Distances too great for direct measurement are measured indirectly by methods explained in Geometry.

The circumference of a circle, and the surfaces and volumes of regular solids are also measured indirectly; certain straight lines (radius, base, altitude, etc.) are first measured and then the quantity sought is computed with the aid of formulas proved in Geometry. The formulas most often used are given in the Appendix at the end of this book.

The volume of a liquid body or an irregular solid may be found by means of a *measuring glass* (Fig. 7).

Angles are measured with *protractors*, *theodolites*, or *sextants*.

11. Weight. If we hold a body in the hand, we feel that it is exerting pressure downwards; if we release the body, it falls to the ground. In short, the body behaves as if the earth attracted it. This attractive force is called *gravity*, and the downward pressure exerted by a body upon its support is called its *weight*. If we place two bodies in the pans of a scale-pan balance (Fig. 4), the arms AB, AC of which are equal in length, and if then the beam BC remains horizontal, we infer that the weights of the bodies are equal; for they exactly counterbalance when acting against each other under the same conditions. This operation is called *weighing*; it enables us to compare the weights of different bodies if suitable units are chosen.

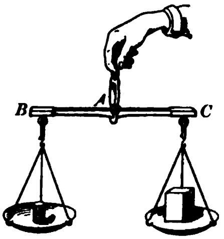


FIG. 4.

12. Mass. The quantity of matter which a body contains is called its *mass*. Although weight and mass are quantities unlike in kind, there exists a simple relation between them. If we hold a glass in the hand and pour into it sand or water, we observe that the weight appears to increase just as fast as the quantity of matter in the glass. We make weight the practical test of mass in all cases, by assuming that two bodies have the same mass if they have the same weight, or more generally, that *mass is directly proportional to weight.*

When we come to the study of the laws of motion (Chap. V) we shall see more clearly the real meaning of mass, and why two, three, four, etc., times the weight necessarily implies two, three, four, etc., times the mass. Meanwhile mass and weight should be carefully distinguished from each other. When we lift a body we feel its *weight* acting against us. When we push horizontally against the bob of a pendulum at rest, it is the *mass* of the bob and not its weight that opposes motion. If the earth ceased to attract bodies they would cease to have weight, but their masses would remain precisely the same as before.

13. Units of Mass and Weight. The English standard of mass is a certain piece of platinum kept at London, and is called a *pound.*

The metric standard of mass is a certain piece of platinum kept at Paris, and is called a *kilogram.*

The kilogram is equal practically to the mass of 1 cubic decimeter (or liter) of pure water at 4° Centigrade.

A kilogram contains 1000 grams. Since a cubic decimeter contains 1000 cubic centimeters, the gram is the mass of 1 cubic centimeter of pure water.

The units of weight in common use are the weights of the units of mass, and have the same names. Thus the downward pressure exerted by a mass of 1 lb. is called a weight of 1 lb.

This double use of the words pound and kilogram, as units for both mass and weight, is a disadvantage, but cannot very well be avoided. Some writers add the word weight when the terms are used to denote weights. Usually the context shows clearly enough the proper meaning.

14. Measurement of Mass and Weight. With the units defined as in the last section, both the mass and the weight of a body are found by weighing the body with a scale-pan balance. We place the body in one pan and add standard weights (really standard masses) to the other pan until there is exact equilibrium. The sum of the standard weights used gives the mass,and also the weight of the body.

This method of estimating weight assumes that the weight of a body does not change with locality. But we know that the weight of a body increases very slowly as we go away from the equator. Now a scale-pan balance cannot measure or even detect changes in weight, because they affect equally the body in one pan and the standard weights in the other. Accordingly, when it is important to take into account the small variations of weight due to change of locality, a scale-pan balance cannot be employed.

15. Density. The mass of one unit of volume of a body is called its *density*. For example, the density of dry pine wood is about 30 lb. per cubic foot; that of sheet copper, 549 lb. per cubic foot, or 8.8 grams per cubic centimeter.

From the definition of density it follows that the mass or weight of a body can be found by multiplying the density by the number of units of volume in the body. It follows also that if the mass (or weight) of a body and its volume are known, its density can be found by dividing the mass by the volume. In other words,

$$\textit{Mass}\text{ or }\textit{weight} = \textit{volume} \times \textit{density}.$$

$$\textit{Density} = \frac{\textit{mass}\text{ or }\textit{weight}}{\textit{volume}}.$$

Density is the weight of unit volume.

LABORATORY EXERCISES.

1. Measure the length of this page, the length of the printed part of the page, and the length of a full line of print on the page,

(1) taking as the unit an inch;
(2) taking as the unit a centimeter.

Measure with an inch scale so that the error in your result may be less than $\frac{1}{32}$ of an inch; and with the centimeter scale so that the error may be less than 0.1 of a centimeter.

2. Measure with a centimeter scale the lengths of the sides of the triangle ABC (Fig. 5). Try to estimate the lengths correct to *tenths* of a millimeter.

3. Find the area of the triangle ABC (Fig. 5). Area $= \frac{1}{2}$ base × altitude.

4. Find the area of the triangular piece of cardboard given you.

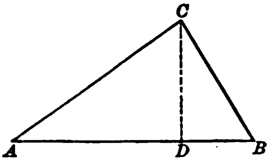


FIG. 5.

5. Describe a circle with a pair of compasses. Then find its circumference and its area.

Measure the radius or the diameter and then apply the formulas:

$$\text{Circumference} = \frac{44 \times \text{radius}}{7}; \quad \text{area} = \frac{22 \times (\text{radius})^2}{7}.$$

6. Find the surface and the volume of a rectangular block of wood.

Measure its three dimensions, and find the product of the three numbers which express the dimensions in terms of the same unit. Each dimension should be measured four times, once along each of the four parallel edges which give the dimension in question, and the arithmetic mean of the results (their sum divided by 4) should be taken as the true value.

The object of making more than one observation of a physical quantity, is to eliminate as far as possible the errors of observation. By taking the mean of several observations we obtain a value which is much more likely to be very near the true value than that of any one actual observation.

7. Find the surface and the volume of a cylinder.

The area of the base may be found as in Ex. 5; or the circumference of the base may be found by rolling the cylinder carefully on paper until it has made just one revolution, and then the radius and area found by means of the formulas given in Ex. 5.

8. Find the surface and the volume of a sphere.

Fig. 6 illustrates a simple method of determining the diameter of the sphere.

The surface and volume of the sphere are to be computed by means of the formulas

$$\text{Surface} = 4\,\pi r^2,\ \text{volume} = \frac{4\,\pi r^3}{3},\ \text{where } r = \text{the radius, and } \pi = \frac{22}{7}.$$

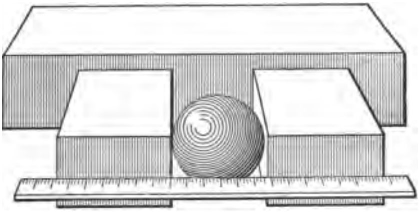


FIG. 6.

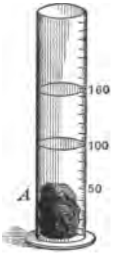


FIG. 7.

9. Find by the displacement of water the volume of a small stone.

Fig. 7 illustrates sufficiently the mode of procedure; when the stone is put into the vessel the level of the water rises from the mark 100 to the mark 160.

10. Find the weight of a given body.

If a platform balance (Fig. 8) is used, weights up to one kilogram can be determined to 0.1 of a gram.

By means of a simple chemical balance the weight of a body not exceeding 300 grams may be found to 0.01 of a gram.

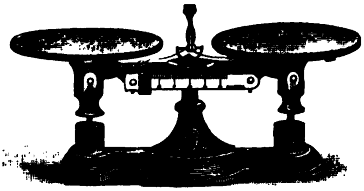


FIG. 8.

11. Find the density of the wood used in Exercise 6.

12. Find the density of the stone used in Exercise 9.

13. Find the density of a piece of wood, using for apparatus only a measuring glass, a metal sinker, and a balance with a set of weights.

CLASS-ROOM EXERCISES.

For numerical data, see the Tables opposite page 1.

1. What part of a kilometer is one millimeter?

2. What is the difference in centimeters between 4 meters and 8 dm.?

3. Reduce 1 square meter to square centimeters.

4. Reduce 16 liters to cubic centimeters.

5. Reduce 10 cubic meters to liters.

6. Mt. Blanc is 4800 meters high. Find the height in feet.

7. Which is the greater, 8 kilometers or 5 miles? Find the difference in feet.

8. Which is the greater, 20 centimeters or 8 inches? Express the difference in millimeters and tenths of a millimeter.

9. The Eiffel Tower in Paris is 300 meters high. Find the height of the tower in feet.

10. A body falls 16.1 feet in one second. What is this distance in centimeters?

11. In London the average height of the barometer at the sea level is 29.96 inches. Find the height in millimeters.

12. How many square millimeters are there in 1 square meter?

13. How many cubic centimeters are there in 1 liter?

14. A certain distance is equal to 89.6 cm. Change the decimal point so that the number shall give the distance in meters.

Write out and in

15. What is the weight in grams of 84 ccm. of water? What is the weight in kilograms of 84 liters of water?

16. A gallon holds 4 quarts. Which is the larger, the quart or the liter? What is the difference in cubic centimeters?

17. What is the weight of 72 ccm. of lead, its density being 11.3 grams per cubic centimeter?

18. If a lump of sulphur weighs 400 grams and its density is 2.1 grams per cubic centimeter, what is the volume of the lump in cubic centimeters?

19. An iron rod is 8 meters long and 2 cm. in diameter. The density of the iron is 7.2 grams per cubic centimeter. What does the rod weigh?

20. A brass cylinder weighs 2.4 kilograms. Its height is 7 cm. and the radius of its base is 3.8 cm. Find the density of brass.

21. A glass ball weighs 600 grams and its diameter is 7.7 cm. Find the density of glass.

22. What is the diameter of a platinum wire weighing 35 grams per meter? The density of platinum is 21 grams per cubic centimeter.

23. A straight piece of fine glass tubing weighs 1.2 grams. After a column of mercury 5 cm. long was drawn up the tube it weighed 2.2 grams. The density of mercury is 13.6 grams per cubic centimeter. Find the mean internal diameter of the tube.

Hint. The mercury in the tube forms a cylinder which is 5 cm. high and weighs just 1 gram. Find first the volume of this cylinder (see p. 9), then the area of its base, then the diameter of the base.

24. If the weight of 1 ccm. of a body in grams is denoted by d, and the weight of 1 cubic foot of water is 1000 ounces, show that the weight of 1 cubic foot of the body is equal to $\frac{1000d}{16}$ pounds.

Solution. Since 1 ccm. of water weighs 1 gram, the number d expresses how many times as heavy as water the body is when equal bulks are compared. Therefore

$$\begin{aligned}\text{Weight of 1 cubic foot of the body} &= d \times \text{weight of 1 cubic foot of water}\\ &= d \times 1000 \text{ oz.}\\ &= \frac{1000\,d}{16} \text{ lb.}\end{aligned}$$

25. One cubic centimeter of marble weighs 2.8 grams. What is the weight of 1 cubic foot of marble in pounds?

26. One cubic foot of granite weighs 165 lb. What is the weight of 1 ccm. of granite in grams?

27. With a 16-lb. weight and a quantity of sand, how could you obtain a quantity of sand weighing exactly 1 lb.?

1,4

(21) ,136

Elementary Ideas about Force.

16. Force Defined. Whenever one body appears to be the cause of motion or change of motion in another body, we say that it exerts *force* upon that body.

Force may be defined as *action exercised upon a body and tending to change its state of rest or motion.* Every force implies two bodies, namely, a body acting and a body acted upon.

A man exerts force when he pulls a door bell or pushes anything along the floor; a bat exerts force when it strikes a ball; water exerts force when it turns a water wheel. An apple falls from the tree to the ground; seeing no other cause for this motion, we say that the earth attracts the apple (*i.e.*, exerts force on it).

Muscular effort gives us our first idea of force. It teaches us that forces differ in magnitude, and also that a force may tend to move a body and yet not actually move it; as, for instance, when we push harder and harder against a table without moving it.

17. Effects of Force. When only one force acts on a body it causes motion or change of motion; but two or more forces may so act on a body that their combined effect is not motion or change of motion of the body as a whole, but relative motion of its parts. In this case the body suffers a change in size or shape, or else breaks up into separate parts.

Thus the beam in Fig. 3 (p. 5) is acted upon by three forces, namely, gravity and the pressures at the two props. The total effect of these three forces is not to move the beam as a whole but to bend it or change its shape. If the props were suddenly removed, the beam would at once begin to move under the action of gravity alone.

The effects of force, therefore, are of two kinds,

(1) Change of motion (either in rate or in direction),
(2) Change of size or shape (including *rupture*).

Change of size or shape is called *strain.*

18. Balanced Forces. Forces which so act on a body that they do not cause change of motion, but a strain of some kind, are said to be *balanced* or *in equilibrium.*

The simplest case is that of two *equal* forces acting on a body in *opposite* directions along the *same straight line.* But any number of forces greater than one may act on a body in such a way that they form a balanced system of forces.

The stretched rubber cord in Fig. 9 and the wood under compression in the vice in Fig. 10 are bodies acted upon by two balanced forces.

The beam in Fig. 3 affords an example of three balanced forces; they are the weight at the middle and the pressures on the supports.

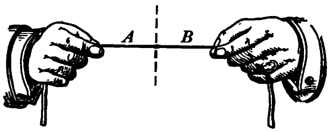


FIG. 9.

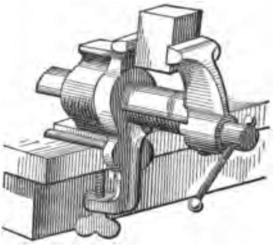


FIG. 10.

The body on which a balanced set of forces is acting need not be at rest, but may be in rapid motion, *provided the motion is uniform.* The forces are balanced if they cause no *change* of any kind in the motion.

For example, a book pushed along the surface of a table at uniform speed is acted upon by two pairs of balanced forces. The weight of the book and the equal upward pressure of the table constitute one pair. The push forward combines with the friction against the table to form the other pair; these two forces are equal, and remain equal so long as the book is moving at a uniform rate in a straight line. Each pair of forces strains the book somewhat, although the strains are too slight to be observed. If the book were a plastic body like jelly or soft dough, they would be seen very easily.

19. Stress and Strain. A pair of balanced forces, regarded as the cause of strain, is called a *stress.*

If the two forces act away from each other, the stress is called *tension* or *pull,* and the strain is called *extension* or *elongation;* if they act towards each other, the stress is called *pressure* or *push,* and the strain is called *compression.*

When a body is strained by tension (as in Fig. 9) or by pressure (as in Fig. 10), the stress exists not merely at the points where the external forces are applied, but all along the line of action between their points of application.

If the rubber cord (Fig. 9) should suddenly break at the dotted line, its parts *A*, *B* would at once begin to move away from each other. This motion is prevented now by a pull of *A* upon *B* and an equal contrary pull of *B* upon *A*; that is, by a pair of balanced forces or a *stress.*

20. Action and Reaction. Stress is a *mutual action* between two bodies; as, for example, between the rubber cord and either hand in Fig. 9, or between the wood and either jaw of the vise in Fig. 10. It is convenient to have names for the two equal forces that make up this mutual action, and they are called the *action* and the *reaction.* Which force is called action and which reaction is a mere matter of convenience.

Thus, in Fig. 9, we may take as the action either the pull of the hand on the cord or the pull of the cord on the hand.

When we exert muscular force upon anything, we seem to create the stress by our own act; hence it is natural to call the force which we exert the action, and the resistance which we meet the reaction.

If, as often happens, we confine our attention to one of the two bodies concerned, we are apt to regard the force exerted upon this body as the *only* force, and to overlook altogether the other half of the stress. But action and reaction always exist together; and they obey a very simple law, namely:

Action and reaction are always equal and opposite.

21. Elements of a Force. The elements of a force are its *magnitude*, its *direction*, and its *point of application*. When these are known the force is known.

The magnitudes of balanced forces are most easily compared by referring them to the force of gravity as a standard. For this purpose the units of weight (pound, gram, etc.) are employed as units of force; and balanced forces of every kind are measured by the weights which they are just capable of supporting. This mode of measuring force is called the *gravitation measure* of force.

The direction of a force is the direction in which motion will take place if other forces do not interfere.

The point of application of a force is really a small extent of surface; but we reduce in thought this surface to a point, in order to study the action of the force to better advantage.

A force is *represented* by a straight line drawn from the point of application in the direction of the force and containing as many units of length as there are units in the force.

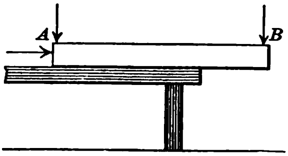


FIG. 11.

In Fig. 11 a beam projects beyond the edge of a table. The influence of direction is shown by pushing against the beam at *A*, first vertically, then horizontally. The influence of the point of application is shown by pushing vertically first at *A* and then at *B*.

22. Mechanics. The science of force is called *Mechanics*. It is divided into two parts: *Statics*, which treats of balanced forces, and *Dynamics*, which treats of unbalanced forces.

The study of balanced forces acting on liquids is called *Hydrostatics*. The study of unbalanced forces acting on liquids is called *Hydrodynamics*. The study of the mechanical properties of gases is called *Pneumatics*.

CLASS-ROOM EXERCISES.

1. Define force, and give examples of forces in action.

2. If a body is acted upon by a force and yet does not move, what is to be inferred? Give an example.

3. What kind of force is exerted when a man lifts his hand to his head? What other force is also acting?

4. Name the forces which cause motion in the following cases: a waterfall, a revolving windmill, a swinging pendulum, a locomotive drawing a train of cars.

5. A book is made to slide down a slate by raising one end of the slate. What force causes the motion? What force tends to stop the motion? See § 35.

6. If you push a sled on ice, it moves for a short time and then comes to rest. What force causes the motion? What force stops the motion?

7. Is friction a force that tends to produce or to destroy motion?

8. What are the two different effects of force?

9. Give an example of a force so acting that

(1) it makes a body move faster and faster.
(2) it makes a body move slower and slower.
(3) it changes the direction of the motion.

10. Give an example of a force so acting as to cause a strain. Is more than one force in action?

11. Give an example of two balanced forces; also one of three balanced forces.

12. A bucket is lowered down a well at a uniform rate of speed. What two balanced forces are in action?

13. A train is moving uniformly. What balanced forces are in action?

14. What kind of stress is exemplified when two railway trains come into collision? when a bucket of water is drawn up a well?

15. Give an instance of a force so acting as to cause rupture. Is more than one force acting?

16. Point out two cases of action and reaction in Fig. 9; and two cases also in Fig. 10.

17. What forces constitute action and reaction in the following cases:

(1) a trunk lying on a floor.
(2) a man pushing against a wall.
(3) a horse drawing a cart at a uniform rate.

18. Describe how you would represent on paper a force of 6 lb. acting due east, and a force of 8 lb. acting on the same point due south.

Elasticity.

23. Elasticity Defined. *Elasticity* is the property in virtue of which a body endeavors to recover its original size or shape when these have been changed by the application of force. The *force* of elasticity is the force with which at any instant the body endeavors to recover its original size or shape.

Thus, a rubber cord when stretched, a strip of steel when bent, and two ivory balls when they collide, display elastic force. On the other hand, putty and clay have practically no elasticity at all.

24. Elastic Limit. A straight piece of dry pine wood, if bent a little and then released, becomes perfectly straight again; if bent still more, the same thing happens; but at length, if the bending is continued, it ceases to become straight again when left to itself, and suffers a permanent change of shape. If the bending is carried much farther, the wood snaps in two.

A body is said to show *perfect* or *imperfect* elasticity according as it does, or does not, recover exactly its original size or shape after the stress that caused the strain is removed. Most bodies are found to be perfectly elastic, provided the strain does not exceed a certain limit. This limit depends on the nature of the material and the way the stress is applied. If the stress exceeds this limit, the body suffers a permanent change in shape or size called a *set*. This limiting stress is called the *elastic limit* of the material for the kind of stress in question.

The civil engineer always aims to design a bridge or other structure so that no piece shall ever be subject to a stress greater than its elastic limit; for, if this should happen, the piece is liable to injury, especially if the stress is applied suddenly.

25. Hooke's Law. The relation between stress and strain is most easily observed by suspending different weights at the lower end of a spiral spring (Fig. 12), and fastening to the spring a pointer which indicates the elongation of the spring by moving over a graduated scale. We find that if weights proportional to the numbers 1, 2, 3, etc., are used, the elongations are proportional to the same numbers.

Whatever the nature of the body or the kind of strain, the following simple law holds true (Hooke, 1676):

Up to the elastic limit, strain varies directly as stress.

When the stress exceeds the elastic limit the strain increases more rapidly than the stress.

The *spring balance* (Fig. 12) consists of a spiral steel spring enclosed in a metal case. A pointer moves over a scale graduated to indicate English or metric units of weight. This instrument can be used not only to weigh bodies, but also to measure balanced forces of all kinds. Any instrument constructed to measure balanced forces by the strains of a steel spring is called a *dynamometer*.

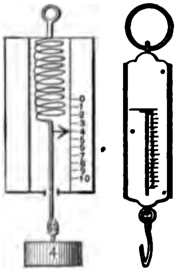


FIG. 12.

26. Tensile Stress. Tensile stress acts in the direction of the length of a bar, and tends to elongate it.

The vertical tie-rods of a railroad bridge and the piston rod of a steam engine during one half of the stroke are examples of bodies under tensile stress.

The following laws have been established by experiment:

(1) The strain varies directly as the stress.

(2) The strain varies directly as the length.

(3) The strain varies inversely as the cross-section; that is, *doubling* the cross-section *halves* the elongation, etc.

27. Modulus of Elasticity. The tensile stress (per unit of cross-section) which would just double the length of a bar of any material, *provided Hooke's Law held good for such an extension,* is taken as the measure of the elasticity of the material, and called its *modulus* of elasticity (Thomas Young).

Illustration. A steel bar 50 ft. long and having a cross-section of 0.1 sq. in. was found to increase in length 0.2 in. under a stress of 1000 lbs. What is the modulus of elasticity of the steel?

Since 0.2 in. is contained in 50 ft. (600 in.) 3000 times, it follows that, *if Hooke's Law held good,* it would take a force of 3000 × 1000 lbs., or 3,000,000 lbs., to double the length of the bar. But this would be the stress on a cross-section of 0.1 sq. in. Therefore the stress per square inch, or modulus of elasticity, is 10 times as much, or 30,000,000 lbs.

28. Compressive Stress, or *pressure,* tends to shorten a bar, and the change in length is called the *compression.*

Compression obeys the same laws as elongation; but unless the bar is very short, other effects than simple compression soon occur, such as *bending, bulging, buckling,* or *splitting.*

29. Bending Stress. A beam resting horizontally on two supports, A and B (Fig. 13), and bent by a heavy weight, is said to be under *bending stress.* The strain is measured by the distance, CD, through which the beam is bent at the point where the load is applied, and is called the *deflection.* The amount of the deflection depends on the nature of the material, and is also governed by the following laws:

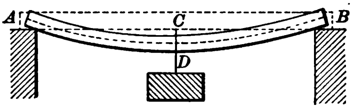


FIG. 13.

(1) The deflection varies directly as the weight.

(2) The deflection varies inversely as the breadth of the beam.

(3) The deflection varies directly as the cube of its length.

(4) The deflection varies inversely as the cube of its depth.

30. Best Shape for Beams. Floor beams and the girders of bridges should be so stiff that they bend very little under the greatest loads they have to support. How can this condition be satisfied with the least outlay of material?

If we examine a horizontal beam supporting a heavy load (Fig. 13), we see that the lower side is extended and the upper side compressed. Between these there must be a portion of the material which is practically under no stress at all, and which therefore contributes little or nothing to the strength and stiffness of the beam.

A line drawn from end to end of the beam through the points of no stress is called the *neutral axis.*

The farther we go from the neutral axis, either upwards or downwards, the greater is the resistance offered by the material to bending stress.

Hence, we infer that the depth of a rectangular beam should be much greater than its breadth (Fig. 14), and that still more is gained by collecting the material in large *flanges* at the top and bottom, joined by a thin *web* (Fig. 15). These conclusions are fully verified by experiment.

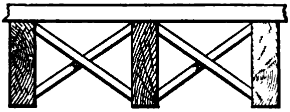


Fig. 14.

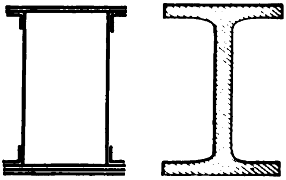


Fig. 15.

We are led to the same conclusions by the laws of § 29.

If we double the breadth of a beam, then by law (2) the deflection is halved, that is, the stiffness is doubled. But if we double the depth, the stiffness by law (4) becomes 8 times as much as before.

31. Twisting Stress, or *torsion,* tends to twist a bar.

The shaft of a screw propeller and the axles that convey power from wheel to wheel in machinery are examples of bodies under twisting stress.

If one end of the bar is fixed, the angle through which the other end turns is called the *angle of torsion.*

It varies directly as the length of the bar.

32. Shearing Stress. Shearing stress tends to make one part of a body slide over another part. Thus the iron bolt, *A* (Fig. 16), is under shearing stress. If the stress is great enough, the middle part of the bolt will be torn away.

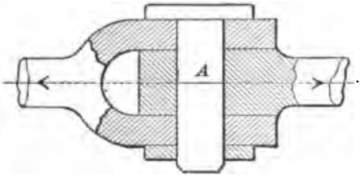


FIG. 16.

Cutting cloth with a pair of shears and punching rivet holes in metal plates are other examples of shearing stress.

33. Plasticity. A solid that shows no elasticity when its shape is changed is said to be *perfectly plastic.*

Putty and wet clay are almost perfectly plastic. Every body that is strained beyond the elastic limit shows more or less plasticity.

A plastic solid is said to be *malleable* if it can be rolled or hammered into a thin sheet, and *ductile* if it can be drawn out into a fine wire.

The order of malleability of the metals is as follows: gold, silver, copper, tin, platinum, lead, zinc, iron. The order of ductility is as follows: gold, silver, platinum, iron, copper, zinc, tin, lead.

Gold is so malleable that a square inch of gold leaf no thicker than letter paper can be hammered out into 50,000 sq. in.

Heat increases both malleability and ductility. For example, wrought iron, when white hot, can be rolled into thin sheets; and glass, when softened by heat, can be drawn out into fine threads.

34. Uses of Elasticity. We have already mentioned the spring balance. A watch would be of no use without a spring to keep the wheels in motion. We apply steel springs in a hundred ways to cause small but necessary return movements (*e.g.*, the spring of an electric bell). Steel springs are in universal use to make our chairs comfortable to sit in; our beds comfortable to lie in; our carriages and cars comfortable to ride in. Any body who has ridden over a rough road in a cart without springs will realize how much steel springs in carriages contribute to our comfort, by converting jolts and jars into smooth wave motions.

The elasticity of India rubber is put to almost as many uses as that of steel. If steel and rubber alone were to lose their elastic properties, our lives would be made extremely uncomfortable.

We may go farther and say that if there were no such thing as elasticity, our very existence from hour to hour would be placed in extreme peril. When a person jumps, the shock to that most delicate organ, the brain, is partly broken by the elastic arches of the feet, and still farther by the elastic fibers and discs in the body, so that there is a gradual arrest of the downward movement of the head. Were it not for this, the brain would be ruptured by a very small leap.

LABORATORY EXERCISES.

1. Verify Hooke's Law by the method shown in Fig. 12.

2. Determine to what extent an India rubber cord obeys Hooke's Law.

3. Verify Hooke's Law for bending stress (§ 29), and determine the elastic limit of the bar which you use.

4. Compare by experiment the forces required to break by bending stress two bars of pine wood alike in all respects except that one is twice as deep as the other.

CLASS-ROOM EXERCISES.

1. Explain, if you can, the reason for laws (2) and (3) in § 26.

2. If a wire breaks under a weight of 12 kilograms, and 4 meters of the wire weigh 0.3 gram, how long must the wire be to break under its own weight when hung up by one end?

3. If the safe pressure which brick can stand is 200 lb. per square inch, and brick is 2.2 times as heavy as water, how high can a brick wall be carried with safety?

4. If a rod 40 ft. long is stretched 2 in. by a force of 200 lb., how much will this force stretch a similar rod 2 ft. long?

5. A bar 8 qcm. cross-section is stretched 3 cm. by a certain force. What would be the elongation if the cross-section were only 1 qcm.?

6. If a weight of 1 ton bends a girder half an inch, how much would a weight of 10 tons bend the girder, if Hooke's Law holds true?

7. If a beam 8 in. wide is bent 3 in. by a certain weight, how much would it be bent if it were 2 ft. wide?

8. If a beam 12 ft. long is bent 2 in. by a certain weight, how much would it be bent if it were 6 ft. long?

9. If a beam 10 in. deep is bent 1 in. by a certain weight, how much would it be bent if the depth were 5 in.?

10. If a bar 60 cm. long, 2 cm. wide, 9 cm. deep is bent 1 mm. by a weight of 30 kg., how much will a bar of the same material 80 cm. long, 6 cm. wide, 3 cm. deep be bent by a weight of 20 kg.?

Solution. Let x = the deflection required.
The change from 30 kg. to 20 kg. makes the deflection $\frac{20}{30}$ as much.
" " " 2 cm. " 6 cm. " " " $\frac{2}{6}$ " "
" " " 60 cm. " 80 cm. " " " $\frac{80^3}{60^3}$ " "
" " " 9 cm. " 3 cm. " " " $\frac{9^3}{3^3}$ " "

$$\text{Therefore, } x = 1 \times \frac{20}{30} \times \frac{2}{6} \times \frac{512}{216} \times \frac{729}{27} = 14.22 \text{ mm.}$$

11. If a pine beam 5 ft. long, 2 in. wide, 3 in. deep, is bent 1 in. by a weight of 600 lb., how much will a pine beam 15 ft. long. 4 in. wide, 6 in. deep be bent by a load of 2 tons?

12. A pine floor beam is to be made 20 ft. long and 8 in. wide. What must be its depth to bend only 2 in. under a weight of 12 tons? See Ex. 11.

Friction.

35. Friction Defined. When we move one body upon another, their surfaces being pressed together by any force, we experience a resistance to the motion; this resistance is called *friction.*

Friction is a force that always tends to prevent or destroy motion, and always acts in the opposite direction to that of the motion.

There are two kinds of friction: *sliding* and *rolling.*

We have sliding friction when we push a book along the surface of a table, and rolling friction when we place rollers under the book and move it along the table.

Rolling friction is usually much less than sliding friction. For this reason we use wheels in carriages, provide armchairs with casters, and place rollers under buildings and large blocks of stone when we wish to move them.

Friction is due to two causes: the pressure between the surfaces in contact and their want of perfect smoothness. The smoothest surface, if viewed through a powerful microscope, is seen to have numerous little projections and cavities. When two bodies press against each other, the projections of one fit into the cavities of the other. Hence arises that resistance to motion which we call friction.

36. Laws of Sliding Friction. The following laws have been established by experiment (Coulomb, 1821):

1. *Friction varies directly as the pressure between the surfaces in contact.*

2. *Friction is independent of the extent of the surfaces.*

3. *Friction is independent of the rate of motion.*

4. *The friction at the instant of starting is greater than the friction in a state of uniform motion.*

Law 2 is true only when the pressure per square inch between the surfaces increases as the area in contact diminishes.

37. Coefficient of Friction. Since friction varies directly as pressure, the fraction whose numerator is the friction and denominator the pressure *has a constant value* for two given surfaces; doubling the denominator, for example, doubles the numerator, and therefore the value of the fraction does not change. The constant value of this fraction for two surfaces is called the *coefficient* of friction for these surfaces.

$$\textit{Coefficient of friction} = \frac{\textit{friction}}{\textit{pressure}}$$

and therefore

$$\textit{friction} = \textit{the coefficient of friction} \times \textit{the pressure.}$$

The coefficient of friction for dry pine wood is about 0.25. Hence, the force required to push a 40-lb. pine block along a horizontal pine board is 0.25 × 40 lb., or 10 lb.

38. Modes of diminishing Friction. In machinery, and in moving bodies generally, friction is a disadvantage, and therefore we try to get rid of it as far as possible.

The usual methods of diminishing friction are:

(1) Making the rubbing surfaces as smooth as possible.

(2) Applying oil and other lubricants.

(3) Substituting rolling friction for sliding friction.

A few values of the coefficient of friction are as follows:

Wood on wood, surfaces dry and rough,	0.50 to 0.80.
Wood on wood, surfaces dry and smooth,	0.20 to 0.45.
Metal on metal, smooth but not oiled,	0.20 to 0.30.
Metal on metal, smooth and well oiled,	0.07 to 0.08.
Metal on metal, oil constantly renewed,	0.05.

The coefficient of friction of a carriage or car is called its *draught*. On a level railway it is about 0.004, or 8 lb. per ton weight.

In delicate galvanometers (instruments for measuring minute electric forces) the moving body is suspended by a fiber of unspun silk; this substitutes, in place of friction, a very small resistance to torsion.

39. The Fixed Pulley. If we raise a weight by pulling on a cord which passes from the weight up over a beam, we find that we have to exert a great deal of extra force to overcome the friction of the cord along the beam.

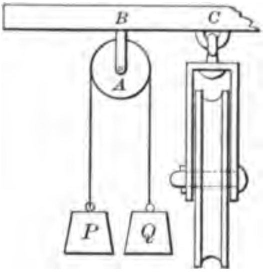


FIG. 17.

This loss from friction is almost entirely avoided by using a very *flexible* cord and a *fixed pulley.*

The pulley is a small wheel with a groove in its rim, turning freely about an axis at right angles to the face of the wheel. If equal weights, P and Q, are attached to the ends of the cord, they will balance each other. If P exceeds Q by a small amount (0.01 of P is enough for a good pulley, well oiled), P will descend and draw up Q; if Q exceeds P by the same fraction, the reverse motion will occur.

The fixed pulley enables us to change the direction of a force as much as we please, and yet make but a very slight change in its magnitude.

40. Uses of Friction. Friction is useful sometimes by producing indirectly a motion which we wish to have produced, and sometimes by destroying a motion which we wish to have destroyed. The following are examples:

(1) The friction of locomotive wheels on a railway track.

(2) The use of leather belts to carry power from wheel to wheel in a factory.

(3) The application of brakes to stop the motion of a railway train.

If friction did not exist, walking would be impossible, nails and screws would not hold in wood, woven fabrics could not be made, and buildings could not be erected; or, if erected, they would fall to the ground on the slightest disturbance.

LABORATORY EXERCISES.

1. Verify the first two laws of friction given in § 36.

Place a well-seasoned, smoothly planed, rectangular block of pine wood on a smoothly planed, horizontal pine board, and apply force to it horizontally, as shown in Fig. 18. Instead of a scale pan and weights, a pull of the hand registered by a spring balance may be used. For law 1 use different weights placed on the block (the weight of which must of course be included). For law 2 place different faces of the block in contact with the board.

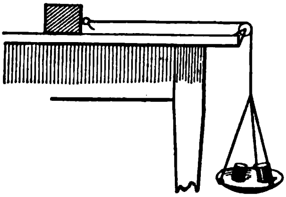


FIG. 18.

Find from your results the coefficient of friction of pine on pine.

CLASS-ROOM EXERCISES.

1. Describe the use made of friction in docking a steamboat at a wharf, if you have seen the thing done.

2. Give examples of sliding and of rolling friction.

3. Why is it harder for a horse to start a heavily loaded cart than to keep it in motion when once started?

4. The current of a river is less rapid near the banks than in the middle of the stream. Can you think of any reason for this?

5. What is the coefficient of iron on iron if an iron cube weighing 200 lb. is made to slide along a horizontal iron rail by a force of 38 lb.?

6. What force would be required to make the iron cube in Exercise 5 slide along the rail if a weight of 250 lb. were placed on top of the cube?

7. What horizontal force is required to push a trunk weighing 300 lb. along a floor, the coefficient of friction being 0.35?

8. What force is required to pull a sled along a smooth surface of ice, if the sled and its load weigh 300 lb. and the coefficient of friction is 0.05?

9. If the coefficient of friction between a wooden beam and the floor is 0.4, and it requires a force of 140 lb. to make the beam slide along the floor, find the weight of the beam.

10. Show that law 2, § 36, is a necessary consequence of law 1.

The Parallelogram Law.

41. Two Concurrent Forces. *Concurrent* forces are forces whose lines of action pass through the same point.

Let two equal weights be connected with a brass ring by means of cords passing over fixed pulleys, as shown in Fig. 19; then we observe that two equal concurrent forces act on the ring in opposite directions, and that the ring remains at rest. But if the forces are not equal (allowing for friction), then the ring is set in motion. In general,

1. *If two equal concurrent forces act on a rigid body in opposite directions, they balance each other.*
2. *If two concurrent forces acting on a rigid body are balanced, they must be equal and opposite in direction.*

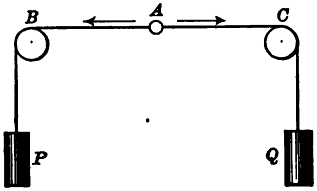


FIG. 19.

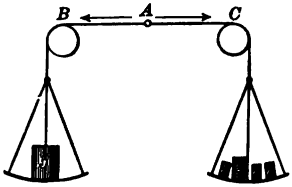


FIG. 20.

If forces of 500 grams toward the left and 200 grams toward the right act on the ring, the ring will move toward the left. But if in any way we make the total force toward the right equal to 500 grams (by adding three forces of 100 grams each, for example, as shown in Fig. 20), then the ring will remain at rest.

If concurrent forces having a common line of action are balanced, the sum of the forces directed one way is equal to the sum of the forces directed the opposite way. If these sums are not equal, their difference is an unbalanced force which will cause motion.

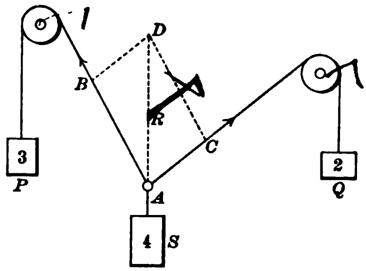


FIG. 21.

42. Resultant and Components. If three concurrent forces, *P*, *Q*, *S*, are made to act on a ring (Fig. 21) in different directions, but so that the ring remains at rest, then any one of the forces, as *S*, must be just balanced by the combined action of the other two forces, *P* and *Q*. Now we know that *S* would be just balanced by a force *R* equal to *S*, but opposite in direction (§ 38); hence the *single* force *R* is exactly equivalent to the *two* forces *P* and *Q*, and might be substituted for them.

A single force (here *R*) equivalent to two or more forces (here *P* and *Q*) is called their *resultant*, and they are called its *components*.

43. Parallelogram of Forces. If we represent the forces *P* and *Q* (Fig. 21) by straight lines *AB*, *AC*, drawn from the common point of application *A*, in the directions of *P* and *Q* respectively, and so drawn that their lengths are proportional to the magnitudes of *P* and *Q* respectively, and if upon *AB* and *AC* as sides we construct the parallelogram *ABDC*, we find that the diagonal *AD* exactly represents the resultant of *P* and *Q* (that is to say, a force *R* equal to *S*, but opposite in direction). Hence, we have the law:

If two concurrent forces are represented by straight lines drawn from their common point of application, the diagonal of the parallelogram constructed upon these lines as sides will represent their resultant.

This truth is known as the *Parallelogram Law* (Newton, 1686).

44. The Resultant in Special Cases. In certain cases the resultant R of two concurrent forces P and Q is easily found.

(1) If P and Q have the same direction, $R = P + Q$.

(2) If P and Q have opposite directions, $R = P - Q$.

(3) If P and Q act at right angles to each other, the parallelogram $ABDC$ (Fig. 22), in which AB represents P, and AC represents Q, is a rectangle; and the triangle ABD, the hypotenuse of which AD represents R, is a right triangle. By a theorem of geometry (see Appendix),

$$\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2,$$

or

$$\overline{AD}^2 = \overline{AB}^2 + \overline{AC}^2 \text{ (since } AC = BD).$$

Therefore, $R^2 = P^2 + Q^2$

whence $R = \sqrt{P^2 + Q^2}$.

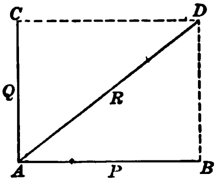


FIG. 22.

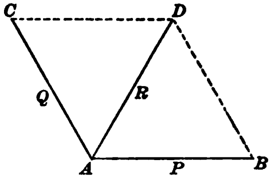


FIG. 23.

(4) If $P = Q$, the parallelogram $ABDC$ (Fig. 23) is a rhombus, and the diagonal AD bisects the angle BAC.

If $P = Q$, and also the angle $BAC = 120°$, the triangle ABD is equilateral, and therefore $R = P = Q$.

45. Three Balanced Concurrent Forces. If three concurrent forces are balanced, the following condition must be satisfied:

The resultant of any two of the forces must be equal and opposite to the third force.

For unless this were true, the resultant would not balance the third force (§ 41), and therefore the two components of the resultant would not balance the third force.

46. Resolution of a Force. By means of the Parallelogram Law a single force may be *resolved* into two components acting in given directions. We construct a parallelogram, the diagonal of which represents the given force, and the sides of which have the given directions of the components.

The resolution of a force into perpendicular components is very often useful. We will take as an example a weight W supported on a smooth inclined plane by another weight P, which is connected with W by means of a cord and fixed pulley, as shown in Fig. 24.

First represent W by a line AB drawn through a certain point A where (as explained later) the entire weight of the body may be considered to act.

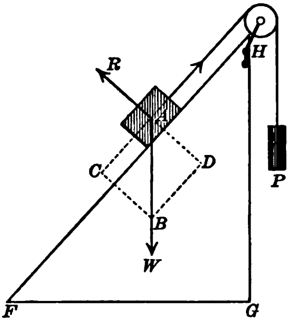


FIG. 24.

Then construct the rectangle $ACBD$ by drawing AC and BD parallel to the length FH of the plane, and AD and BC perpendicular to FH. The components of W are represented by AC and AD.

The component AC pulls the body down the plane and is balanced by P, and therefore P is equal to AC; AD pushes the body directly against the plane, and is balanced by the reaction R of the plane.

The triangles ABC, FGH are equiangular, and are therefore similar (see Appendix); therefore,

$$\frac{AC}{AB}=\frac{GH}{FH}, \text{ and } \frac{BC \text{ (or } AD)}{AB}=\frac{GF}{FH}.$$

$$\text{Or, } \frac{P}{W}=\frac{\textit{height of plane}}{\textit{length of plane}}, \text{ and } \frac{R}{W}=\frac{\textit{base of plane}}{\textit{length of plane}}.$$

47. Friction on Inclined Plane. Friction exists on every actual inclined plane. If the angle of inclination *GFH* of the plane (Fig. 25) is small enough, friction alone will prevent a body from sliding down the plane.

Let the angle *GFH* be increased till the body, left to itself, is *just on the point of sliding;* then the component *AC* of the weight of the body must be just equal to the friction *AE*. Since the pressure on the plane $= AD = BC$, therefore (§ 37),

$$\frac{AC}{BC} = \text{coefficient of friction.}$$

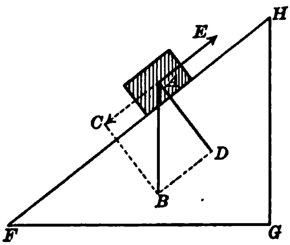


FIG. 25.

Since the triangles *ABC*, *FGH* are similar,

$$\frac{AC}{BC} = \frac{GH}{GF} = \frac{\text{height of plane}}{\text{base of plane}}.$$

Therefore, *coefficient of friction* $= \dfrac{\textit{height of plane}}{\textit{base of plane}}$.

48. Any Number of Concurrent Forces. If any number of concurrent forces act on a rigid body, we can find by the Parallelogram Law the resultant *R* of any two of them, then the resultant of *R* and a third force, and so on. In this way we can reduce the forces to *one* force, which is the resultant of the whole system. If the final resultant is equal to 0, the system is a balanced system.

It is clear also that if the resultant of all the forces *but one* is equal and opposite to that one, the system is a balanced system (§ 41). Conversely,

If a system of concurrent forces is balanced, any one force is equal and opposite to the resultant of all the other forces.

LABORATORY EXERCISES.

1. Verify by experiment the Parallelogram Law.

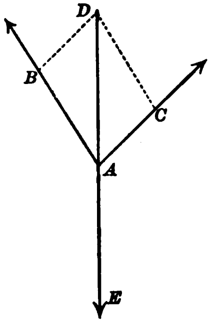


FIG. 26.

In Fig. 21 (p. 30) the forces are supposed to be $P = 300$ grams, $Q = 200$ grams, $S = 400$ grams. When the ring A is at rest, fasten behind the ring a piece of cardboard, and on it mark a point in the line of action of each force; this determines on the cardboard the directions of the three balanced forces. Lay the cardboard on a table, draw lines through A in the directions of the forces, and lay off lengths AB, AC, AE proportional to their magnitudes. Complete the parallelogram $ABCD$, draw AD, and find its value by measurement; AD and AE should be equal in length and opposite in direction.

2. Find by the method suggested in § 47 the coefficient of friction between two smooth surfaces of dry pine wood.

CLASS-ROOM EXERCISES.

1. Forces of 6 lb. and 8 lb. act on a point. Find their resultant (1) if they act in the same direction, (2) if they act in opposite directions, (3) if they act at right angles.

2. Find the resultants of the following pairs of forces acting at right angles: (1) 3 lb. and 4 lb.; (2) 5 lb. and 12 lb.; (3) 8 lb. and 15 lb.

3. Two forces of 20 lb. each act on a point at right angles. Find their resultant. What angle does it form with each force?

4. The resultant of two concurrent forces acting at right angles is 25 lb. One of the forces is 7 lb. Find the other force.

5. Resolve a force of 16 lb. into two perpendicular components, one of which shall be three times as large as the other.

6. Resolve a force of 60 lb. into two perpendicular components, making equal angles with the given force.

7. Three concurrent forces have the values 5, 7, 16. How must they act if their resultant is 4? 18? 28? 20? (See § 44.)

8. A 100-lb. weight is supported by two cords, each making an angle of 30° with the horizon. Find the tension of each cord.

9. What force parallel to a smooth inclined plane 40 ft. long and 24 ft. high will support on the plane a body weighing 1 ton?

10. The base of a smooth, inclined plane is 12 ft. and its height 5 ft. What force acting parallel to the plane will support on it a weight of 260 lb.? What will be the pressure on the plane?

Note. In the language of engineers the *grade* of an incline means the ratio of the height to the length of the incline, and is commonly expressed in the percentage form. Thus, a "4 per cent grade" means that in walking 100 ft. up the incline you rise 4 ft. Similarly, the *pitch* means the ratio of the height to the base.

11. What force is needed to support 1 ton on a smooth incline, if the grade is 8 per cent?

12. It is desired to lay a track along rising ground so that (neglecting friction) a force of 100 lb. should just cause a weight of one ton to ascend the incline. What must the grade be?

13. The pitch of a plane is 0.25, and the coefficient of friction between a certain body and the plane is 0.33. If the body is placed on the plane will it slide down or remain at rest?

14. Which is steeper, a 3 per cent grade or a 3 per cent pitch?

15. A body weighing 1000 lb. is supported in front of a wall by a tie-rod AB and a brace AC, as shown in Fig. 27. The angle $ACB = 45°$. Find the pull of the tie-rod and the push of the brace.

~~*Hints.*~~ The forces acting at A are 1000 lb. downwards, the pull of the rod in the direction AB, and the push of the brace in the direction AD. Take AE to represent 1000 lb. and complete the rectangle (here a square).

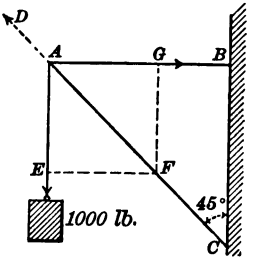


FIG. 27.

16. Three concurrent, balanced forces act, one toward the west, another toward the south, and the third toward the northeast. If the third force is 6 lb., find the other two.

17. A cord attached to two pegs A and B, fixed in the ceiling of a room, supports a ring weighing 50 lb. at C so that ACB is a right angle. Find the tension of the cord.

$25\sqrt{2}$

18. A car on a railway track is pulled by a horizontal force equal to 100 lb. in a direction making with the rails an angle of 60°. What is the force tending to make the car move forward?

Law of Moments.

49. Turning Effect of Force. If a uniform bar of wood AB (Fig. 28) is capable of turning freely about an axis passing through its middle point C, the bar will remain at rest in any position; therefore, its own weight may be left out of account. If we hang equal weights P, Q at points D, E, equally distant from C, the bar also remains at rest.

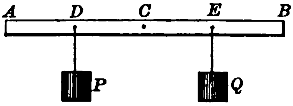


FIG. 28.

If we increase P, the end A of the bar falls; if we diminish P, the end A rises. If we push P away from C, the end A falls; if we push P towards C, the end A rises.

Hence, the turning effect of a force depends not alone on the magnitude of the force, but also on the *distance* of its point of application from the axis, increasing as this distance increases, diminishing as this distance diminishes.

A rigid bar capable of turning about a fixed axis is called a *lever*, and the axis is called the *fulcrum*. Levers are used for the purpose of raising a weight or overcoming a resistance Q by means of an applied force P.

Levers may be divided into three classes, according to the relative positions of P, Q, and the fulcrum F, as shown in Fig. 29.

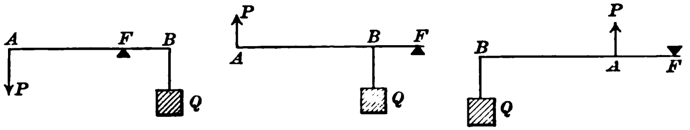


FIG. 29.

In the *first* class of levers, F is between P and Q; in the *second* class, Q is between P and F; in the *third class*, P is between Q and F.

50. Law of the Lever. Let us now determine by experiment the condition of equilibrium of a lever acted upon by two forces that tend to turn the lever about its axis.

(1) Let a uniform lever *AB* (Fig. 30) be so mounted on an axis *C* that its own weight may be disregarded. Let a weight of 8 oz. be hung 12 in. from the fulcrum on one side, and a weight of 6 oz. 16 in. from the fulcrum on the other side. The lever remains at rest, and we note the fact that 8 × 12 = 6 × 16. Let the distances of these weights from the fulcrum be now changed, but so that the equilibrium of the lever is maintained. If we measure the new distances and multiply each force by its arm, we find that the products, as before, are equal.

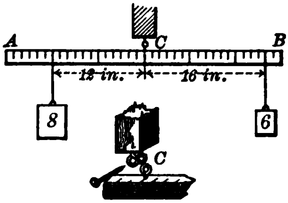


FIG. 30.

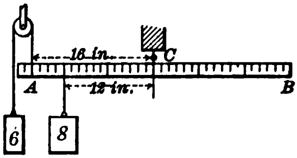


FIG. 31.

(2) Apply the same weights on the same side of the fulcrum in opposite directions (Fig. 31). When there is equilibrium, the products, found as before, are equal.

(3) Apply the same weights to a bent lever *ACB*, with the fulcrum at *C* (Fig. 32). When there is equilibrium, measure the lengths of the arms *CD*, *CE*. Again we find that the products obtained by multiplying each force by its arm are equal.

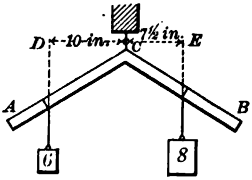


FIG. 32.

If a force acts on a lever, the perpendicular distance from the fulcrum to the line of action of the force is called its *arm*.

When a lever is in equilibrium under the action of two forces, *the products found by multiplying the number of units in each force by the number of units in its arm are equal.*

This truth is called the Law of the Lever (Archimedes, 250 B.C.).

51. Moment of a Force. The product of a force and its arm with respect to an axis is called the *moment* of the force with respect to that axis; and measures the effect of the force to produce rotation about the axis.

Evidently the two factors of a moment, namely, force and distance, are of equal importance in determining the turning effect; doubling either doubles the turning effect, and doubling both quadruples it.

The law of the lever, if we denote the two forces by P and Q, may now be briefly expressed as follows:

$$\textit{Moment of } P = \textit{moment of } Q.$$

52. Use of a Lever. It follows from the law of the lever that a force Q will be balanced by a smaller force P, provided the arm of P be made as many times greater than the arm of Q as P is times smaller than Q; for if this condition is satisfied, the moments of P and Q are equal.

Thus, a weight of 500 lb. applied 2 ft. from the fulcrum will be just balanced by a weight of 100 lb. applied 10 ft. from the fulcrum; for $100 \times 10 = 500 \times 2$.

Herein lies the practical utility of a lever. If we wish to raise a heavy weight Q, we apply, by means of a lever, a small force P, and secure equilibrium by giving to P a sufficiently long arm; then, if we increase P very slightly (just enough to overcome friction at the fulcrum), motion takes place and Q rises. The ratio of Q to P is called the *mechanical advantage* of the lever.

$$\textit{Mechanical advantage} = \frac{Q}{P} = \frac{\textit{arm of } P}{\textit{arm of } Q}.$$

Thus, in the example above mentioned, the mechanical advantage is found by dividing either 500 by 100 or 10 by 2, and is equal to 5. That is to say, 1 lb. so acts that it supports 5 lb.

53. Parallel Forces. Forces whose lines of action are parallel are called *parallel* forces.

The lever in Fig. 33 is under the action of three balanced parallel forces, a weight of 6 lb. acting at *A*, a weight of 8 lb. acting at *B*, and an upward pull at *C*. If this upward pull at *C* is registered by a spring balance, and the weight of the lever itself is left out of account, the pull is found to be just 14 lb., or equal to the sum of the weights acting downwards. And if we compute the moments of *any two* of the forces with respect to the point of application of the third force, we shall find that they are equal.

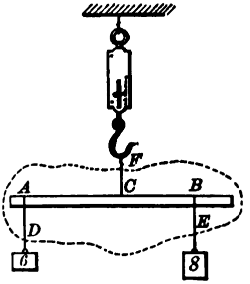


FIG. 33.

Now imagine the lever transformed into any rigid body, shown by the dotted line; this change (if we disregard the weight of the body) will not affect the equilibrium. Neither will it be affected if the points of application of the forces are changed to others in their respective lines of action (say from *A* to *D*, from *B* to *E*, from *C* to *F*). We now have a rigid body acted upon by three balanced parallel forces.

If we call the two forces farthest apart the *outer* forces, and the other force the *inner* force, the conditions satisfied by three balanced parallel forces acting on a rigid body may be stated as follows:

1. *The outer forces act in the same direction, and the inner force acts in the opposite direction.*
2. *The sum of the outer forces equals the inner force.*
3. *The moments of any two of the forces about the point of application of the third force are equal.*

54. General Law of Moments. Let $ABCD$ (Fig. 34) be a square piece of board (whose weight we neglect) mounted so as to turn freely about an axis passing through its middle point O. Let four forces, P, Q, R, S, act at the corners A, B, C, D, respectively, in such a manner that the board remains at rest. The arms of these forces with respect to the axis are OE, OF, OG, OD, respectively.

There is a *fifth* force in action, namely a pressure on the axis O; but, so far as the turning effect about O is concerned, this force may be disregarded, for it has no arm and therefore no turning effect about O at all.

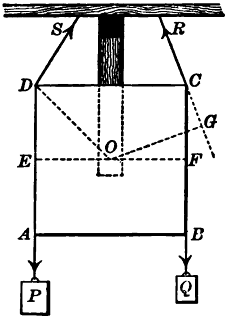


FIG. 34.

The forces P and R tend to make the body turn about the axis in one direction, while the forces Q and S tend to make it turn in the opposite direction. Since the body remains at rest, the combined turning effect of P and R must be just equal to the combined turning effect of Q and S; in other words, the sum of the moments of P and R with respect to O must be equal to the sum of the moments of Q and S, or

$$P \times OE + R \times OG = Q \times OF + S \times OD.$$

If a rigid body capable of rotating about an axis is in equilibrium under the action of forces whose lines of action lie in a plane perpendicular to the axis, the sum of the moments of the forces which tend to turn the body in one direction is equal to the sum of the moments of the forces which tend to turn the body in the opposite direction.

It can be proved that this law holds true if the moments of the forces are taken with respect to *any point* in the plane of the forces.

LABORATORY EXERCISES.

1. Verify the Law of the Lever by experiments like those shown in Figs. 30 and 31, p. 37.

2. Verify the Law of Moments (p. 40), using a lever with at least two weights acting on each side of the fulcrum.

CLASS-ROOM EXERCISES.

1. What kind of lever is a pair of scissors? a pair of nut crackers? the treadle of a lathe? the oar of a boat? a pump handle? a claw hammer drawing out a nail? a pair of sugar tongs?

2. The arms of a lever of the first kind are 9 in. and 25 in. What weight acting on the longer arm will balance 125 lb. acting on the shorter arm? What is the pressure on the fulcrum? 4

3. Weights of 1 lb. and 12 lb. balance on a lever. The longer arm is 3 ft. long. What is the length of the shorter arm? 20"

4. An oar is 12 ft. long and the rowlock is 2 ft. from the handle. If the oarsman pulls with a force of 100 lb., what force is exerted at the rowlock in propelling forward the boat? 2

5. Find the mechanical advantage of the levers in Exs. 2, 3, and 4.

6. Why is a piece of wire to be cut with shears placed near the rivet?

7. A bar 9 ft. long supports at its ends weights of 16 lb. and 20 lb. Find the position of the fulcrum.

Hint. Let $x =$ one arm; then $9 - x =$ the other arm.

8. A horizontal rod, hinged at A, has two 8-lb. weights, acting 12 in. and 18 in. from A. What upward force 16 in. from A will produce equilibrium?

9. A stiff pole, 12 ft. long, sticks out horizontally from a vertical wall. It would break if 20 lb. were hung at the end. How far along the pole may a boy weighing 80 lb. venture with safety?

10. A bar 44 in. long is in equilibrium with weights of 5 lb. and 6 lb. hung at the ends. Find the position of the fulcrum.

11. A lever of the second class, 10 ft. long, has to be applied so that a force of 20 lb. will overcome a resistance of 180 lb.; what must be the distance from the resistance to the fulcrum?

12. Two men pull each with a force of 36 lb. on oars 7 ft. long, the rowlocks being 2 ft. from the handle. Find the total force exerted on the boat.

13. A horizontal bar, whose weight we neglect, 12 ft. long, rests on two props, A and B. A weight of 160 lb. is hung 3 ft. from A. Find the pressure on each prop.

Hint. Treat the bar as a lever, having for a fulcrum first A and then B.

14. Two men, A and B, carry a load of 200 lb. on a pole between them. The men are 5 ft. apart and the load is 2 ft. from A. What part of the load does each man bear?

15. A rod 5 ft. long rests on two props. Weights of 4, 6, 8, and 10 lb. are hung from the rod at distances of 1, 2, 3, ~~and 4 ft.~~ respectively from one prop. Find the pressure on each prop.

NOTE. Two parallel forces are said to be *like*, if they act in the same direction; *unlike*, if they act in opposite directions.

If P and Q denote two parallel forces, A and B their points of application, and R their resultant acting at C in the line AB, solve the following problems:

16. P and Q like, $P = 4$ lb., $Q = 8$ lb., $AB = 9$ in.; find R and AC.
17. P and Q like, $P = 7$ lb., $Q = 2$ lb., $AB = 3$ ft.; find R and AC.
18. P and Q unlike, $P = 4$ lb., $Q = 6$ lb., $AB = 1$ ft.; find R and AC. 3
19. P and Q unlike, $P = 3$ lb., $Q = 9$ lb., $AB = 2$ ft.; find R and AC. 3

20. A street lamp weighing 100 lb. is supported by a bracket projecting 4 ft. from a wall, as shown in Fig. 35. The tie-rod AC is attached to a point C in the wall 4 ft. above the brace AB. Find the pull along the tie-rod and the push along the brace.

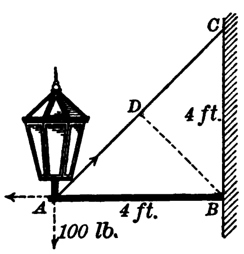


FIG. 35.

Let x = pull of tie-rod, y = push of brace.

Since there is equilibrium, the moment of x about B = the moment of 100 lb. about B.

The arm of x is BD. By Geometry

$$BD = DC,$$

and $$\overline{BD}^2 + \overline{DC}^2 = \overline{BC}^2.$$

$$\therefore 2\,\overline{BD}^2 = 16, \text{ whence } BD = 2\sqrt{2}.$$

$$\therefore 2\sqrt{2} \times x = 400, \text{ whence } x = 141 \text{ lb.}$$

By taking the moments about C

$$4y = 100 \times 4, \text{ whence } y = 100 \text{ lb.}$$

21. Solve Ex. 20 by the Parallelogram Law.

22. Solve Ex. 15, p. 35, by the Law of Moments.

Gravity Acting on Solids.

55. Direction of Gravity. The direction of the force of gravity at any place is called the *vertical* direction; it is indicated to the eye by a *plumb line*, that is, a string supporting a small weight in a state of rest.

A line or plane perpendicular to a vertical line is said to be *horizontal.*

Vertical lines, if produced, meet near the earth's center; therefore, two vertical lines are not parallel (Fig. 36). Since, however, the earth's center is nearly 4000 miles distant, two vertical lines near each other may be regarded as parallel, without sensible error.

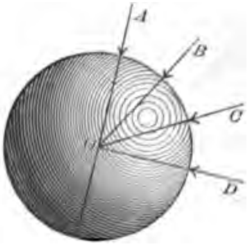


FIG. 36.

For instance, two vertical lines which are 1 meter apart form an angle of only 0.03 of a second. So small an angle as this cannot be directly measured with our most perfect instruments.

A plumb line at London is very nearly opposite in direction to a plumb line at New Zealand, and nearly at right angles to a plumb line on the island of Ceylon in India.

56. Weight. The force exerted by gravity upon a body is called the *weight* of the body.

The weight of a body, as already stated (§ 14), varies from place to place; it is less at the equator than at the poles by about one half of one per cent; and it is also a little less at the top of a mountain than at the sea level. A scale-pan balance, from its nature, cannot detect these changes in weight, and a spring balance is not sensitive enough to measure them with accuracy. But the pendulum is an instrument which can measure them, as will be explained at another time. For all ordinary purposes these changes may be neglected.

57. Center of Gravity. Upon each part of a body, however small, gravity is acting; so that the entire action constitutes a system of innumerable small forces, sensibly parallel in direction, each force having its own point of application. But all these points of application may be reduced to *one*, as we will now show.

From § 53 it appears that when two weights *P*, *Q* (Fig. 37) act on a rigid body at any points *A*, *B*, they are just balanced by a force $P+Q$ acting upwards at a point *C* so situated that the moments of *P* and *Q* about *C* are equal. It follows that we may replace *P* and *Q* by a single weight equal to $P+Q$, and acting downwards at *C* without affecting the equilibrium of the body.

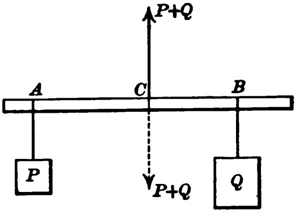


FIG. 37.

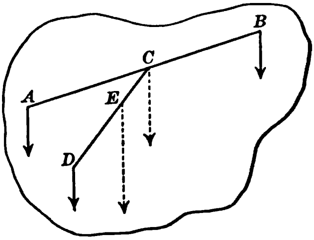


FIG 38.

Now, conceive a body divided into any number of parts (whose size we may leave out of account), and gravity acting on each part. Replace the forces acting at *A* and *B* (Fig. 38) by a force equal to their sum acting at *C*; then replace this force and that acting at *D* by a force equal to their sum acting at *E*, and so on. Finally, we must arrive at *one* point where *the entire weight of the body may be considered to act.* This point is called the *center of gravity* of the body.

If we imagine a system of *strictly* parallel forces acting on a body, a force on each particle, the fixed point of application of their resultant is called the *center of mass* of the body.

58. Regular Bodies. If a body has a regular shape and is uniform in density, its center of gravity coincides with its center of figure. The position of the center of figure in some cases is obvious, and in other cases can be found by geometrical reasoning.

For the sake of brevity the letters C. G. are often used in place of the words center of gravity.

The C. G. of a thin, uniform, rectangular board *ABCD* (Fig. 39) is the intersection *G* of the diagonals *AC* and *BD*.

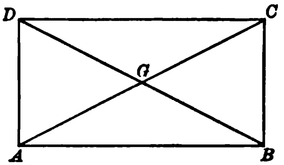


FIG. 39.

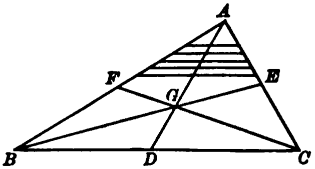


FIG. 40.

The C. G. of a uniform triangle *ABC* (Fig. 40) lies in the line *AD* joining the vertex *A* to the middle point *D* of the opposite side *BC*. For we may conceive the triangle made up of lines parallel to *BC*. Each of these lines will be bisected by *AD*; therefore its C. G. lies in *AD*; therefore the C. G. of the triangle lies in *AD*. By the same reasoning it must also lie in *BE* (*E* being the middle point of *AC*). Therefore it must be at *G*, the intersection of *AD* and *BE*.

It is proved in Geometry that $AG = \frac{2}{3}\ AD$ and $BG = \frac{2}{3}\ BE$.

59. Weight of a Lever. When we wish to take into account the weight of a lever, we must treat this weight as a force acting at the center of gravity of the lever.

For example, let a uniform lever *AB* (Fig. 41) be 32 in. long and weigh 2 lb., and let the fulcrum *F* be 4 in. from *A*. How many pounds *x* applied at *A* will keep the lever horizontal?

The C. G. of the lever is at *C*, 12 in. from *F*. Therefore, taking moments about *F*, we have

$$4x = 12 \times 2, \text{ whence } x = 6 \text{ lb.}$$

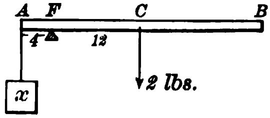


FIG. 41.

60. Kinds of Equilibrium. If the center of gravity of a body is supported, the whole body is supported. If the center of gravity is not supported, *it will descend to the lowest position possible under the circumstances.*

Let us apply this principle to a body capable of turning about a fixed axis. The body is in equilibrium if the vertical line through its center of gravity passes through the axis; for in this case the weight of the body will be balanced by the pressure of the fixed axis.

Now, suppose the body to be slightly disturbed. The equilibrium is said to be *stable, unstable,* or *neutral,* according as the body, when left to itself, returns to its original position, moves still farther from its original position, or remains at rest in its new position.

These three kinds of equilibrium are all illustrated in Fig. 42, where *A* represents the axis and *G* the center of gravity. The equilibrium will evidently be stable, unstable, or neutral, according as the center of gravity is below, above, or at the point of support.

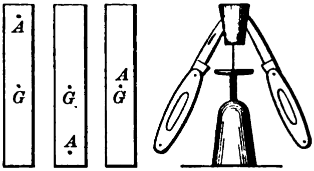


FIG. 42.

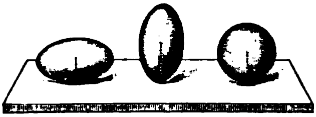


FIG. 43.

The definitions just given of the three kinds of equilibrium apply also to bodies having curved surfaces and resting upon a horizontal plane (Fig. 43). In this case the center of gravity of the body is always above the point of support.

In all cases, if a body is in stable equilibrium, the center of gravity is at the *lowest* position possible under the circumstances; if in unstable equilibrium, the center of gravity is at the *highest* possible position.

61. Stability of Equilibrium. Suppose a body with a flat base is placed on a horizontal plane (Fig. 44). The vertical line drawn through its center of gravity may fall either within or without the base. In the first case the center of gravity is supported, and the body is in stable equilibrium. In the second case the center of gravity is not supported, and there is no equilibrium at all; the body will topple over.

In such cases as a table or a chair the base is understood to be the area enclosed by a string stretched round the legs at the floor.

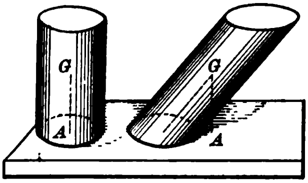


FIG. 44.

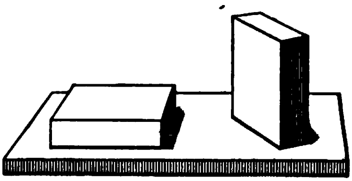


FIG. 45.

The amount of stability possessed by a body resting on a horizontal plane varies in different cases. It increases with the weight of the body, and with the distance through which the center of gravity of the body has to be raised in order to make the body tip over.

An iron cube, for example, has more stability than a wooden one of the same size, because it weighs more.

A rectangular block of wood (Fig. 45) has greater stability when resting on its largest face than when resting on either of the others, because its C. G. is then in the lowest position and must be raised farther before the block will overset.

The Leaning Tower at Pisa in Italy is a very interesting case of stability of equilibrium. It is a round tower 188 ft. high, built of white marble, with eight circular stories which gradually diminish in diameter from the base towards the top. A plumb line suspended from the top strikes the ground 15 ft. from the base; but the C. G. is so low down that a vertical line through it falls within the base.

62. The Balance. The subject of equilibrium is illustrated by the conditions which a good balance must satisfy.

(1) A balance should be *true;* that is, the beam should remain horizontal when equal masses are in the pans.

This requires that the arms be exactly equal in length and the pans exactly equal in weight.

(2) A balance should be *stable;* that is, the beam if disturbed from the horizontal position should quickly return to it. Stability is secured, provided

(*a*) The position of the center of gravity of the beam is *below* the fulcrum (§ 60).

(*b*) The straight line *AB* (Fig. 46) joining the points of suspension of the pans, does not pass *above* the fulcrum *F*.

If *AB* passes above *F*, the addition of masses to the pans will raise the C. G. of the beam and added masses, and may raise it to a point above *F*, in which event the beam would be in unstable equilibrium and would overset. It is best to have *AB* and *F* in a straight line, as seen in the figure.

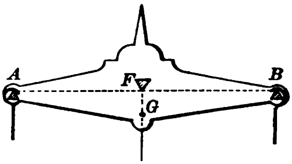


FIG. 46.

(3) A balance should be *sensitive;* that is, the beam should turn through an angle easily visible when the masses in the pans differ very slightly from each other.

The conditions that conduce to high sensitiveness are:

(*a*) The center of gravity of the beam must be *very near* the fulcrum.

(*b*) Long arms, a light beam, and very little friction.

Double Weighing. This is a method of obtaining the true weight of a body by the use of a false balance (balance with unequal arms).

Place the body in one pan and add shot or sand to the other till there is equilibrium. Then replace the body by standard weights till there is again equilibrium. The sum of the standard weights gives the true weight of the body. Why?

LABORATORY EXERCISES.

1. With a wooden bar, a meter stick, and a set of weights, find by experiment the weight of the bar and the position of its C. G. (See Ex. 8 below.)

2. Find (1) by experiment, (2) by construction (as explained in § 58), the C. G. of a triangular piece of cardboard. Method (1) depends on the fact that if the cardboard be suspended by one of its vertices, the vertical line through the point of suspension will pass through the C. G. of the cardboard (Fig. 47).

If the results of (1) and (2) do not agree, what is the reason?

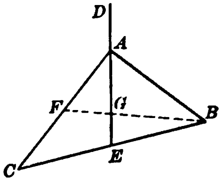


FIG. 47.

CLASS-ROOM EXERCISES.

1. A ladder is raised from a horizontal to a vertical position by making it turn about one of its ends. Why does the force required to raise it become less as it approaches the vertical position?

2. Why is a wagon going along an uneven road more likely to overset if loaded with hay than if loaded with an equal weight of iron?

3. A uniform rod 1 ft. long and weighing 4 lb. has 6-lb. and 8-lb. weights fastened at its ends. At what point will it balance? 5 6

4. A uniform bar 3 ft. long and weighing 6 lb. has 3 rings, each weighing 3 lb., at distances 3 in., 15 in., and 21 in. from one end. About what point of the bar will the system balance? 15 in.

5. A uniform beam 10 ft. long and weighing 140 lb. turns on a pivot 4 ft. from one end where hangs a weight of 41 lb. What weight must be hung at the other end to maintain equilibrium?

6. A uniform rod 2 ft. long and weighing 5 lb. has a weight of 1 lb. placed at one end. Find the position of the C. G. of the whole. 10

Hint. The required position is evidently that of the fulcrum on which the whole will balance.

7. A uniform rod 14 ft. long balances at a point 4 ft. from one end when 8 lb. is hung at that end. Find the weight of the rod. 14

8. A wooden bar *AB*, 40 in. long, balances on a pivot *C* 10 in. from *A* when a weight of 10 lb. hangs at *A* and 2 lb. at *B*. If the weight at *A* is changed to 4 lb., the bar balances at a point 15 in. from *A*. Find the weight of the bar and the position of the C. G.

6 lb. weight

16 2/3

REVIEW QUESTIONS ON CHAPTER I.

1. Define the terms mass and weight so as to make clear the difference in their meanings.

2. Describe briefly the scale-pan balance and the spring balance. Why is a scale-pan balance unable to indicate variations in the weight of a body?

3. What is meant by the term *stress?* What is meant by the term *strain?* Give illustrations of the meanings of stress and strain. What is meant by *tensile stress?* What is meant by *bending stress?* What is meant by *shearing stress?*

4. If a body is acted upon by two or more forces, how can you tell whether the forces form a balanced system or not?

5. State and illustrate the law of action and reaction.

6. What are the *elements* of a force? How is a force represented?

7. State Hooke's law. How is it illustrated in tensile stress? in bending stress?

8. Why is it better to make the depths of floor timbers greater than their widths?

9. Mention some of the uses of elasticity.

10. Mention some of the uses of friction.

11. State the laws of sliding friction.

12. What is meant by saying that the coefficient of friction of pine on pine is 0.25? How much force will be required to make a pine plank weighing 60 lb. slide on a pine floor?

13. State the parallelogram law. If two concurrent forces act on a body, how can you find the magnitude and direction of a third force that will exactly balance the two given forces?

14. Under what conditions will the resultant of two concurrent forces have the greatest value? Under what conditions will the resultant of two concurrent forces have the least value?

15. What is the law of equilibrium of a simple lever acted upon by two forces? Describe a lever such that a mechanical advantage equal to 2 is gained by its use.

16. State the laws of equilibrium for three parallel forces.

17. State the law of moments.

18. Define the center of gravity of a body. How may it be found in the case of a triangle?

19. Define and illustrate stable, unstable, and neutral equilibrium.

20. Upon what conditions does stability of equilibrium for a body resting on a plane depend?

CHAPTER II.

FLUID PRESSURE.

Gravity Acting on Liquids.

63. Liquids Compared with Solids. The most important difference between solids and liquids is that the particles of a solid are held firmly together by a force called *cohesion*, while in a liquid *the force of cohesion is very weak*, so that the particles give way in any direction with the greatest ease; as is seen, for example, when we thrust our hand into water.

Another difference is that a liquid, when set in motion, is not brought to rest by friction in the proper sense of the word, but by a kind of resistance called *viscosity* (see p. 159).

Again, solids differ greatly from one another in compressibility, but *liquids are nearly incompressible.* Experiment shows that a pressure of 3000 lb. per square inch will compress 100 cubic inches of water to only 99 cubic inches.

Moreover, while solids show elasticity in very different degrees, *all liquids have perfect elasticity of volume.* For instance, if 100 cubic inches of a liquid are compressed to 99 cubic inches, and then the pressure is removed, the liquid will at once expand to its original volume, 100 cubic inches.

Owing to the weakness of cohesion and the absence of true friction, a liquid, if subjected to pressure, behaves very differently from a solid, as we shall proceed to explain.

The behavior of fine, dry sand gives us an idea of the nature of a liquid. It can be poured from vessel to vessel like water; and it will run out in a stream through a hole bored in the side of a vessel.

64. Direction of Liquid Pressure. If a flat sheet of metal is pushed edgewise through water, the resistance to the motion is very small; this shows that there is no friction to speak of between the metal and the liquid.

Assuming no friction, it follows that the pressure of a liquid at rest against the sùrface with which it is in contact *must be perpendicular to that surface at every point.*

For if at any points the pressure were *not* perpendicular, the liquid at those points would move along the surface; that is, the liquid would not be in a state of rest.

65. Pascal's Law. Imagine a closed vessel (Fig. 48) filled with a liquid (whose weight we disregard), and provided with small pipes, equal in cross-section, and closed by pistons *A*, *B*, *C*, *D*, fitting tightly but capable of moving without friction.

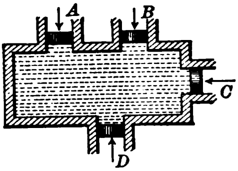


FIG. 48.

Now suppose a force of, say, 2 lb. to be applied to the piston *A*; then each of the other pistons will move outwards, unless prevented from so doing by applying to it a force of just 2 lb. In general,

A pressure applied to any portion of the surface of a liquid is transmitted in all directions, and is exerted unchanged in amount upon every equal portion of the surface of the containing vessel (Pascal, 1648).

Conceive the pistons *B* and *C* brought together, making one piston twice as large as either; then a force of 4 lb. would be exerted upon this piston. And if *B*, *C*, and *D* were united into one piston a force of 6 lb. would be exerted, and so on. That is, we have the general law,

The pressure upon any area is proportional to the area.

66. The Hydraulic Press. In the hydraulic press, the Law of Pascal is usefully applied. By working a pump, provided with a plunger piston of small size, water is driven into a large cylinder, where it slowly forces upwards a plunger piston of large size, thereby compressing whatever is placed between the platform of the piston and the fixed cross beam at the top of the press (Fig. 49).

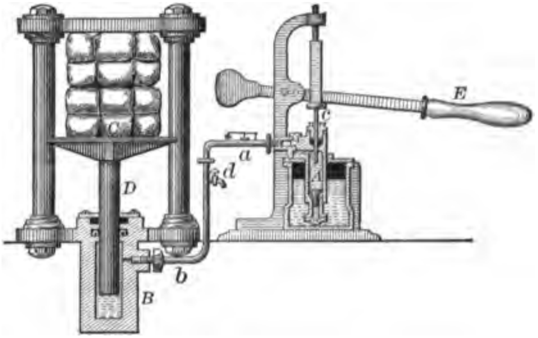


FIG. 49.

Suppose that the cross-section of the press piston is 200 times as great as that of the pump piston; then a downward pressure of 1 lb. on the pump piston will exert an upward pressure of 200 lb. on the press piston. In other words, the effect of the applied force is multiplied 200 times.

In this connection Pascal's own words may be quoted: "A vessel full of water is a new principle in mechanics, and a new machine for the multiplication of force to any required extent, since one man will by this means be able to move any given weight."

If the pressures on the two pistons are registered by dynamometers, and friction is allowed for, the hydraulic press supplies a convincing proof of the truth of Pascal's law.

The hydraulic press is used for compressing such substances as paper, cotton, and tobacco; for extracting juices from plants and oil from seeds; for testing the strength of iron girders and chains; and for raising enormous weights.

67. Free Surface. If a liquid acted upon by gravity is at rest, the *free* surface, or surface in contact with the atmosphere, is horizontal.

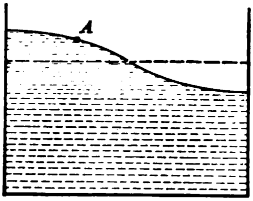


FIG. 50.

For if the surface were *not* horizontal, the higher molecules would be like bodies on a perfectly smooth inclined plane. Therefore they would descend, and the liquid would *not* be in a state of rest; and the motion would not cease until the surface became horizontal.

A horizontal surface is said to be *level*. Strictly speaking, it is a curved surface because the earth is round. The curvature is too slight to be noticed on a small pond, but it is apparent enough on the ocean. What phenomena make it evident?

68. Pressure Due to Gravity. A liquid at rest exerts pressure in consequence of its own weight, and this pressure obeys the following laws:

1. *The pressure at any point is equal in all directions.*

2. *The pressure at any point varies directly as the depth of the point below the free surface of the liquid.*

NOTE. The pressure of the atmosphere on the free surface is neglected.

3. *The pressure is the same at all points in the same horizontal plane.*

In these laws the phrase *pressure at any point* is to be understood as meaning *the pressure exerted upon one unit of area* at the point in question. Thus, the pressure at a point 20 cm. below the free surface of water is equal to 20 grams per square centimeter.

These laws may all be verified by experiment (see p. 60). They may also be established by reasoning based on the nature of a liquid, as we proceed to show.

Law 1 is Pascal's law in another form. If it were not true, every molecule subjected to unequal pressures would move in the direction of the greatest pressure; that is, the liquid would not be in a state of rest. But the liquid is at rest. Therefore the law is true.

Law 2 may be proved as follows. Consider a vertical cylinder *AB* of liquid with unit cross-section (Fig. 51). This cylinder receives no support from the lateral pressure upon its curved surface, because this pressure is horizontal in direction (§ 64). Therefore the downward pressure at *B* is just equal to the weight of the cylinder. This weight varies directly as the height *AB*, because the cylinder is uniform in density and cross-section. Therefore the pressure at *B* varies as *AB*.

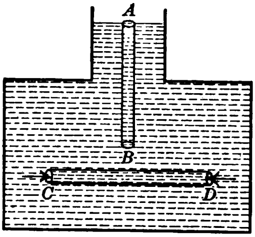


FIG. 51.

Law 3 is established by similar reasoning. Let *C* and *D* be two points in the same horizontal plane. Consider a liquid cylinder of unit cross-section, having the line *CD* as axis. Since the weight of this cylinder and the pressure upon its curved surface act vertically, they cannot prevent horizontal motion. Therefore the pressures at *C* and *D* must be equal to each other.

This proof holds equally good whether the points *C* and *D* are, or are not, directly below the free surface of the liquid. Therefore the pressure of a liquid upon the base of the vessel which contains it is *independent of the shape* of the vessel. The pressure depends on three things only:

(1) The *area* of the base,

(2) The *depth* below the free surface,

(3) The *density* of the liquid.

69. Pressure against Any Surface. If the surface pressed by a liquid is not horizontal, the pressure varies from point to point, increasing with the depth of the point. The mean or average pressure will be at the mean or average depth, and this is the depth of the middle point of the surface, or the center of gravity of the surface regarded as a thin uniform plate.

It can be proved that *the total pressure on the surface is the same as if the pressure were uniform at all points, and equal to the actual pressure at the mean depth of the surface.*

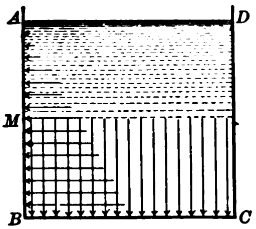


FIG. 52.

Thus, in a cubical vessel full of water (Fig. 52), the total pressure against one side of the vessel is just half as great as the pressure upon the base; for although the areas pressed are equal, the mean depth of the side is only half the uniform depth of the base. The arrows in the figure show the relative values of the pressure at different points.

The increase of pressure with depth is made evident when water is allowed to issue in jets through openings in the side of a vessel full of water (Fig. 53). The lower the opening, the greater the velocity of the jet. If the vessel is suspended by a cord it will move in the direction opposite to that of the jets. This is because the lateral pressure no longer exists at the openings *A* and *B*, but still remains acting upon the vessel at the opposite points *C* and *D*.

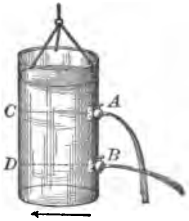


FIG. 53.

In the *lawn sprinkler* we see this unbalanced lateral pressure causing motion in a circle.

The increase of pressure with depth is very rapid. Water weighs 62½ lb. per cubic foot; hence, the pressure per square foot at a depth of 100 feet amounts to more than 3 tons.

This law of pressure explains why dams and the sides of reservoirs have to be made much stronger at the bottom than at the top, and why water rushes with great violence through a leak in the bottom of a ship.

70. Measurement of Pressure. The pressure of a liquid upon the horizontal base of a vessel with vertical sides is obviously equal to the weight of the liquid; hence (see § 15),

pressure = volume of liquid × its density.

Since the liquid mass has the shape of a prism or cylinder, its volume = its base × its height; therefore,

pressure = area pressed × depth × density.

In this formula we have merely to substitute for the word *depth* the words *mean depth,* and it follows from § 69 that the formula will then hold good for any surface.

If P denote the pressure, a the area pressed, h its mean depth (or depth of any point in case the surface is horizontal), d the density of the liquid; then the following formula will apply to all cases:

$$P = a \times h \times d.$$

If the liquid is water, and metric units are used, then $d = 1$ gram per cubic centimeter; if English units are used, $d = 1000$ oz. per cubic foot.

Suppose a cubical vessel whose edge is 20 cm. is closed at the top, except at one place where a vertical tube is fitted, whose height is 40 cm. and cross-section is 10 qcm. Let both vessel and tube be full of water.

Area of base or one side = 400 qcm.
" " pressed part of top = 390 "
Mean depth of base 60 cm., of top 40 cm., of side 50 cm.
Therefore, pressure on base = 24,000 grams
" " top = 15,600 "
" " side = 20,000 "
Also, weight of all the water = 8,400 "

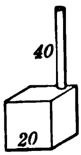


FIG. 54.

The fact that the pressure on the base is much greater than the weight of all the water seems at first sight paradoxical. But the paradox vanishes when we take into account the *upward* pressure on the top. The weight of the water is simply the difference between the downward pressure on the bottom and the upward pressure on the top.

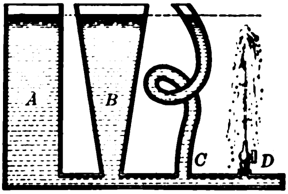


FIG. 55.

71. Liquids in Communicating Vessels. If two or more communicating vessels contain the same liquid, the level of the liquid when at rest will be the same in each vessel; or, more briefly stated,

A liquid maintains its level.

For if the level were higher in *A* than in *B* (Fig. 55), the pressure in the connecting tube from *A* towards *B* would be greater than the opposite pressure from *B* towards *A*. Therefore the liquid would flow from *A* to *B*; and this motion would not cease till the level was the same in both vessels.

If a short vertical tube is inserted into the connecting tube at any point *D*, the liquid will spout up in a jet, rising nearly to the level of the liquid in the vessels, but not quite reaching it on account of the resistance of the air.

Among the applications of the principle that water seeks its level is the *glass gauge* (Fig. 56), which indicates the height of the water in a boiler.

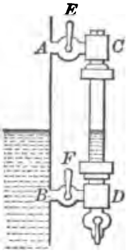


FIG. 56.

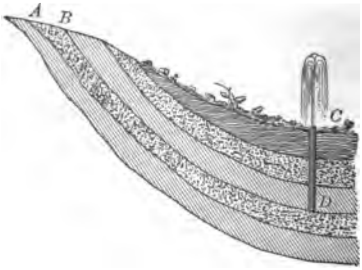


FIG. 57.

Another useful application is the Artesian well (Fig. 57), so called from Artois, France, where these wells were first bored. Artesian wells are wells of small diameter bored into the earth till a water-soaked layer of soil is reached, situated between layers impervious to water.

Artesian wells have been bored in the Desert of Sahara, and at a moderate depth (about 200 ft.) a plentiful supply of fresh water has been found. The Artesian well at Grenelle near Paris is 1800 ft. deep. It is the first deep Artesian well ever bored. The work of boring it extended from 1834 to 1841. On Feb. 26, 1841, the boring rods suddenly sank several yards, and within a few hours a column of water spouted up at the rate of 600 gallons per minute, and at a temperature of 29° C.

The deepest Artesian well in the world is at Sperenberg near Berlin; its depth is 4194 ft.

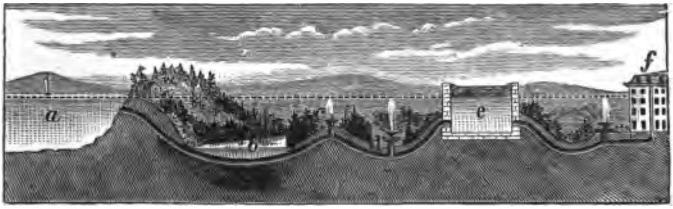


FIG. 58.

The method of supplying a city with water from a lake or reservoir situated higher than the city is another application. In Fig. 58 the water flows in an aqueduct from the lake *a* under the river *b*, over the hill *c*, through the valley *d*, into the reservoir *e*, from which it is carried by pipes to the streets of the city. If the level of the lake is not high enough, it becomes necessary to pump the water into the reservoir.

If we pour into the arm *AB* of a U-tube (Fig. 59) first some mercury and then some water, we find that we must make the water column in *AB* 13.6 cm. high in order to raise the mercury in the other arm 1 cm. above the level *BC*. The reason is this: the two columns *AB* and *CD* must be equal in weight; and this requires that *AB* be equal to 13.6 times *CD*, because mercury is 13.6 times as heavy as water. In general,

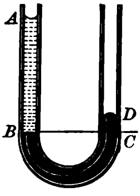


FIG. 59.

The heights of the balancing columns of two liquids in a U-tube are inversely as their densities.

LABORATORY EXERCISES.

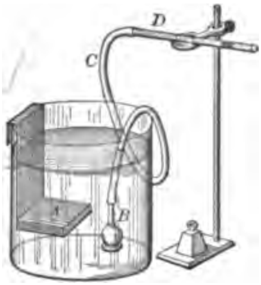


FIG. 60.

1. Verify the laws of liquid pressure stated on p. 54.

The apparatus sketched in Fig. 60 may be used. The larger the vessel of water, the better. The mass of lead *A* is placed so as to be between a part of the water and the free surface. The pressure gauge consists of a 'thistle tube' *B* with its lower end tightly closed by thin sheet rubber, and connected by a rubber tube *C* with an open horizontal glass tube *D* containing a short column of water. In the case of Law 2 all that this apparatus can show is that pressure increases with depth.

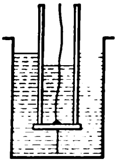


FIG. 61.

2. Show by the method suggested in Fig. 61 that the pressure upward at any point in a liquid is equal to the pressure downward.

The movable base of the inner cylinder must fit the sides of the cylinder water-tight. It will be found to give way before the level of the liquid inside is as high as the level outside. Why should we expect this, provided the upward pressure is equal to the downward pressure?

CLASS-ROOM EXERCISES.

If the kind of liquid is not mentioned, assume it to be water.

1. A vessel full of water is fitted with a tight cork. Why is it that a slight blow on the cork may suffice to break the vessel?

2. In a hydraulic press the areas of the pistons are 500 sq. in. and $\frac{1}{4}$ sq. in. What pressure on the small piston will exert a pressure of 1 ton on the large piston?

3. The diameters of the pistons in a hydraulic press are 32 in. and 2 in., respectively, and the pressure on the small piston is 10 lb. What pressure is exerted by the large piston?

Hint. Remember that the areas are as the *squares* of the diameters.

2 8 1 7 0

4. If in Ex. 3 the force of 10 lb. is applied at the end of a pump handle 5 ft. long, and the pump piston is attached to the handle 2 in. from the fulcrum, what power will be exerted?

Here the mechanical advantage of the lever is 60 ÷ 2, or 30, and that of the press alone (found as in Ex. 3) is 256.

Therefore the pressure exerted $= 30 \times 256 \times 10$ lb. $= 38.4$ tons.

5. In a hydraulic press the radius of the press piston is 3 ft., and that of the pump piston is 2 in. The pump handle is 5 ft. long, and the piston is attached to the handle 1 in. from the fulcrum. What force must be applied to the pump handle to produce a pressure of 243 tons?

6. Find the pressure (in kilograms per square centimeter) 1 km. below the surface of fresh water.

7. A house is supplied with water from a reservoir 241 ft. above the ground. Find the pressure per square inch on a tap 25 ft. above the ground.

8. A trough 6 meters by 2 meters contains water 3 meters deep. Find the pressure on the bottom of the trough. 36

9. A mill dam is 40 feet long, and the water is 15 feet deep. Find the pressure upon the dam.

10. A trough 20 cm. long, 10 cm. wide, 8 cm. deep is full of mercury. Find the pressure on its base and its sides if the density of mercury is 13.6 grams per cubic centimeter.

11. At what depth in water will the pressure on a horizontal square whose side is 50 cm. be 50 kg.? 25 cm.

12. The lid of a vessel full of water is a square whose side is 20 cm. A pipe, leading from a side of the vessel, is filled with water to a height of 3 meters above the lid. How many kilograms must be placed on the lid to prevent the escape of the water?

13. A hole 6 in. square is made in a ship's bottom 20 ft. below the water line. What force is required to hold a piece of wood tightly over the hole? 3/

14. Find the pressure on an isosceles triangle in water, held vertically with its base in the surface, the base being 50 cm. and the height 30 cm.

15. A cube whose edge is 20 cm. long is sunk till its top which is horizontal is 60 cm. below the surface. Find the pressure on one of its vertical sides. ?

16. A cubical box whose edge is 8 cm. is full of water, and a pipe opening into the side of the cube contains water to a height of 48 cm. above the bottom of the box. Find the pressure on the base, top, and one side of the box. Compare the pressure on the base and the weight of the water in the box, and explain why they are not equal.

Law of Archimedes.

72. Buoyant Force. It is easier to lift a stone under water than in air. If we place a piece of wood under water it will rise to the surface and float. When we enter a bath we apparently weigh next to nothing.

Every solid appears to lose weight when immersed in a liquid. How great is this apparent loss?

To answer this question by experiment, we may use a balance with a pan having a hook on its lower side from which bodies may be suspended (Fig. 62). Such a balance is called a *hydrostatic* balance. Place in one pan a hollow metal cylinder *A*, and suspend below the pan a solid metal cylinder *B* which just fits into *A*. Equipoise them by placing weights in the other pan (not shown in the figure). Then immerse *B* in water; the equilibrium is destroyed. But if we fill *A* with water the equilibrium is restored. Therefore the upward push of the water on *B* must be just balanced by the weight of the water in *A*; that is, by a quantity of water just equal in volume to that of the cylinder *B*.

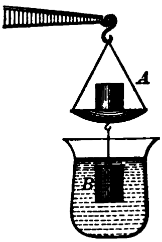


FIG. 62.

A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced.

This force is called the *buoyant* force of the liquid, and its point of application is the center of gravity of the liquid displaced. Hence, the center of gravity of the displaced liquid is often called the *center of buoyancy.*

This important law was discovered by the Greek philosopher Archimedes about 240 B.C. The story goes that he made the discovery while in a bath. Filled with joy, he rushed out of the bath into the streets, crying *eureka* (I have found it).

73. Cause of Buoyant Force. To understand the cause of buoyant force, consider a cylinder 1 qcm. in cross-section and 4 cm. high immersed in a liquid with its axis vertical (Fig. 63). The pressure at each point of the curved surface is balanced by an equal pressure at a point on the opposite side of the cylinder. The upward pressure on the base and the downward pressure on the top act upon equal areas (1 qcm.), but the depth of the base is 4 cm. more than that of the top. Therefore the upward pressure exceeds the downward pressure by the weight of a liquid column 1 square centimeter in cross-section and 4 cm. high; that is, a liquid column having the same volume as that of the immersed cylinder. The result would evidently be the same whatever be the dimensions of the cylinder.

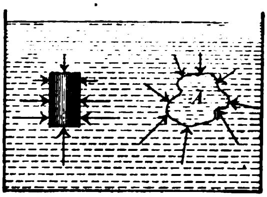


FIG. 63.

But without analyzing the pressures we can show that the law must hold true for any body whatsoever.

Imagine any portion of the liquid *A* to become a solid; this will not affect the pressures exerted upon it by the liquid which surrounds it. The portion *A* is kept at rest by these pressures. Hence, they must have a resultant equal to the weight of *A* and acting upwards through the center of gravity of *A* (§ 41). If now in place of this solidified liquid we substitute wood, iron, or any substance, this resultant, which is simply the buoyant force, will be unaltered, and remain equal to the weight of the liquid displaced by *A*.

Therefore a body completely immersed in a liquid will sink or rise to the surface according as its weight is greater or less than the weight of an equal bulk of the liquid; in other words, according as its density is greater or less than the density of the liquid.

Hence, iron sinks in water while cork floats. But a solid lighter than a liquid will not rise in it unless the liquid can gain access to the under surface of the solid so as to exert pressure upwards. A simple experiment of Pascal proves this. He placed a piece of wood in close contact with the smooth bottom of a vessel and then filled the vessel with mercury (Fig. 64). The wood did not rise; on the contrary, it was held fast to the bottom by the weight of the mercury above it. This explains why ships, which have settled down on a mud bank at low tide, sometimes will not rise when the tide rises.

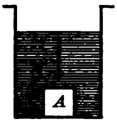


FIG. 64.

74. Floating Bodies. The Law of Archimedes applies just the same whether the solid is wholly or partially immersed in the liquid. If the solid floats, the buoyant force must be equal to the entire weight of the solid. Hence, for a floating body the law may be stated as follows; and in this form it is known as the *Law of Flotation:*

Weight of a floating body = weight of the liquid displaced.

In order that a floating body may be in equilibrium the center of gravity of the body and the center of buoyancy must be in the same vertical line.

Many illustrations of buoyant force and flotation might be mentioned.

1. The density of an egg is a little greater than that of fresh water, but a little less than that of very salt water. Hence, an egg sinks in fresh water, but floats in salt water (Fig. 65, *A* and *B*). But if we make a mixture of the fresh and salt water in such proportions that the egg and the mixture are alike in density, then the egg will remain at rest in any part of the liquid (Fig. 65, *C*).

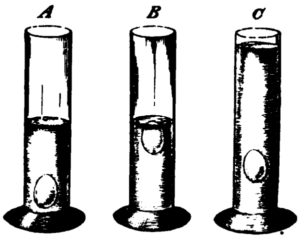


FIG. 65.

2. Warm water rises through cold water because warm water is lighter than cold water, and cream rises to the surface of milk because cream is lighter than milk.

3. The average density of the human body is nearly the same as that of water, being slightly greater when the lungs are empty and slightly less when they are full of air. Hence, a man can float by turning on his back in the water and filling his lungs with air.

4. An empty tin box will float, although the metal is much heavier than water. This is because its hollow shape enables it to displace a volume of water much greater than that of the metal alone.

5. In order to raise a sunken vessel, *lighters*, so full of water that they can barely float, are moored over it at low tide and attached to it by chains (Fig. 66). The water is pumped out of the lighters when the tide is rising. The buoyant force of the water and the tide, acting together, raise the vessel. Wooden piles which have been driven into the bed of a river during the construction of a bridge are removed in a similar way.

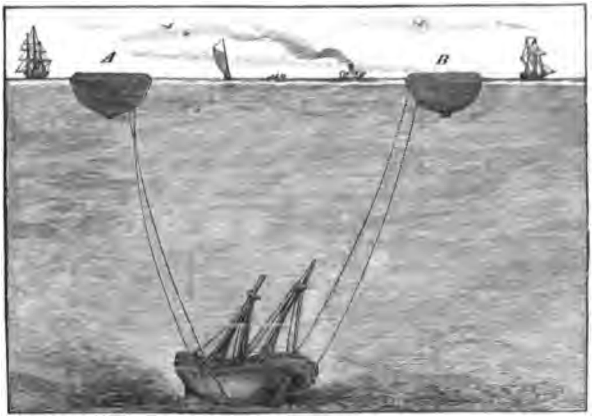


FIG. 66.

6. *Floating docks* are contrivances for raising ships out of the water for the purpose of making repairs. The dock is sunk by filling it with water, and the ship is floated upon it and made fast. Then the water is pumped out of the dock, which being thus made much lighter rises, and lifts the ship out of the water.

7. The lower the center of gravity of a floating body the greater is the stability of the equilibrium. For this reason ballast is put into the bottom of a vessel, and a plank floats with the greatest stability when placed flat upon the water.

75. Specific Gravity. The *specific gravity* of a substance is the *number* that expresses the ratio which the weight of a given volume of the substance bears to the weight of an equal volume of a standard substance.

For solids and liquids pure water at 4° Centigrade is the standard substance; so that for all solids and liquids

$$\textit{Specific gravity} = \frac{\textit{weight of the body}}{\textit{weight of equal bulk of water}}.$$

For the words 'specific gravity' the abbreviation sp. gr. is often used.

The importance of specifying temperature in defining specific gravity will appear later.

If the centimeter and gram are the units used, then, since 1 ccm. of water at 4° C. weighs 1 gram, the denominator of the above fraction expresses the volume of the water, and therefore that of the body, in cubic centimeters. But the weight of a body divided by its volume gives its density (§ 15). Therefore, when these units are used, the *specific gravity and the density of a substance are numerically equal.*

If English units are used, and we take 1 cubic foot of water as weighing 1000 oz., then density (in oz. per cu. ft.) = sp. gr. × 1000.

Example. Iron is 7.8 times as heavy as water; therefore,
sp. gr. of iron = 7.8; density = 7.8 g. per ccm. = 7800 oz. per cu. ft.

The following are methods of finding specific gravities.

Solid heavier than water. Suppose that a piece of copper weighs 89 grams in air and 79 grams in water. The loss, 10 grams, is the weight of an equal bulk of water. Therefore the specific gravity of copper $= 89 \div 10 = 8.9$.

Solid lighter than water. Flotation method. Suppose that a piece of wood floats so that we can observe what part of its total volume V is immersed; let it be four fifths. Let x denote the specific gravity of the wood; then, weight of the wood $= Vx$ (V being expressed in cubic centimeters); also, weight of water displaced $= \frac{4}{5}V$. Therefore (by Law of Flotation) $Vx = \frac{4}{5}V$, whence $x = \frac{4}{5}$.

Solid lighter than water. Sinker method. Suppose that a piece of wood weighs 40 grams in air, that a lead sinker weighs 50 grams in water, and that the two tied together weigh only 30 grams in water. Then the buoyant force on the wood not only wholly supports the wood, but takes away 50 — 30, or 20 grams, from the weight of the sinker when immersed alone. Hence, the buoyant force upon the wood, or weight of an equal bulk of water, is 40 + 20, or 60 grams. Therefore the specific gravity of the wood $= \frac{40}{60} = \frac{2}{3}$.

Solid soluble in water. In this case we have to use a liquid whose density is known, and in which the solid is not soluble. Suppose a lump of ice weighs 84 grams in air and 14 grams in kerosene the density of which is 0.77 grams per cubic centimeter. Then the specific gravity of ice with respect to kerosene is 84 ÷ 70, or 1.2. Hence, 1 ccm. of ice weighs 1.2 times as much as 1 ccm. of kerosene, or 1.2 × 0.77 grams, or 0.924 grams. Therefore the specific gravity of ice = 0.924.

Liquids. First method. Weigh a bottle, first empty, then full of water, then full of the liquid. Let the weights be 80 grams, 200 grams, and 260 grams, respectively. Then the weights of equal volumes of water and of the liquid are 120 grams and 180 grams. Therefore the specific gravity of the liquid = 180 ÷ 120 = 1.5.

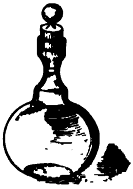


FIG. 67.

Bottles called *specific gravity flasks*, with perforated glass stoppers, are made for the purpose of performing this experiment with great accuracy (Fig. 67).

Liquids. Second method. Suppose a piece of glass weighs 330 grams in air, 230 grams in water, and 150 grams in sulphuric acid. The glass loses 100 grams in water and 180 grams in acid. These are the weights of equal volumes of water and the acid. Therefore the specific gravity of the acid = 180 ÷ 100 = 1.8.

Liquids. Third method. The specific gravity of a liquid is often found by means of a *hydrometer.*

The *common* hydrometer (Fig. 68, *A*) consists of a closed glass tube having a bulb at the lower end loaded with shot or mercury so as to keep the instrument upright. The tube is graduated by marking the points to which the instrument sinks in liquids of known densities; the less the density, the deeper the instrument sinks. The specific gravity of a liquid can then be found by floating the hydrometer in it and simply observing the mark to which it sinks. These hydrometers are used for testing the purity of alcohol, acids, milk, etc., and are graduated with special reference to their use.

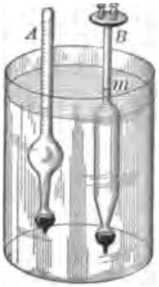


FIG. 68.

Fahrenheit's hydrometer (Fig. 68, *B*) applies the law of flotation somewhat differently. There is a mark *m* on the narrow part of the tube. Suppose that the instrument itself weighs 100 grams, and that it is made to sink to the mark *m*, in water by adding to the pan at the top 22 grams, and in olive oil by adding 20 grams. Then equal volumes of water and olive oil weigh respectively 122 grams and 120 grams. Therefore the specific gravity of olive oil = 120 ÷ 122 = 0.91.

LABORATORY EXERCISES.

Exercises in determining specific gravities by the methods just explained should be performed.

In explaining the methods nothing was said about practical difficulties and sources of error. The student should take note of these to the best of his ability and include them in his record of the experiment.

For example, air bubbles will collect on the surface of an immersed solid, and affect its apparent weight under water. They should be brushed away; or the water may be boiled beforehand to expel the air.

The platform balance (p. 11) may be used for these experiments. For this purpose it must be mounted firmly a foot or more above the table, and the solid suspended by a loop of thread passing over one platform.

CLASS-ROOM EXERCISES.

1. What effect is produced on the weight of the vessel of water in Fig. 62 when the cylinder *B* is immersed in it? Explain.

2. Explain the use of life preservers.

3. When a ship passes from fresh water into salt water, will it sink deeper or rise higher? Why?

4. Why does an iron steamship float when iron is nearly 8 times as heavy as water?

5. Fishes are provided with an air-bag which they can inflate or contract at will. Show how this enables them to rise or sink in the water.

6. Will lead (sp. gr. 11.3) sink or float in mercury (sp. gr. 13.6)?

7. Find the weight of a cubic meter of petroleum (sp. gr. 0.79). 790 Kg. [illegible]

8. Four cubic feet of cork weigh 60 lb. Find the sp. gr. of cork. [illegible]

9. What is the buoyant force on 1 cdm. of lead under water? [illegible]

10. Find the volume of 1 kg. of cast iron (sp. gr. 7.2). What does the iron weigh under water? 138.9 [illegible] 861 [illegible] grams.

11. If a piece of ivory weighs 95 grams in air and 76 grams in water, find its volume and its specific gravity. 19 cu. cm. [illegible]

12. A piece of glass weighs 24 grams in air and 16 grams in water. Find its volume and its specific gravity. [illegible]

13. What will 1 cubic decimeter of stone (sp. gr. 2.5) weigh in water? If this stone is placed in mercury (sp. gr. 13.6), how much of it will project above the surface of the mercury? 1.5 Kg. [illegible]

14. A block of wood is placed in a vessel just full of water. It floats half submerged, and 100 ccm. of water run out. Find the weight, volume, and specific gravity of the wood. 100 gms. 200 cu. cm. [illegible] sp. gr.

15. An iceberg has the form of a cube. Its height above the water is 30 ft. Find its entire height (sp. gr. of ice 0.918, that of sea water 1.025). [illegible]

16. A piece of pomegranate wood (sp. gr. 1.35) is fastened to a block of pine (sp. gr. 0.65) of equal bulk. Will the two float or sink in water?

17. A cube of wood floating in water supports a weight of 200 grams. When the weight is removed the cube rises 2 cm. Find the size of the cube. x = 10 cm, [illegible]

18. A barge with vertical sides, floating in fresh water, is 30 ft. long and 20 ft. wide. An elephant is driven upon the barge. When all is quiet, it is found that the barge has sunk just 4 in. Find the weight of the elephant. 1250 lbs.

19. A man weighs 75 kg. and his volume exclusive of his head is 72 cdm. How many cubic decimeters of cork (sp. gr. 0.25) are required to keep the man floating with his head above water? 4 cu. [illegible]

20. A barge with vertical sides sinks to ⅓ of its depth when unloaded and to ¾ of its depth when loaded. If the barge weighs 4 tons, find the weight of the cargo. 5 tons.

21. A lighter 20 ft. long, 8 ft. wide, 5 ft. deep, and weighing 2 tons is filled with water, so that it is nearly sinking, and then fastened by chains to a wreck beneath. Find the lifting power exerted on the wreck when all the water is pumped out of the lighter. 48 tons.

22. A cylinder floats in water with 4½ in. of its length immersed. To what depth will it sink in a liquid whose specific gravity is 0.915?

23. A piece of glass weighs 47 grams in air, 22 grams in water, and 25.8 grams in alcohol. Find the specific gravity of alcohol.

24. A body weighs 160 grams in air, 110 grams in water, and 80 grams in sulphuric acid. Find the specific gravity of the acid.

25. A Fahrenheit's hydrometer, weighing 100 grams, requires 250 grams in the pan to sink it to the marked point in naphtha and 350 grams to sink it to this point in water. Find the specific gravity of naphtha.

26. A piece of copper sulphate weighs 3 oz. in air and 1.86 oz. in oil of turpentine (sp. gr. 0.88). Find the specific gravity of copper sulphate.

27. A piece of gold weighs 9.7 grams. A flask full of water weighs 95 grams. The gold is dropped into the flask, displacing some of the water. The flask and its contents now weigh 104.2 grams. Find the specific gravity of gold.

28. A piece of wood weighs 120 grams in air. A piece of lead weighs 30 grams in water. Both weigh 20 grams in water. Find the specific gravity of the wood.

29. A solid weighs 4 lb. in air. A sinker weighs 8 lb. in water. Both weigh 6 lb. in water. Find the specific gravity of the solid.

30. A solid weighs 100 grams in air and 85 grams in water. What will it weigh in a liquid whose specific gravity is 0.8?

31. A nugget of gold and quartz weighs 10 oz. The specific gravity of gold is 19.4, that of quartz is 2.1, and that of the nugget is 6.4. Find the weight of the gold in the nugget.

32. A body weighs 18 lb. and its specific gravity is 3.5. If the body is suspended by a string, find the tension of the string (a) when the body is immersed in water, (b) when it is immersed in a liquid whose specific gravity is 2.

33. The specific gravity of pure milk is 1.03. The specific gravity of an article sold as pure milk is found to be 1.02. Prove that 33 per cent of this so-called milk is water.

Atmospheric Pressure.

76. Gases Compared with Liquids. Gases like liquids have weight, and such perfect freedom from molecular friction that they obey the laws of Pascal and Archimedes.

But gases differ from liquids in three important respects:

(1) They behave as if their molecules repelled one another.
(2) They are very compressible.
(3) They expand much more rapidly when heated.

The proof of (1) is seen in many familiar phenomena. If we prick a hole anywhere in a distended bladder, the air will rush out. If we place a closed rubber bag, containing a little air, under the receiver of an air pump, and pump the air out of the receiver, the bag will swell out to a great size (Fig. 69).

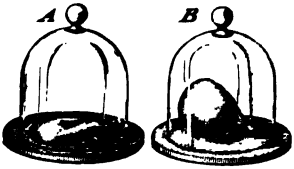


FIG. 69.

In fact, a given quantity of gas tends to expand indefinitely in all directions, and must therefore be kept in a vessel closed on all sides.

The great compressibility of a gas has already been mentioned (see § 2), and will soon be studied further.

The effects of heat upon a gas are of extreme importance, and will be considered at another time.

At present, the fact is to be noted that *the density of a gas varies with the pressure and the temperature;* and both elements should be specified, unless otherwise understood, when the value of the density of a gas is stated.

The density of dry air at 76 cm. and 0° Centigrade is 1.293 grams per liter and about 1¼ oz. per cubic foot.

The specific gravity of a gas is usually referred to that of dry air, as the standard, at the same pressure and temperature.

77. Pressure of the Atmosphere. The fact that water will rise in a tube, when the air is removed from the tube, was known in ancient times and utilized by the invention of pumps and siphons. But the explanation was not known; it was simply said that "nature abhors a vacuum."

The true reason was discovered by Torricelli, an Italian, in the year 1643. Reflecting on the known fact that water will not rise higher than 34 ft., he was led to suspect that the water column is supported by the pressure of the atmosphere due to its own weight; whence he reasoned that a column of mercury which is 13.6 times as heavy as water would stand no higher than 30 in., because the quotient obtained by dividing 34 ft. by 13.6 is 30 in. He then performed the simple experiment which has justly become famous in the history of science (Fig. 70).

Having filled with mercury a glass tube, about a yard long and closed at one end, he closed the open end with his finger, inverted the tube, and plunged it into a cup of mercury. On removing his finger, the mercury column settled a little, leaving a vacuum at the top, but remained in equilibrium at a height of about 30 in. (76 cm.).

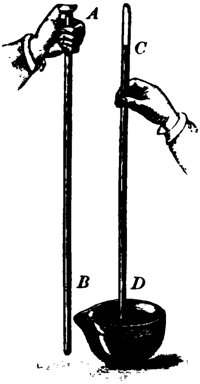


FIG. 70.

When the French philosopher, Pascal, heard of this experiment, he caused it to be performed at the top of a mountain where there is less air above the tube. The mercury column, as Pascal had predicted, was found to stand several inches lower than when the experiment was performed in the valley below.

In this way Pascal verified the conclusion already reached by Torricelli, that it is the pressure of the atmosphere which sustains the column of mercury in the tube.

The same falling of the mercury column is observed when a tall bell-jar is placed over the inverted tube, and the air removed from the jar by means of an air pump.

In Torricelli's experiment the mercury column is supported by the pressure of the atmosphere exerted upon the surface of the mercury in the cup and transmitted through the mercury to the tube. The height of the column will be the same whatever be the size of the tube; for although the weight of mercury to be supported increases with the cross-section of the tube, the upward pressure which supports it increases in the same ratio (§ 68).

The pressure of the atmosphere is due to its weight. Torricelli's experiment, therefore, proves that the weight of the atmosphere is practically equal to that of an ocean of mercury surrounding the earth to the height of 30 in. or 76 cm.; or, since mercury is 13.6 times as heavy as water, equal to that of an ocean of fresh water 34 ft. or 10.34 meters high. Expressed in ordinary units, this pressure amounts to 14.7 lb. per square inch, or 1034 grams per square centimeter. Any gaseous pressure equal to this is called an *atmosphere.*

This pressure is exerted all over the outside of a body, and on the inside too if the air can gain admittance; so that usually it is not felt or noticed in any way. But if the pressure is removed from one side of a body, the other side will experience the whole unbalanced pressure of nearly 15 lb. per square inch; and if the air is removed from a closed vessel, the outside will experience a crushing pressure, and the vessel will collapse unless very strong.

Since the atmosphere is in contact with the free surface of a liquid the pressure at the surface is not zero (as assumed in § 68), but 14.7 lb. per square inch.

A man's body exposes to the atmosphere an average surface of about 18 sq. ft.; hence he experiences a total pressure of about 19 tons. We are accustomed to this pressure inside and outside and do not notice it. But if we ascend a high mountain or go down under the water in a diving bell, the *change* of pressure is very quickly felt and may cause vertigo, heart trouble, or other serious consequences.

78. The Barometer. The column of mercury supported by the atmosphere varies slightly in height from day to day, and even from hour to hour. Therefore the pressure of the atmosphere undergoes changes. A Torricellian tube mounted so that these changes can be accurately observed is called a *barometer*. A *siphon* barometer is shown in Fig. 71. The short arm is open to the air. The long arm is closed, and contains the empty space. Between the arms is a scale so graduated that the sum of the readings at the levels of the mercury columns in the two arms gives the true difference of levels; that is, the height of a column of mercury which exerts the same pressure as the atmosphere.

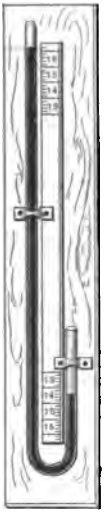


FIG. 71.

The *Fortin* barometer is in common use at the weather bureau stations. The vessel containing the mercury has for its base a buckskin bag enclosed in a brass cylinder. Against this bag a thumb screw presses from below. By turning this screw the level of the mercury can be raised or lowered till it just touches the end of a fine ivory pointer. The height of the mercury column is then read off on a scale at the top of the tube.

Barometric changes are caused by changes in the temperature of the air, by currents of air, and by variations in the amount of aqueous vapor in the air. Certain slight changes (seldom amounting to more than a tenth of an inch) are found to be periodical. The greater changes due to the alternate passage of areas of stormy and fair weather follow no known laws. Nevertheless the close connection between barometric changes and changes in the weather make the barometer of great use as a weather indicator.

A decided fall of the mercury generally precedes foul weather, while a rise often indicates the approach of fair weather. At sea a very sudden fall is regarded as a sure sign of an approaching hurricane.

Lines of equal barometric pressure at any one time, drawn on a map or chart, are called *isobaric lines* or *isobars.* By their aid the barometric condition of a large region can be seen at a glance. The forecasts of the weather, issued daily by the Weather Bureau, are largely based on information obtained in this way.

Isobars often form closed curves enclosing a region of barometric depression. In case the change of pressure within this region is very rapid, air will rush violently in from all sides with a spiral motion, and the region will experience a revolving storm called a *cyclone.*

LABORATORY EXERCISES.

1. Two Magdeburg hemispheres (Fig. 72), when fitted together, are easily separated while the air is within them, but when most of the air is removed great force is required to pull them apart. They receive their name from Magdeburg, Prussia, where Otto Guericke, the inventor of the air pump, first used them to show the pressure of the air.

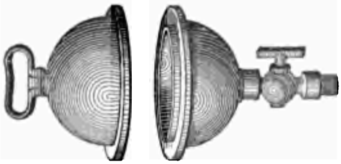


FIG. 72.

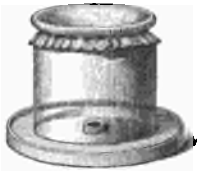


FIG. 73.

2. When the air is removed from a wide-mouthed vessel over which a piece of bladder is fastened air-tight, at first the bladder is pushed inwards (Fig. 73), and finally it bursts with a loud report.

3. Boil some water in a flask, and while the water is boiling place over the mouth of the flask a ripe banana, end on, having first cut a few slits in the peel. Remove the lamp. Very soon you see the peel thrown aside in strips, while the banana enters the flask. Explain. Instead of a banana a hard-boiled egg with the shell removed may be used.

4. Fill a glass brimful of water, cover carefully with a piece of cardboard, and then invert. The water does not run out. Explain.

5. Fill a *pipette* (Fig. 74) with water by suction. By keeping the finger pressed upon the upper end, the pipette can be carried about without the water escaping. By loosening the finger a little the water can be made to escape by drops. Explain these phenomena. What is the explanation of suction?

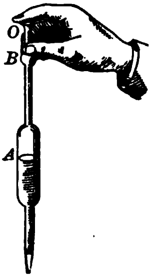


Fig. 74.

The pipette is used to remove a liquid from a vessel which we do not wish to disturb, or to put a liquid into a vessel drop by drop.

6. Boil water in a closed flask provided with a safety tube *A* (Fig. 75) and conduct the steam through a glass tube into cold water at a lower level. Remove the source of heat. What happens? Explain. Repeat the experiment without a safety tube. What happens? Explain.

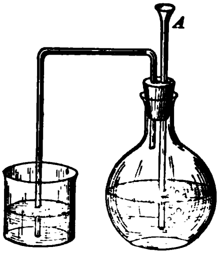


Fig. 75.

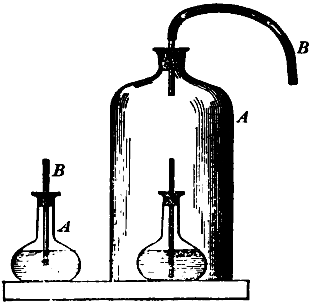


Fig. 76.

7. Blow air into the bottle *A* through the tube *B* (Fig. 76). On removing the mouth, the water spouts out in a jet through *B*. When the jet subsides, place the bottle under the receiver of an air pump and exhaust the air. The jet recommences. Explain.

8. Take two small bottles, and fit one of them with a perforated rubber stopper. Fill this bottle nearly full of water, and connect the two bottles by a bent tube reaching nearly to the bottom of each one. Place the receiver of an air pump over the bottles, and exhaust the air. What happens? Readmit air. What happens? Explain.

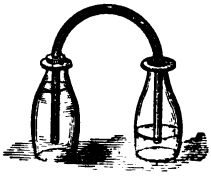


Fig. 77.

CLASS-ROOM EXERCISES.

1. Find the specific gravity of air referred to water. The density of air at 76 cm. and 0° C. is 1.293 grams per liter.

2. Find the density at 76 cm. and 0° C. of hydrogen gas and of carbonic acid gas. Sp. gr. of hydrogen referred to air = 0.069; and that of carbonic acid gas = 1.529.

3. If 1 liter of coal gas at 76 cm. and 0° C. weighs 0.78 grams, what is the specific gravity of coal gas referred to air?

4. Find the weight of the air in a room 6 meters long, 5 meters wide, and 4 meters high.

5. What is the pressure of the air in grams upon 1 qcm. when the barometer stands at 74 cm.?

6. What is the pressure of the air in pounds upon 1 sq. in. when the barometer stands at 29 in.?

7. If the bladder in Fig. 73 has a diameter of 20 cm., with what force is it pushed in when the pressure of the air below it has been reduced to 152 mm., the pressure above it being 760 mm.?

8. The piston of a steam engine has a diameter of 10 in., and the steam exerts a pressure upon it of 5 atmospheres. Find the effective force when the other side of the piston is exposed to the atmosphere.

9. How high will a barometer filled with alcohol stand when a mercury barometer stands at 76 cm.? Sp. gr. of alcohol = 0.8; and that of mercury = 13.6.

10. During a storm a mercury barometer falls from 30 to 29 inches. Through what distance would a water barometer fall under the same conditions?

11. Two glass tubes are arranged in a vertical position so that their lower ends dip into water and kerosene respectively, while their upper ends are connected to a mouth-piece. Air is then sucked out of the tubes till the height of the water column is 260 mm. and that of the kerosene column is 329 mm. Find the specific gravity of kerosene (§ 71).

12. If a barometer filled with glycerine reads 325 in. when a mercury barometer stands at 30 in., find the specific gravity of glycerine, and state the principles involved in the calculation.

13. A barometer tube, with a cross-section of 1 qcm., dips into a cistern of mercury whose cross-section, excluding the tube, is 10 qcm. If the mercury in the tube falls 1 cm., what is the real change in the pressure of the atmosphere?

14. If the density of air, like that of water, were uniform and equal to that of air at the sea level, how high would the atmosphere extend? Assume the height of a water barometer to be 34 ft.

Boyle's Law.

79. Volume and Pressure. Experiment proves that if a given mass of gas be compressed to *half* its original volume, the pressure exerted by the gas at every point is *doubled.*

If 12 cubic inches of gas under atmospheric pressure are compressed first to 6, then to 4, then to 3 cubic inches, the pressure will increase to 2, 3, 4 atmospheres, respectively. If we multiply the volume in each case by the corresponding pressure, the resulting products are all equal.

$$12 \times 1 = 6 \times 2 = 4 \times 3 = 3 \times 4.$$

This law holds true only provided *the temperature of the gas does not change.* It may be thus stated:

At constant temperature the product of the pressure and the volume of a given mass of gas is constant.

To express the law by symbols, let P and P' denote the pressures corresponding to the volumes V and V'; then

$$P \times V = P' \times V'.$$

This law appears to have been discovered independently by Robert Boyle in England (1662), and by Mariotte in France (1676); hence in France it is always called Mariotte's law. For the experimental mode of verifying it, see p. 84.

Sometimes the law is stated by saying that "pressure varies inversely as volume"; but in applying the law, this form of statement is less convenient than the one given above.

A very good example of Boyle's law is seen in the action of the common popgun (Fig. 78). A pellet A closes one end of a tube and a piston B fits tightly into the other end. When the piston is pushed quickly in, the force of the compressed air drives the pellet out.

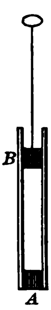


FIG. 78.

The increase of air pressure by compression was known to Ctesibius (150 B.C.), and applied by him in the construction of air guns. The popgun is the simplest form of an air gun.

80. Measurement of Gaseous Pressure. The pressure of a gas, like that of a liquid, is measured by the pressure exerted upon the unit of area, and is expressed,

(1) by ordinary gravitation units (pounds or grams);

(2) by the height of a column of mercury or of water that would exert an equivalent pressure; or

(3) by atmospheres.

Thus, the average pressure of the atmosphere is 14.7 lb. per square inch, or 30 in. of mercury or 34 ft. of water. The pressure exerted upon a cannon ball, when the cannon is fired, is more than 1000 atmospheres.

Instruments made for the purpose of registering gaseous pressure are called *pressure gauges* or *manometers.*

The action of a simple *open-air* manometer is evident from Fig. 79. The *compressed-air* manometer (Fig. 80) is so constructed that when the mercury stands at the same level in both arms, the air in the closed arm is under atmospheric pressure. If the gas exerts greater pressure than this, mercury is forced into the closed arm, compressing the confined air according to Boyle's law. The scale is graduated so that the pressure may be read in atmospheres.

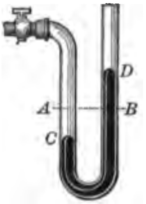


FIG. 79.

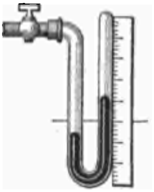


FIG. 80.

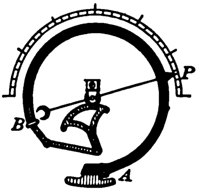


FIG. 81.

Steam pressure is generally measured by *Bourdon's pressure gauge* (Fig. 81). It consists of a copper tube *AB* bent nearly into a circle, closed at the end *B*, and connected at the other end *A* with the boiler. As the pressure of the steam within this tube increases, the tube tends to straighten out, and the motion of the free end *B* is communicated to a pointer *P*, which moves over a scale graduated by applying known pressures. When the pressure diminishes, the elasticity of the tube restores it to its original shape.

The safety valve of a boiler (Fig. 82) is a contrivance to prevent the steam from exerting a pressure dangerous to the safety of the boiler. A metal plug *D* which exactly fits a circular hole in the top of the boiler is joined to a beam *ABC* at the point *B*. The fulcrum is at *A*, and any desired amount of weight may be hung at the other end *C*. When the moment of the pressure of the steam is greater than the moment of the weight at *C*, the plug rises and allows the steam to escape.

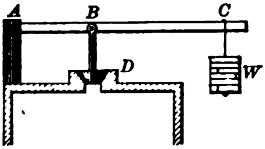


FIG. 82.

81. Density and Pressure. If a given volume of gas be compressed to half its original bulk, the density will be doubled; each cubic centimeter must contain twice as many molecules as before. But the pressure is also doubled by Boyle's law. Hence, *the density varies directly as the pressure.*

This is also true if, instead of compressing the gas, we force into the vessel which contains it a quantity of gas equal to that already there. In all cases, *if the temperature remains constant,* the density and the pressure vary in the same ratio.

But when a gas is heated, its density and its pressure follow different laws. If the gas is allowed to expand freely, its pressure will remain the same, but its density will diminish. If the gas is prevented from expanding, its pressure will increase, while its density will remain unchanged.

It will now be clear why, in order to compare the densities of gases, a standard pressure and standard temperature have to be selected. The standards chosen are the pressure of 76 cm. of mercury and the temperature of 0° Centigrade (that of melting ice).

The fact that density varies as pressure explains why the atmosphere decreases in density as we ascend. The pressure decreases because there is less air above us to cause pressure, and the density decreases at the same rate. At a height of 3 miles both are only about half as great as at the level of the sea.

82. The Diving Bell. An interesting application of Boyle's law is found in the *diving bell* (Fig. 83). It is a bell-shaped body made of cast iron with walls so thick that it will sink in water, even if full of air, when lowered mouth downwards by means of a chain. As the bell sinks, the pressure on the air inside increases and its volume diminishes, the water rising higher and higher in the bell.

When the volume of air in the bell is half as great as at the surface of the water, the pressure must be twice as great, or two atmospheres; therefore the distance AC from the level of the water in the bell to the free surface must be 34 ft.

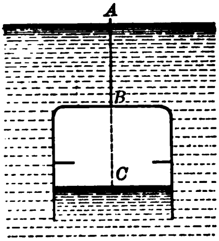


FIG. 83.

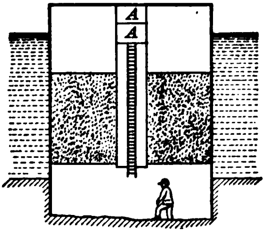


FIG. 84.

In practice, however, the bell is kept entirely free from water by forcing air into it by means of a condensing pump at the surface ; this is necessary in order that workmen may perform the operations for which the bell is employed.

The tension of the chain which supports the bell is equal to the weight of the bell less the weight of the water which it displaces.

Imagine now a hollow cylinder to be constructed so long as to reach from the surface of the water to the foundations of the piers of a bridge. Such an elongated diving bell is called a *caisson* (Fig. 84). The water is kept out by pumping in air, and the air is retained by air locks through which the workmen and materials can pass.

83. The Balloon. In the ascent of a balloon the Laws of Archimedes and of Boyle are both illustrated.

First, the law of Archimedes. A balloon rises in air for the same reason that a cork rises in water, because its weight is less than the weight of the fluid which it displaces. The difference between these two weights is the *lifting power* of the balloon.

Hot air, hydrogen, and coal gas have all been used for filling balloons. Their comparative values, as regards lifting power, may be inferred from the following data:

A cubic meter of air at 76 cm. and 0° C. weighs about 1.29 kg.
A cubic meter of hydrogen at 76 cm. and 0° C. weighs about 0.09 kg.
A cubic meter of coal gas at 76 cm. and 0° C. weighs about 0.75 kg.
A cubic meter of air at 76 cm. and 200° C. weighs about 0.75 kg.

Thus, a balloon containing 100 cubic meters of coal gas has a lifting power of 129 — 75, or 54 kg.; if the silk, car, etc. weigh 53 kg., the balloon will slowly ascend. If, however, this balloon were filled with hydrogen instead of coal gas, the lifting power would be 129 — 9, or 120 kg., and it would be able to carry up a man weighing anything less than 67 kg. (about 147 lb.).

Hot-air balloons were first constructed by the Montgolfier Brothers in Paris (1783). In them men for the first time ascended into the air. But their great bulk and still more their liability to catch fire are serious objections to their use. Then hydrogen was tried, and after that coal gas. Hydrogen supplies great lifting power with small size, but it is expensive and is also very liable to leak through the walls of the balloon. For military purposes, where it is desired to hold a balloon captive at a moderate height, small size is of prime importance on account of the action of the wind; hence, in such cases balloons are filled with hydrogen which is carried highly compressed in steel flasks at a pressure of about 100 atmospheres. But for most purposes coal gas is preferred from its cheapness and the facility with which it can be obtained.

When a fully inflated balloon rises through the air, the external pressure upon it is continually diminishing while the internal pressure of the gas remains the same. Hence, the stress on the material of the balloon increases and may become great enough to rupture it. For this reason a balloon is not fully inflated at the start, and therefore, as it ascends, Boyle's law comes into application. The balloon increases in bulk in the same ratio as the external pressure diminishes until it is fully distended. During this period the lifting power of the balloon remains practically constant; for the increase in the volume of the air displaced by the balloon is just counterbalanced by the decrease in its density.

If the balloon, after being fully distended, continues to ascend, its lifting power will diminish, until finally a point is reached where the weight of the air displaced is only equal to the weight of the balloon. When this point is reached, the balloon ceases to ascend.

The aëronaut is always provided with ballast to throw out, and he can also allow gas to escape by opening a valve. By these means he can to a certain extent control the motion of the balloon in a vertical direction. No satisfactory means for steering a balloon horizontally has yet been discovered.

FIG. 85.

The highest balloon ascent on record is that made by Glaisher and Coxwell in England, Sept. 5, 1862. The barometer fell to 7 in. and this shows that the height must have been about 7 miles. At this altitude both aëronauts lost the use of their limbs, and Mr. Glaisher became unconscious. Fortunately Mr. Coxwell was just able to open the valve by seizing the cord with his teeth and dipping his head. The balloon then began to descend, and the men revived.

During the siege of Paris by the Germans in 1870, sixty-four balloons left the city, carrying with them besides passengers about 3,000,000 letters weighing nearly 10 tons. Two of the balloons were lost at sea, five fell into the hands of the Germans, and one landed in Norway.

A *parachute* is a sort of gigantic umbrella, which by exposing a great extent of surface to the air enables an aëronaut to descend safely from a great elevation to the ground.

LABORATORY EXERCISES.

1. Verify Boyle's Law by experiment.

A stout glass tube, bent as shown in Fig. 86, the long arm open, the short arm closed, is fastened in a vertical position so that a little mercury at the bend shall stand at the same level AB in both arms. The distance AC represents the volume V of the air confined in the short arm, and the height of the barometer at the time gives the pressure to which it is subjected. Mercury is now poured into the open arm till the level rises in the short arm to some point A_1 and in the long arm to B_1. The difference of levels between A_1 and C gives the reduced volume V_1 of the confined air; and the difference of levels between A_1 and B_1 + the height of the barometer at the time gives the pressure P_1. Then more mercury is added, and V_2 and P_2 determined, etc.

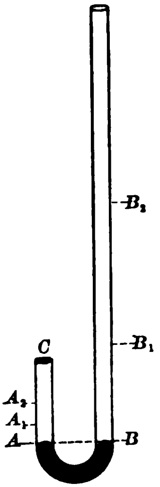


FIG. 86.

Then compare the products VP, V_1P_1, V_2P_2, etc.; they should be very nearly equal.

The temperature of the air should be kept constant.

Careful experiments have proved that no known gas rigorously obeys Boyle's Law, and that the deviations from the law become very considerable when the pressure is so high or the temperature so low that the gas is on the point of condensing to a liquid.

CLASS-ROOM EXERCISES.

1. A chemist generates a quantity of hydrogen gas which at 0° C. and under a pressure of 95 cm. measures 800 ccm. Find its volume at 76 cm.

2. A bladder holds 30 cubic inches of air under a pressure of 15 lb. per square inch. Find the size of the bladder if the pressure is reduced to 5 lb. to the square inch.

3. A bladder contains 640 ccm. of air under the pressure of 76 cm. At what pressure will the volume of the bladder be 480 ccm.?

4. A bladder containing 3 cubic feet of air at atmospheric pressure is sunk to a depth of 80 ft. in water. To what bulk is the bladder compressed, the water barometer standing at 32 ft.?

5. If the level of the mercury in the open arm of the manometer in Fig. 86 is 57 cm. higher than in the other arm, what is the pressure exerted by the gas in the vessel?

5 **6.** If the air in a compressed-air manometer is compressed to one fifth of its volume under atmospheric pressure, what pressure does it exert?

7. What will 1 liter of air weigh at 0° C., under a pressure of 4 atmospheres, if 1 liter of air at 0° C. and 76 cm. weighs 1.29 grams?

8. What is the volume of 1 gram weight of air at 76 cm. and 0° C.? What is the volume at 38 cm. and 0° C.?

9. A bubble of air rises to the surface from the bottom of a lake 150 meters deep. The volume of the bubble at the start is 1 ccm., and the water barometer at the time reads 10 meters. What is the volume of the bubble on reaching the surface?

10. At what depth in water would a bubble of air be compressed so as to become as dense as water, the height of the water barometer being taken as 10 meters and the density of air being 1.29 grams per liter?

Let x = required depth; then $x + 10$ represents in meters of water the pressure required; then apply the principle that density is proportional to pressure (§ 81).

11. A cylinder, open at the top, is inverted and immersed in water. Find the depth at which the cylinder will be half full of water; two thirds full; three fourths full.

12. A cylindrical diving bell, 15 feet high, is sunk in water so that the top is 51 feet below the surface. Find how high the water will rise within the bell.

13. A cylindrical diving bell, 12 feet high, is lowered until the water within the bell rises to a height of 8 feet. How far is the top of the bell below the surface of the water?

14. If a small hole is made in the top of a diving bell under water, will water flow in or air flow out?

15. The standard balloon used in the siege of Paris, 1870, had a capacity of about 70,000 cubic feet. The density of air = 1¼ oz. per cubic foot. Sp. gr. of coal gas (referred to air) = 0.4; and that of hydrogen = 0.07. Taking the weight of the balloon, car, etc. as 1000 lb., find the lifting power (1) if coal gas is used, (2) if hydrogen gas is used.

16. The capacity of a balloon is 30,000 cubic feet. Weight of silk, car, etc. 560 lb. Sp. gr. of the gas used = 0.45. Weight of 1 cubic foot of air = 1.2 oz. Find the lifting power of the balloon.

17. If you ascend 10.5 meters above the level of the sea, the barometer falls 1 mm. Why does it fall? If you rise 10.5 meters higher, will the barometer fall more or less than 1 mm.? Explain.

Pneumatic Instruments.

84. Air Pump. The air pump shown in Fig. 87 will answer for performing the ordinary experiments. It consists of a metal cylinder *B*, called the *barrel*, fitted with a piston, and having at the lower end two short tubes *A* and *C*, within which self-acting conical valves work so that air can enter the barrel through *A* and leave it through *C*.

To remove the air from a *receiver R*, the tube *A* is connected with *R*, and the piston is drawn up.

The air in the barrel expands, the valve in *A* opens, and air rushes from the receiver into the barrel, until the air which at first filled only the receiver fills both receiver and barrel. When the piston is pushed down, the valve in *A* closes, that in *C* opens, and the air in the barrel is pushed out into the atmosphere. Thus at each complete stroke of the piston a barrel full of air of less and less density is ejected into the atmosphere.

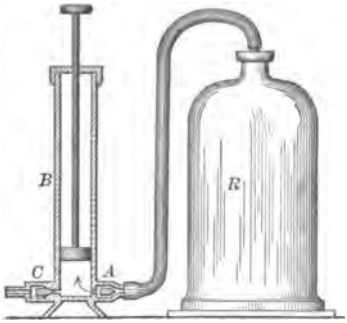


Fig. 87.

It is evident, however, that by this process a perfect vacuum cannot be obtained, even supposing no leakage; for by each stroke only a *part* of the air is removed, and the remainder is left in the receiver. Practically, leakage cannot be wholly prevented, and the rate of leakage increases as a vacuum is approached. When the air leaks in as fast as it is expelled by pushing down the piston, or when the pressure of the air in the receiver becomes too feeble to open the valve in *A*, further exhaustion becomes impossible.

If the ratio of the volume of the barrel to that of the receiver is known, it is easy to compute theoretically the degree of exhaustion after any number of strokes.

Suppose, for example, that the barrel holds 3 liters and the receiver 7 liters, and that the height of the barometer is 76 cm. When the piston is raised the 7 liters of air in the receiver expand to 10 liters, and the pressure by Boyle's law diminishes to $\frac{7}{10} \times 76$, or 53.2 cm. When the piston is pushed down, 3 liters of air are expelled, the pressure remaining unchanged. When the piston is raised the second time the same proportional change of pressure is produced; so that after two strokes the pressure becomes $\frac{7}{10} \times \frac{7}{10} \times 76$, or 37.24 cm.; and so on. After n strokes the pressure will be $(\frac{7}{10})^n \times 76$ cm. The density is reduced in the same ratio (§ 81).

The actual degree of exhaustion is often registered by a *mercury gauge.* It consists of a tube the lower end of which dips into a cup of mercury while the upper end is connected with the air in the receiver. As the air is removed from the receiver, the mercury rises in the tube.

85. Compression Pump. If the tube C (Fig. 87) is connected with a closed vessel, and the tube A is left open to the air, the air pump becomes a *compression* pump, or *condenser.* At every stroke a barrel full of air is taken from the atmosphere and driven into the closed vessel. If the latter is ten times as large as the barrel, ten full strokes will raise the pressure of the air within it to two atmospheres.

A simple condenser is used for inflating the pneumatic tires of bicycles. Compression pumps driven by water or steam are employed to supply air, condensed to the requisite pressure, to divers, diving bells, and caissons; also to furnish air under high pressure for driving machinery, especially boring machines in mines and in tunnelling operations, the air after driving the borer assisting in the ventilation. Although pumps for exhausting air have been (and still are) of great use in the study of physics, pumps for compressing air are now much more important from an industrial point of view.

86. The Suction Pump. The common *suction* pump (Fig. 88) consists of a barrel *B* in which a piston *P* moves up and down, and which is connected below with a pipe leading to the well. At the top of this pipe, and in the piston, are valves, both opening upwards. Describe and explain the action of the pump.

The distance from the lower valve to the water in the well must not exceed 34 ft. Why? Practically this distance must be considerably less than 34 ft. on account of friction and leakage.

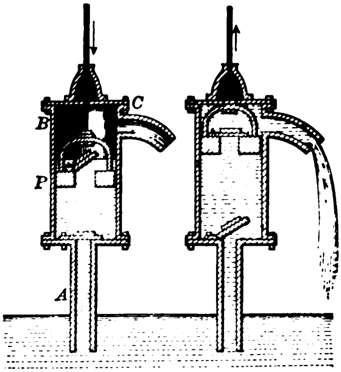


FIG. 88.

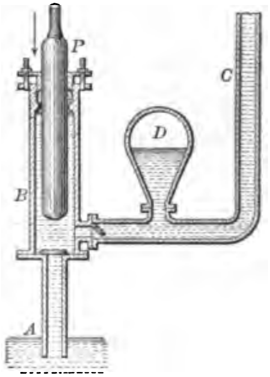


FIG. 89.

Force necessary to raise the piston. Let a denote the section of the piston, P the atmospheric pressure upon this area, h the height of the water column above the piston, h' the height of the water column below the piston; then, using metric units,

$$\text{downward pressure on top of piston} = P + ah$$
$$\text{upward pressure on bottom of piston} = P - ah'.$$

The downward pressure exceeds the upward by the amount $a\,(h + h')$, or the weight of a column of water whose base is the section of the piston and whose height is the distance through which the water is raised. To raise the piston, this weight + the friction of the piston must be overcome.

87. The Force Pump. In this pump (Fig. 89) the piston has no valve, and the delivery tube *C* has a valve opening outwards. Describe and explain the action.

To make the flow continue during the ascent of the piston, the elastic force of air is often utilized by the addition of an air chamber, or *air dome* (Fig. 89, *D*). The inflow of water into the air dome is intermittent, but the outflow is continuous; for the air never ceases to be compressed, and it exerts a continuous pressure.

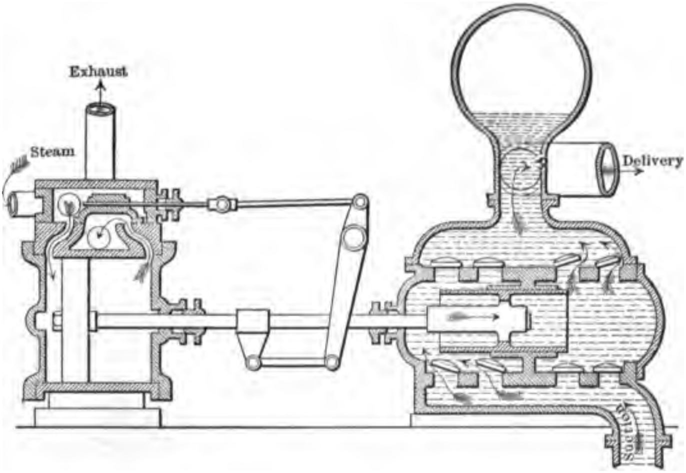


FIG. 90.

Another use of the air dome is to prevent sudden changes in pressure from straining the barrel and piping.

Steam fire engines and steam pumps are always provided with large air domes; moreover, a constant stream of water is still further ensured by making the pump double acting, as is very clearly illustrated in Fig. 90.

88. The Siphon. The *siphon* (Fig. 91) is a bent tube used for transferring liquid from a vessel to a lower level without disturbing the vessel.

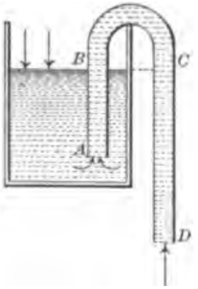


FIG. 91.

To set it in action we place it, as shown in the figure, with the shorter arm dipping in the liquid and then suck the air out of the longer arm. The flow now begins, and continues till the level of the liquid sinks to the bottom *A* of the short arm.

The atmospheric pressure acts upward at the end *D* of the long arm, and also at the level *B* within the short arm; it has, however, a greater weight of liquid to support in the long arm than in the short arm. Hence, at the top of the siphon there is an unbalanced force acting towards the long arm; this force causes the flow.

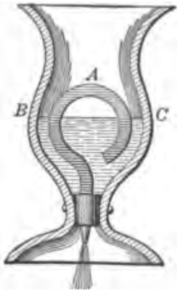


FIG. 92.

FIG. 93.

The *vase of Tantalus* (Fig. 92) illustrates the action of an intermittent siphon. Water poured into it remains till the level rises to the point *A*; then the siphon suddenly begins to act, and the vessel is rapidly emptied.

The action of an *intermittent spring* (Fig. 93) is explained in the same way.

89. Induced Air Currents. If we take two glass tubes *A* and *B* with ends drawn out to fine openings, pass them through a large cork as shown in Fig. 94, place the lower end of *B* in water, and blow air through *A*, the water will rise in *B* and be thrown out in a fine spray. The swift-moving current of air in *A*, combined with the atmospheric pressure on the water below, removes the air from *B* as effectively as a good air pump.

A stream of fluid (either liquid or gas) in rapid motion tends to push or drag the adjacent air along with it, and if the stream flows past the end of a tube, it will draw air out of the tube. Currents of air caused in this way have been called *induced* air currents.

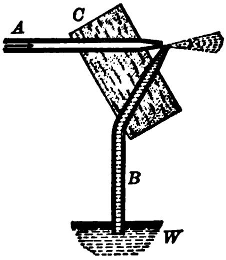


FIG. 94.

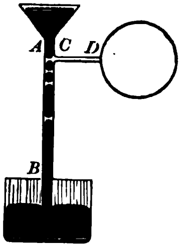


FIG. 95.

Sprengel's air pump (Fig. 95), by which an exhaustion to a millionth of an atmosphere can be obtained, works on this principle. Mercury is allowed to flow down a long tube *AB*, and a side branch *CD* connects the tube with the vessel to be exhausted of air. The mercury flows down in a series of drops, and air from the vessel never ceases to expand into the intervals between the drops. Consequently the air is swept down with the mercury to the bottom of the tube.

The *atomizer*, used by dentists and physicians, utilizes the same principle, and it is also usefully applied in the locomotive engine by discharging the jets of waste steam into the chimney so as to cause a strong draft through the fire under the boiler.

CLASS-ROOM EXERCISES.

1. Air is pumped out of a vessel till the mercury in a gauge connected with the vessel rises to a height of 68 cm. The barometer at the time reads 76 cm. What is the pressure of the air remaining in the receiver? What is its weight, if the air in the receiver originally weighed 19 grams?

2. What must be the capacity of the barrel of an air pump in order that one half of all the air in a receiver holding 4 liters may be removed by a single stroke?

3. If the receiver of an air pump is twice as large as the barrel, what fraction of the air in the receiver will be removed by each complete stroke of the piston?

4. If the volumes of the receiver and barrel of an air pump are 5 cubic feet and 1 cubic foot respectively, and the original pressure of the air is 30 in., what will the pressure be after 3 strokes?

5. The barrel of a condenser has a capacity of 1 liter, and a hollow iron vessel has a capacity of 12 liters. How many strokes are required to raise the pressure of the air in the vessel to 10 atmospheres?

6. The distance from the spout of a pump to the level of the water in the well is 20 ft. The area of the piston is 10 sq. in. Find the force (neglecting friction) required to raise the piston.

7. The area of the piston of a force pump is 15 sq. in., and the water is raised to a height of 64 ft. above the piston. Find the force required to push down the piston.

8. How would the action of a siphon be affected by taking it to the top of a mountain?

9. What would happen if a small hole were made in the short arm of a siphon in action? in the long arm?

10. What is the greatest depth a vessel can have in order that it may be emptied of water by means of a siphon?

11. If a vessel 3 ft. deep is full of mercury, can it all be emptied by means of a siphon? Why?

12. In what respect would the action of a siphon at the bottom of a caisson differ from its action in the free atmosphere?

13. A vessel of water is under the receiver of an air pump, and a piece of wood is floating in the water. Will the wood rise higher or sink deeper if the air is removed from the receiver? Explain.

14. What effect on the wood in Ex. 13 would be produced if air were pumped into the receiver? Explain.

REVIEW QUESTIONS ON CHAPTER II.

1. What is Pascal's law? How is it applied in the hydraulic press?

2. Prove that the pressure exerted by a liquid at rest under gravity increases uniformly with the depth.

3. Describe a vessel so shaped that when filled with water the pressure upon the base shall be much greater than the weight of the water.

4. What is the hydrostatic paradox? Explain it.

5. Give an example of unbalanced lateral pressure and its effect.

6. A mill dam has the shape of a rectangle. After a heavy rain the water rises to double its previous height, and the surface of the pond is also doubled. What change takes place in the pressure on the dam?

7. How is the total pressure against a mill dam found?

8. State the law of Archimedes. How is it verified by experiment?

9. Give the general proof of the law of Archimedes.

10. Apply the law of Archimedes to a floating body.

11. How much weight will a cubic foot of any substance appear to lose when immersed in water?

12. Oil floats on water but sinks in alcohol. Iron sinks in water but floats on mercury. Explain these facts.

13. Define density and specific gravity, and point out the essential difference in their meaning. Why is the density of a substance in the metric system numerically equal to its specific gravity?

14. Describe a method of finding the sp. gr. of a solid that will sink.

15. Describe a method of finding the sp. gr. of a solid that will float.

16. Describe a method of finding the sp. gr. of a liquid.

17. In determining the specific gravity of a solid the experimenter pays no attention to the fact that a large number of air bubbles were clinging to the solid when it was weighed under water. Will the result he obtains be too large or too small? Explain.

18. What is the weight of 1 liter of dry air under standard conditions? What are the standard conditions?

19. Describe Torricelli's experiment. What does it prove?

20. Describe a siphon barometer. What does it measure?

21. Describe the nature of the connection between the readings of a barometer and the state of the weather.

22. How will the reading of a barometer be affected if the tube is not in a vertical position?

Hint. The downward pressure of the mercury depends on its *vertical* height.

23. How do we ascertain the height of a mountain by a barometer?

24. If a barometer reads 30 in. at the sea level and 20 in. at the top of a mountain, will it read more or less than 25 in. at a point halfway up the mountain?

25. A man tries to draw vinegar out of a cask, and finds that it will not run out. On removing the bung from the cask the vinegar runs out freely. Explain.

26. A circular piece of soft moist leather with a string attached to its center is called by boys a *sucker*. When the sucker is pressed down upon a flat-faced heavy stone, it is found that the stone can be raised by pulling the string. Explain this phenomenon (Fig. 96).

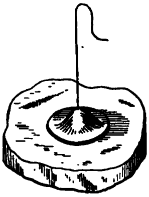


FIG. 96.

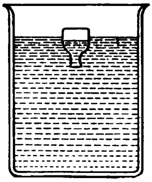


FIG. 97.

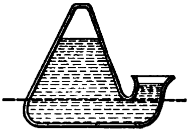


FIG. 98.

27. State Boyle's Law, and illustrate its meaning by a numerical example.

28. If a diving bell is lowered to the same depth, first in fresh water, and then in salt water, which liquid will rise higher in the bell? In which case is the tension of the chain the greater?

Explain with a diagram:

29. The construction and action of an air pump.

30. The construction and action of a suction pump.

31. The construction and action of a force pump.

32. The construction and action of a siphon.

33. Water is introduced into a small bottle to such an extent that the bottle will just float when inverted in water. When the bottle is pushed down some distance in the water, it will sink to the bottom and remain there. Explain (Fig. 97).

34. A lump of lead and a large wooden ball balance each other when suspended from the pans of a balance. A receiver is set over the balance and the air is exhausted. The wooden ball descends. Explain.

35. Explain the action of the *pneumatic inkstand* (Fig. 98).

36. Explain the action of a blacksmith's bellows.

CHAPTER III.

HEAT.

Change of Volume.

90. Temperature. The bodies that surround us act upon our skin so that they feel sometimes warm or hot, sometimes cool or cold. This property of bodies we call their *temperature,* and the cause of the property we call *heat.*

Hot and cold bodies also produce various other effects. Thus, a hot iron ball rapidly melts a piece of ice that is near it, changes into steam a drop of water which touches it, and if very hot glows with red or white light. On the other hand, a very cold body freezes water in contact with it, condenses steam, and makes all bodies around it cooler, at the same time becoming warmer itself.

Since bodies differ in temperature, a method of comparing temperatures is needed. Our sensations of heat and cold are of little value for this purpose, because they depend not only on the temperature of the body which we touch, but also on the state of the skin at the time.

Thus, if we dip one hand in cold water, the other into hot water, and then both hands into lukewarm water, the first hand will feel warm, and the second hand cold.

The air in a cellar feels warm in winter but cool in summer. A thermometer shows that the temperature of the air is lower in winter than in summer. The air in the cellar *feels* warm or cool according as we are accustomed to the sensation of colder or warmer air out of doors.

A method of comparing temperatures which is independent of our sensations is found in *expansion,* or the changes which heat causes in the volume of a body.

91. Expansion of Gases. A bladder containing some air when held near a hot stove swells out and becomes full and tense; if taken to a cool place it soon collapses.

The effect of heating air is very easily shown by closing a glass flask by a cork through which passes a fine tube containing a short column of water. If we merely apply our hands to the glass, the column quickly ascends (Fig. 99). If the flask is heated when closed air-tight, either the cork is blown out or the flask breaks.

A gas expands rapidly when heated; if confined so that it cannot expand, its pressure rapidly increases.

If a glass tube having a bulb at one end is held with the other end under water, and the bulb is warmed, what will happen (Fig. 100, *A*)? What will then happen as the bulb cools (Fig. 100, *B*)? Explain.

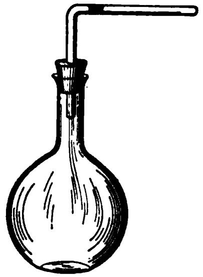


FIG. 99.

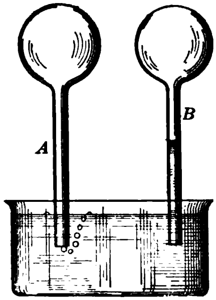


FIG. 100.

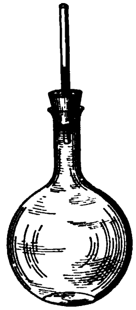


FIG. 101.

92. Expansion of Liquids. If a glass flask completely filled with water and closed by a cork through which a tube passes is heated, a column of water will slowly rise up the tube (Fig. 101). If alcohol instead of water is used the rise is more rapid.

Liquids expand when heated, but less rapidly than gases; and the rate of expansion varies for different liquids.

Ice-cold water when heated to boiling expands about 4%; alcohol expands more than twice as fast as water, mercury about half as fast.

93. Expansion of Solids. A metal rod *AB* (Fig. 102) with flat ends which just fits a gauge *CD* will not fit the gauge after it has been heated over the flame of a lamp; it has increased in length. A metal ball (Fig. 103) which when cold will just pass through a metal ring will not pass through the ring when heated. On the other hand, a ball which will just rest on a cold ring will fall through if the ring is made hot enough.

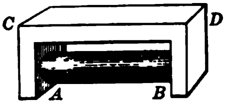


FIG. 102.

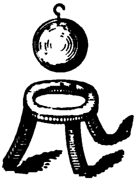


FIG. 103.

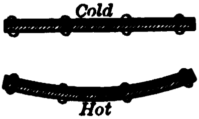


FIG. 104.

If two straight pieces of iron and zinc are riveted together (Fig. 104) and then strongly heated, the compound bar will assume a curved shape, the zinc being on the convex side. This proves that zinc expands faster than iron.

Solids expand slowly when heated; and the rate of expansion varies for different solids.

Among solids, metals expand most rapidly; and the force with which they expand when heated and contract when cooled is enormous, being the same as the stress required to elongate or compress them by an equal amount. Very often provision for the expansion has to be made, and in other cases the force of expansion or contraction is utilized in some way.

Illustrations. 1. The rails of a railroad in cold weather are laid with small spaces between them to allow for expansion in summer.

2. Iron bridges are never fixed at both ends; usually one end rests on metal rollers.

3. Furnace bars are placed loosely in the brickwork, and pipes for carrying hot water have telescopic joints which allow one pipe to slide in the other where they are joined together.

4. Iron tires are fitted when red-hot to the woodwork of a carriage wheel. On cooling they contract and become firmly fastened to the wheel.

5. The bolts in boiler plates are put in red-hot and then hammered down to heads. As they cool they draw together the plates with great force.

6. The contraction of long iron rods in cooling has been utilized to draw back into place the walls of a building that have bulged outwards. Explain more fully how you would do this.

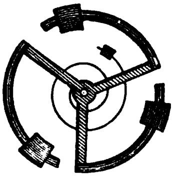


FIG. 105.

7. In the balance wheel of a watch (Fig. 105) an ingenious application of the unequal expansion of metals is made. Any cause which throws the matter of the wheel farther away from the center diminishes the rate of vibration and makes the watch go slower. The heat of summer does this by expanding the radius of the wheel. To offset this expansion, the rim of the wheel is made of several separate pieces, each piece fixed at one end but free at the other, the free end being loaded; moreover, each piece is composed of two metals, of which the more expansible is placed outside. Then on a rise of temperature it is evident (see Fig. 105) that the loaded ends will approach the center. By a proper arrangement of parts this effect may be made exactly to counteract the effect produced by the increase in the radius of the wheel.

94. The Thermometer. The measurement of temperature is the most important application of expansion. Instruments for this purpose are called *thermometers.* For ordinary purposes the expansion of mercury is employed. The mercury is enclosed in a glass tube of small uniform bore, which has a bulb at one end and the other end closed. On the tube, or the supporting frame, is a scale of equal parts which measures the rise or fall of the mercury.

In order to fill a thermometer tube with mercury the bulb is first filled by the process illustrated in Fig. 100 (explain more fully); then the bulb is heated till the mercury by expanding fills the tube; then the upper end is closed by melting the glass. When the mercury contracts a nearly perfect vacuum is left in the upper part of the tube.

The graduation of a thermometer is based on the assumption that the temperature difference between melting ice and water boiling under atmospheric pressure is constant. The bulb is first placed in melting ice, and the point to which the mercury sinks is marked; then both bulb and tube are surrounded by steam at a pressure of 76 cm., and the point to which the mercury rises is marked. These two points are called the *fixed points*. They are marked 0° and 100° on the Centigrade scale, and 32° and 212° on the Fahrenheit scale; and the interval between them is divided into 100 equal parts or *degrees* in one case, and into 180 equal parts in the other. Each scale is then extended beyond the fixed points as far as may be desired. The degrees below 0° are regarded as negative and the sign — is prefixed to them.

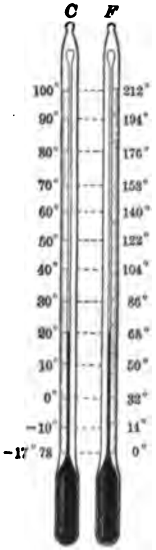


FIG. 106.

The Centigrade scale is in general use for scientific purposes, and the Fahrenheit scale is used for ordinary purposes.

Since 100° C. = 180° F., therefore 1° C. = $\frac{9}{5}$° F., and 1° F. = $\frac{5}{9}$° C. Hence it is easy to change a reading from one scale to the other. But in reducing we must remember that 0° C. corresponds to 32° F.

For example, $68°\text{ F.} = \frac{5}{9}(68 - 32)°\text{ C.} = 20°\text{ C.}$
$15°\text{ C.} = \frac{9}{5} \times 15° + 32°\text{ F.} = 59°\text{ F.}$

Glass bulbs contract very slowly for months or even years. Hence a mercury thermometer is liable through age to read too high, the error being usually less than 1°. To ensure accurate observations, the fixed points should be redetermined by experiment from time to time, and corrections, if necessary, applied to the scale.

For measuring temperatures below the freezing point of mercury (− 39° C.) a thermometer filled with alcohol is used.

95. Maximum Density of Water. Water exhibits a remarkable peculiarity in its rate of expansion. When heated from 0° C. to 4° C. it contracts slightly in bulk. When heated beyond 4° C. it expands continuously until it boils, the rate of expansion slowly increasing with the temperature. Therefore, water has its maximum density at 4° C. Hope's experiment demonstrates this fact very clearly.

Hope filled a metal cylinder (Fig. 107) with water at a temperature of about 10° C., and inserted near the top *A* and the bottom *C* delicate thermometers. Then he surrounded the middle portion *B* with pounded ice. The mercury in the lower thermometer gradually fell to 4° C., and during this period the upper thermometer was almost stationary. Then the lower thermometer became stationary, while the upper one began to fall and continued to fall till it reached 0°. During the first period (while the water at *B* was falling in temperature from 10° to 4°) the water was growing denser and sinking, being replaced by the warmer water from below; during the second period (from 4° to 0°) the water at *B* was expanding and rising to the top of the vessel.

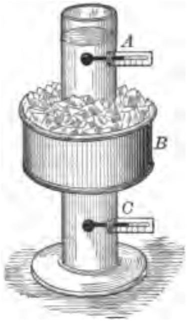


FIG. 107.

Hence in winter, as the water on the surface of a pond cools, it becomes heavier and sinks, while the warmer water below rises. But this circulation ceases when the temperature of the whole mass reaches 4° C. From this time the colder water remains on the surface till it freezes. A short distance below the ice the temperature of the water is always very nearly 4°.

If the water is flowing, however, the motion equalizes the temperature, and ice forms first on the sides and bottom where there is the least motion (ground ice).

Arctic navigators have observed that seals (warm-blooded animals) come to the surface to use their blow holes for breathing as seldom as possible in very cold weather. They find the warmer temperature of 4° C., prevailing in deep water, more agreeable.

96. Coefficient of Expansion. The *coefficient of linear expansion* of a body is the small fraction by which its length is increased when heated from 0° to 1° C.

The *coefficient of cubical expansion* of a body is the small fraction by which its volume is increased when heated from 0° to 1° C.

The rate of expansion of solids is sensibly the same at all temperatures; hence, if k denote the coefficient of linear expansion of a solid, l its length at 0°, and l' its length at $t°$, the whole increase in length will be lkt, and

$$l' = l + lkt = l(1 + kt).$$

If a cube whose edge is one unit long is heated from 0° to 1° the volume of the cube will increase from 1 to $(1 + k)^3$ or $1 + 3k + 3k^2 + k^3$. Now k is always a very small fraction; hence, the much smaller fractions $3k^2$ and k^3 may be neglected without sensible error. This makes the increase in the volume of the cube simply $3k$. Hence, the coefficient of cubical expansion of a solid is, for all practical purposes, equal to *three times* the coefficient of linear expansion.

If, however, a solid is crystalline in structure, or is under stress, the expansion will not be the same in all directions, as assumed above.

A few values of k: Glass 0.000009; platinum 0.000009; iron 0.000012; copper 0.000017; brass 0.000019; lead 0.000028; zinc 0.000029.

A comparison of the values of k for glass, platinum, and copper explains why a platinum wire can be fused into glass without the glass cracking when it cools, while if copper is used the glass always cracks.

In the case of liquids and gases cubical expansion only can be measured. Since the vessel which contains a liquid expands as well as the liquid, we must distinguish between the *real* expansion of the liquid and its *apparent* expansion. The apparent expansion is less than the real by the expansion of the containing vessel. The apparent expansion of mercury which we observe in a thermometer is about 15 per cent less than its real expansion.

97. Law of Charles. Imagine a tube of uniform bore and closed at one end (Fig. 108) to contain a column of dry air, separated from the atmosphere by a pellet of mercury. Let the column of air be 273 mm. long at 0° C. Careful experiments prove that this column will expand against atmospheric pressure just 1 mm. for each degree that the temperature is raised; and also that the same is true if any other gas is used in place of air. Hence, the coefficient of expansion of all gases is equal to $\frac{1}{273}$, or 0.00366.

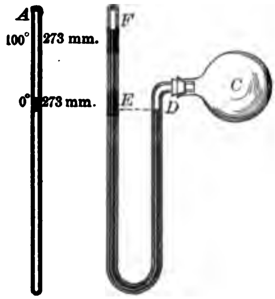


FIG. 108.

At constant pressure the volume of a gas increases uniformly with the temperature, and the rate of increase is the same for all gases. (Charles, 1787, Dalton, 1802.)

If a gas is confined in a vessel *C* (Fig. 108) to which an open-air manometer is connected, and is heated from 0° C. to 100° C., but prevented from expanding by pouring mercury into the open arm of the manometer, the pressure of the gas is found to increase by $\frac{1}{273}$ of the pressure at 0° for each degree.

The pressure of a gas, when the volume is constant, increases uniformly with the temperature at the same rate as the volume when the pressure is constant.

Either piece of apparatus sketched in Fig. 108, if provided with a scale properly graduated, would be an *air thermometer.*

Air, as a substance for measuring temperature, has two decided advantages over mercury or any other liquid or solid, namely:

(1) It expands much more rapidly.

(2) Its rate of expansion is uniform throughout the scale.

But every time an air thermometer is read the height of the barometer has to be observed and allowed for. Hence, for nearly all purposes mercury thermometers are preferred.

98. Absolute Zero. If we apply the law of Charles to the contraction of a gas on cooling, we are led to a very remarkable result. Suppose that a gas at 0° C. is cooled under constant pressure. For each degree that the temperature falls, the gas contracts by $\frac{1}{273}$ of its volume at 0°. Hence, if the cooling were continued till a temperature of − 273° was reached, and if the law held good up to this point, the volume of the gas would be reduced to nothing. In point of fact, so low a temperature as − 273° has never been reached, and before it was reached the gas would undoubtedly become a liquid, when of course the law of Charles would no longer hold true.

This temperature of −273° C. is, however, one of great theoretical importance in the study of gases. It is called the *absolute zero* of temperature; and temperatures, reckoned from this point as the zero point, are called *absolute temperatures.* Any reading on the Centigrade scale is reduced to the equivalent reading on the absolute scale by adding 273.

The law of Charles, if we use absolute temperatures, is stated by saying that *the volume of a gas under constant pressure varies directly as the absolute temperature;* and the laws of Charles and Boyle (§§ 97, 79) may be combined into a single concise formula, as follows:

Let V, P, T denote the volume, pressure, and absolute temperature, respectively, of a gas. First let T be changed to T', keeping P constant. Then V assumes a new value U, and by the law of Charles

$$V : U = T : T'. \tag{1}$$

Now let P be changed to P', keeping the temperature constant. Then U assumes a new value V', such that by Boyle's law

$$UP = V'P'. \tag{2}$$

Eliminating U from (1) and (2), we obtain

$$\frac{VP}{T} = \frac{V'P'}{T'}, \tag{3}$$

a most useful formula in practical problems on gases.

LABORATORY EXERCISES.

1. Find the coefficient of linear expansion of brass.

One form of apparatus for this purpose is sketched in Fig. 109. The brass rod is enclosed in a glass tube *A* so that one end rests against a firm support *B*, while the other end touches a piece *C* connected with a small glass tube which can rotate about an axis at *D* and is pointed towards a distant scale. This tube should be blackened with soot and provided with a hair at the end to aid the eye in estimating the motion on the scale. By this arrangement the small increase in the length of the brass rod is magnified so that it may be observed readily. The rod is heated to 100° C. by allowing steam to enter at *E*.

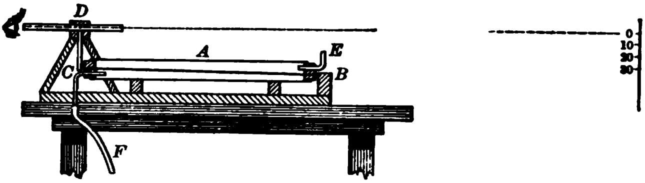


FIG. 109.

CLASS-ROOM EXERCISES.

1. What would happen to the compound bar in Fig. 104 if it were cooled?

2. Why is a glass stopper often loosened by pouring hot water on the neck of the bottle?

3. The longer a pendulum is the slower it vibrates. Hence, show why a clock loses time in summer and gains in winter.

4. A *compensation* pendulum is a pendulum so made that its length shall remain constant in spite of changes of temperature. The principle of action in Graham's *mercury* pendulum is illustrated in Fig. 110. Observe that the effect of expansion upon the mercury in the vessel at the end of the pendulum is to raise its center of gravity a little. Hence, explain how the compensation is effected.

5. What temperature C. is equivalent to 68° F.?

6. What temperature F. is equivalent to 15° C.?

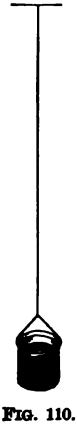


FIG. 110.

7. Change 0° F. to the equivalent Centigrade reading.

8. Change −40° F. to the equivalent Centigrade reading.

9. An iron steam pipe is 60 ft. long at 0° C. How long does it become when steam at 110° passes through it? Value of k, 0.000012.

10. The distance between two marks on a brass bar is 1 meter at 20°. What is the distance at 80°? Value of k, 0.000017.

Note. The full method of solving this question would be, first, to find the distance at 0°, and then at 80°. But k is so small that for all practical purposes the change in distance between 20° and 80° may be taken as equal to $100 \times 0.000017 \times (80 - 20) = 0.102$ cm.

11. A steam pipe is made in sections each 15 ft. long. Assuming that the variation in temperature ranges from 10° C. to 110° C., find how much play must be allowed at each joint. $k = 0.000012$.

12. The iron rails of a railway are each 30 ft. long. What space must be left between two consecutive rails to allow for expansion for a range of temperature of 50° C.?

13. Describe the behavior of water if heated from 0° C. to 100° C.

14. Two thermometers, one filled with mercury and the other with water, are marked so as to agree at the two fixed points, but their readings are found to differ at temperatures between the fixed points. Explain.

15. Compare water, mercury, and air as thermometric substances.

16. Find the weight of 1 liter of boiling water, the coefficient of expansion of water being 0.00046.

17. The sp. gr. of mercury at 0° is 13.6. What is its sp. gr. at 200°? Coefficient of real expansion of mercury = 0.000182.

18. To what temperature must a gas be heated at constant pressure in order that its volume may be twice what it is at 0°?

19. A certain quantity of gas measures 360 ccm. at 30° C. What will be its volume at 0° C.?

20. A glass vessel full of air at 0° under atmospheric pressure is gradually heated. If the vessel can safely stand a pressure of 4 atmospheres, to what temperature may it be heated?

21. An open glass flask holds 1 liter of air weighing 1.293 grams at 0°. What weight of air will it contain at 100°?

22. A chemist generates 60 liters of gas at 10° C. and a pressure of 70 cm. Find the volume of the gas at 0° C. and 76 cm.

23. The pressure on a gas is doubled and the temperature raised from 25° to 116° C. How is the volume altered?

24. If the volume of a gas is doubled, what change in the temperature is required in order that the pressure may remain unaltered?

Change of State.

99. Fusion. If we heat paraffine in which a thermometer is inserted (Fig. 111), the mercury rises to 46° C. and then stops. At the same time the paraffine begins to melt. After the paraffine is all melted the mercury continues its ascent. If the source of heat is removed the mercury falls to 46° and becomes stationary, while the paraffine changes back to a solid; then the mercury begins to fall again. Thus paraffine is a solid below 46° C. and a liquid above 46° C.

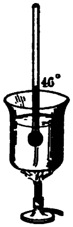


FIG. 111.

When a solid changes to a liquid it is said to *melt, fuse,* or *liquefy;* when a liquid changes to a solid it is said to *freeze, congeal,* or *solidify.*

Many solids become soft or plastic before melting. By reason of this property glass can be moulded in a great variety of forms, and two pieces of wrought iron when heated white-hot can be welded together.

Some melting points. Ice 0°; wax 66°; lead 330°; zinc 360°; copper 1100°; cast iron 1200°; wrought iron 1600°; brass (an alloy of copper and zinc) 950°. Alcohol freezes at −130°, and becomes so viscid (oily) before it freezes that it will not run out of a bottle.

In the case of ice, which contracts on melting, the melting point is lowered by 0.0074° C. for each increase of pressure equal to 1 atmosphere.

The general laws of fusion, established by experiment, are:

1. *The melting point of a given substance under constant pressure is constant, but differs for different substances.*

2. *The temperature of a substance does not change while the process of melting is going on.*

3. *The melting and the solidifying points for the same substance are the same.*

4. *Pressure raises or depresses the melting point according as the solid expands or contracts on melting.*

100. Change of Volume on Melting. Most bodies expand on melting and contract on solidifying. But water, cast iron, bronze, and type metal are important exceptions. They *expand* on solidifying. This is the reason why ice floats on water, and sharp castings can be made with cast iron.

If ice were heavier than water it would sink as fast as it formed. The consequence in a cold climate would be that a few weeks of severe weather would suffice to change all the water in the ponds and lakes into solid masses of ice. Once transformed into ice, the water would mostly remain ice; for the heat of summer would only be able to melt a thin layer at the surface.

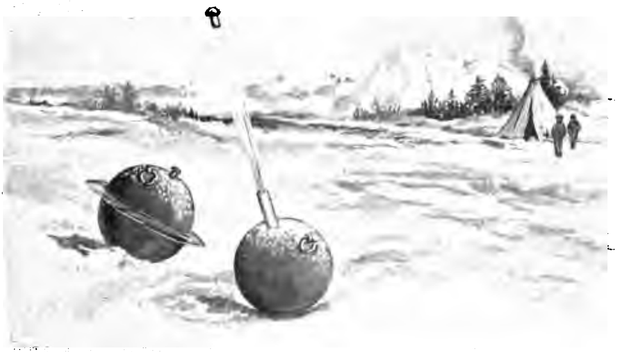


FIG. 112.

When water freezes it expands with great force. The experiments of Major Williams at Quebec proved this very conclusively. He filled an iron bombshell with water, closed the hole with an iron plug, and exposed the shell to the intense cold of a Canadian winter night. In the morning the plug was found 100 yards away, and a cylinder of ice 8 inches long protruded from the hole. At another trial the plug remained firm, but the shell cracked, and a sheet of ice was forced out along the crack as seen in Fig. 112.

101. Regelation. If we take two pieces of ice at 0° and press them firmly together under water (even warm water), we soon find that they are frozen together. This curious phenomenon is known as *regelation.*

Regelation is explained by the fact that the melting point of ice is lowered by pressure (§ 99, Law 4); whence it follows that if two pieces of ice at 0° are pressed together, a small quantity of ice along the pressed surface is obliged to melt. But heat is required to melt ice, and in this case the necessary heat, not being otherwise supplied, is withdrawn from the film of water in contact with the edges of the pressed surfaces. The consequence is that this film freezes and binds the two pieces of ice together.

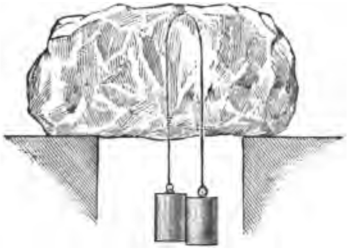


FIG. 113.

Regelation is illustrated every time a snowball is made. To make good hard snowballs the snow must be on the point of melting; if very cold it will not bind, but behaves like fine salt.

By means of regelation Professor Tyndall explained why ice at the melting point and under pressure is a plastic substance, and how it is possible for an Alpine glacier to accommodate itself to the channel through which it moves. Some of his experiments in support of his views were very striking.

For example, he took a straight piece of ice, and, by pressing it through a series of moulds, each more curved than the last, finally turned it out as a semicircular ring.

He also supported two weights by a copper wire which passed over a slab of ice, as shown in Fig. 113. After an hour or so the wire had cut its way through the ice, but the ice remained unbroken. The pressure melted the ice below the wire, but the water thus formed froze again above the wire, because the pressure on it was removed. The heat required to melt the ice below the wire was obtained from the freezing of the water above the wire.

102. Vaporization. If water in an open vessel is left in a warm room it gradually disappears; it changes to aqueous vapor and diffuses through the air of the room. If we slowly heat the water, the change to vapor takes place more and more rapidly, but is confined to the surface of the liquid till the temperature reaches 100° C.; then the change begins to occur throughout the whole mass of the liquid, and the water is said to *boil.*

The passage of a substance from the liquid to the gaseous state is called *vaporization;* the converse change is called *condensation* or *liquefaction.*

Vaporization takes place in two ways: *evaporation,* which is the quiet formation of vapor at the surface of the liquid; and *boiling* or *ebullition,* which is the rapid formation of bubbles of gas throughout the whole mass of the liquid.

There are a few substances which pass directly from the solid to the gaseous state without first becoming liquids. Camphor is an example. In such cases the change is called *sublimation* or *volatilization.*

103. Evaporation. Experiments show that the rate of evaporation of a liquid

(1) *Depends on the nature of the liquid.*

(2) *Increases with the temperature.*

(3) *Increases with the extent of the free surface.*

(4) *Is increased by a continual change of the air in contact with the liquid.*

(5) *Is increased by diminishing the surface pressure.*

Ether evaporates faster than water (1); pools of water dry up faster when the sun shines than when it is cloudy (2); water spread over a floor very soon dries up (3); the action of the wind rapidly dries muddy roads and wet clothes (4).

Water evaporates at all temperatures. When air at any temperature contains all the aqueous vapor it can hold at that temperature it is said to be *saturated.* If the temperature rises, the air can hold more vapor; if the temperature falls, a part of the vapor condenses and forms a cloud, or descends to the ground as rain, snow, or hail.

104. Ebullition. The boiling of water may be observed by heating water in a glass flask (Fig. 114) closed by a cork through which passes a tube for the escape of steam, and a thermometer for recording the temperature.

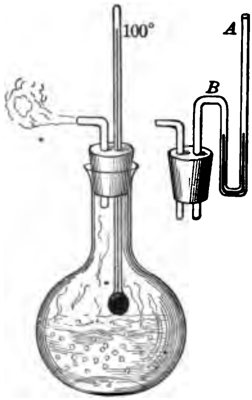


FIG. 114.

If heat is applied to the bottom of the flask the water steadily rises in temperature. After a time small bubbles form on the bottom and sides of the flask, and a little later rise to the surface and escape; these are bubbles of air held in solution by the water. Then bubbles of steam begin to form on the bottom and start to ascend, but meeting with colder water they condense. If the condensation is very rapid, we hear a rattling noise called *simmering.*

Finally the bubbles of steam are seen to rise to the surface and burst, throwing the water about with a soft rolling noise. The water is now boiling, and a stream of steam begins to pour out of the open tube with a hissing sound.

The thermometer now reads 100° C. or a little more. But whatever the reading is, it remains constant while the boiling continues; not till all the water is boiled away will the mercury rise any higher.

The pressure of the steam is equal to (or a trifle greater than) the atmospheric pressure at the time. This may be proved by substituting for the thermometer a manometer *AB* containing mercury. The level of the mercury is the same in both arms, one of which *A* is exposed to the air, and the other *B* to the steam in the flask.

Boiling points under a pressure of 76 cm. Carbonic acid — 75° C.; common ether 35° C.; alcohol 78° C.; water 100° C.; mercury 350° C.; sulphur 430° C.; zinc 1040° C.

The general laws of ebullition are the following:

1. *A liquid boils when the pressure of its vapor becomes greater than the pressure on the surface of the liquid.*

2. *The boiling point of the same liquid under the same conditions is constant.*

3. *The temperature of a liquid while boiling under the application of heat from an external source is constant.*

4. *The boiling point of a liquid rises if the pressure is increased, and falls if the pressure is diminished.*

Franklin illustrated the influence of pressure on the boiling point as shown in Fig. 115. Water is made to boil in a strong glass flask, and then the flask is tightly corked, the lamp at the same time being removed. The flask is inverted, as seen in the figure, and as soon as the boiling ceases cold water is sprinkled over the flask. The water at once begins to boil again, because the pressure on its surface is reduced by the condensation of a portion of the steam.

On the other hand, under a pressure of 2 atmospheres water boils at 120.6° C., and under a pressure of 10 atmospheres the boiling point is 180° C.

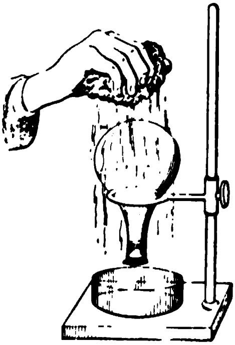


FIG. 115.

Law 4 explains why two mercurial thermometers will not agree in their readings unless their boiling points were determined under the same pressure. It also explains why water boils at a lower temperature on a mountain than at the sea-level. Thus, water boils at the city of Mexico (height 7500 feet) at 92.3° C. and at Quito (height 9500 feet) at 90.1° C.

A simple and fairly accurate rule for estimating heights by boiling points is to multiply 290 meters, or 950 feet, by the difference between the boiling point and 100° C.

Hence, water boiling in an open vessel is not equally hot in all places; at Quito, for example, it is not hot enough to cook potatoes.

105. Liquefaction. A liquid gives off vapor at all temperatures, but for a given temperature there is a maximum pressure which the vapor of that liquid cannot exceed; if you try to increase it by compression the vapor condenses.

This pressure or *tension* (as it is often termed) increases with the temperature. For instance, in the case of water it is 4.6 mm. of mercury at 0° C., 10 mm. at 12°, 92 mm. at 50°, 233 mm. at 70°, 535 mm. at 90°, 760 mm. at 100°, 2 atmospheres at 120.6°, 4 atmospheres at 144°, etc., etc.

Now suppose that we have a quantity of aqueous vapor at 90° and under a pressure of 92 mm., and that we wish to condense it to a liquid. Two methods may be employed.

(1) Keeping the temperature constant, we may increase the pressure from 92 mm. to 535 mm., the maximum tension for 90°; then the least attempt to increase the pressure further will cause condensation.

(2) Keeping the pressure constant, we may cool the vapor to 50°, at which point the maximum tension is only 92 mm.; then any further cooling will cause condensation.

By the joint application of pressure and cold even such gases as oxygen and nitrogen (the chief constituents of the air we breathe) have been liquefied.

Faraday (1823) liquefied carbonic acid gas, and also obtained it in a solid form. Under atmospheric pressure it liquefies at − 79° C.; at 0° C. the maximum tension of the gas is 38 atmospheres. More recently (1877) oxygen, nitrogen, and hydrogen, formerly called the permanent gases, were liquefied by Cailletet at Paris and Pictet at Geneva; and their results have since been confirmed. Under atmospheric pressure the liquefying point of oxygen appears to be about − 182° C., and that of nitrogen − 194° C. That of hydrogen is still uncertain.

The words *gas* and *vapor* have no settled difference in meaning. The term vapor is usually applied to the gas given off by a liquid at any temperature below the ordinary boiling point of the liquid. Substances, like oxygen and hydrogen, which at all ordinary pressures and temperatures are far removed from the point at which they liquefy are always called gases.

106. Distillation. This process illustrates both vaporization and condensation. It is employed for two purposes:

(1) To remove solid impurities dissolved in a liquid.

(2) To separate two liquids whose boiling points differ.

The liquid or mixture of two liquids is heated in a vessel called the *retort;* the vapor given off flows to another vessel called the *condenser,* where, by the application of cold, it is condensed to the liquid form again. The whole apparatus is called a *still.* A simple still is shown in Fig. 116.

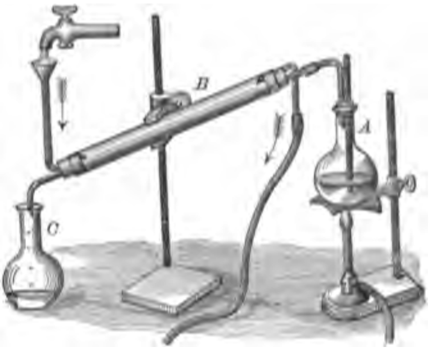


FIG. 116.

In Fig. 116 the vapor of the liquid passes from the retort *A* to the "water jacket" *B*, where it is condensed by cold water applied as shown in the figure. The condensed liquid is collected in the receiver *C*.

Distillation is the ordinary method employed by chemists and apothecaries for the purpose of obtaining perfectly pure water.

If a mixture of water and alcohol (boiling point 78° C.) is placed in a retort and raised to a temperature between the boiling points of the two liquids, the vapor which leaves the retort is mostly alcohol. If this vapor is allowed first to pass through a hot receiver most of the aqueous vapor will condense. The remainder passes on to the condenser. The water can be still further eliminated from the alcohol by repeating the process. This is called *fractional* distillation.

LABORATORY EXERCISES.

1. Determine the melting point of wax, and try to ascertain whether wax contracts or expands when it melts.
2. Demonstrate the effect of freezing on water by filling a test tube with water and surrounding it by a mixture of salt and snow.
3. Perform Tyndall's experiment of passing a wire through ice (p. 108).
4. Perform Franklin's experiment described on p. 111.

CLASS-ROOM EXERCISES.

1. State facts illustrating the force with which water expands when it freezes.

2. A cannon is filled with water and the mouth tightly sealed up. What will happen (1) if the cannon is surrounded by a freezing mixture? (2) if the cannon is surrounded by fire?

3. Why is iron a good metal for making castings? Why are coins stamped and not cast?

4. Explain the adhesion of two pieces of ice when they have been pressed together and the pressure is removed.

5. A piece of ice at 0° C. is heated under atmospheric pressure to 120° C. Trace the changes in volume which occur.

6. Is heat absorbed or set free in the process of evaporation? Illustrate your answer by an experiment.

7. Why is a windy day the best for drying clothes?

8. Why is it impossible to cook certain vegetables in an open vessel on the top of a high mountain? How can they be cooked there?

9. How is the boiling point of water affected by taking the water to the bottom of a deep mine?

10. In order to free syrup from water without discoloring the sugar it is necessary to make the water boil at a temperature of about 150° F. How can this be done?

11. In order to extract gelatine from bones it is necessary that the water in which the bones are boiled should have a temperature considerably above 100° C. How can this be accomplished?

12. Why may a vessel made of pewter or other easily fusible metal be safely placed on a hot stove provided it contains some water?

13. Water may be boiled in a bag made of thin, strong paper without burning the paper. How do you explain this?

14. If in filling a barometer tube a drop of water enters with the mercury, what effect will this have on the reading of the barometer? Why? Would the effect be greater or less if ether instead of water had been admitted? Why?

15. A cylinder is fitted with a piston, and the confined space within the cylinder is just saturated with water vapor at 20°. Describe and explain what will happen in the following cases:

(1) If the volume is increased by pulling up the piston.
(2) If the volume is diminished by pushing down the piston.
(3) If the temperature is raised to 40°, the volume remaining unchanged.
(4) If the temperature is reduced to 10°.

Calorimetry.

107. Heat as a Quantity. When two bodies of unequal temperatures are placed in contact, the temperature of the warm body falls and that of the cold body rises, till both have the same temperature. They are then said to be in a state of *thermal equilibrium.*

If we mix 1 kilogram of water at 20° C. with 1 kilogram of water at 50° C. so quickly that no sensible amount of heat escapes into the air, we obtain 2 kilograms of water at 35° C. Here the cool body gains 15° and the warm body loses 15°.

If we mix 2 kilograms of water at 20° C. with 1 kilogram of water at 50° C. we obtain 3 kilograms of water at 30° C. The 2 kilograms gain 10°, and the single kilogram loses 20°. If we multiply each mass by its change in temperature the products are equal ($2 \times 10 = 1 \times 20$).

The product of any mass of water and its change in temperature is called the *quantity* of heat which it gains or loses. In the examples given the *unit* of heat is the quantity of heat required to raise the temperature of 1 kilogram of water 1° C.

The unit of heat commonly used in Physics is the quantity of heat required to raise the temperature of 1 *gram* of water 1° C. This unit is called a *calorie.*

Heat is a quantity in the same sense that weight or force is a quantity; namely, something capable of measurement and subject to the operations of addition, subtraction, etc.

In the first case of mixture mentioned above, one body loses 15 units of heat and the other gains 15 units. In the second case the loss of one body and the gain of the other are 20 units each. In all cases of mixture we assume that the quantity of heat lost by one body is gained by the other.

Careful experiments show that the quantity of heat required to raise the temperature of 1 kilogram of water 1° C. is slightly greater at high temperatures than at low ones. But for most purposes the variation is unimportant.

108. Specific Heat. If we mix 1 kilogram of water at 41° C. and 1 kilogram of mercury at 10° C. the temperature of the mixture will be 40° C. The water has lost only 1 unit of heat, but this is sufficient to raise the temperature of the mercury 30°. Therefore, 1 kilogram of mercury requires only $\frac{1}{30}$ of a unit of heat to raise its temperature 1° C.

The quantity of heat required to change the temperature of a unit of mass of any substance 1° C. is called the *specific heat* of that substance. Since we always use the same unit of mass both for the substance and in defining the heat unit employed, we may also define specific heat as the ratio of the quantities of heat required to heat equal masses of the substance and of water 1° C. Thus, the specific heat of mercury is the fraction $\frac{1}{30}$, or 0.033.

Water has the greatest specific heat of any substance (except hydrogen). It takes about four times as much heat to heat a given mass of water 1° C. as to heat an equal mass of the solid earth. Hence, the ocean acts as a great moderator of temperature. During the hot season it absorbs a vast amount of heat without rising much in temperature, and during the winter months it slowly gives up this heat to the land and the air. This explains why maritime countries enjoy a more equable climate than regions situated far from the sea. There are islands in the Pacific Ocean where the temperature does not vary more than 5° or 6° during the whole year.

Specific heats. Water 1; ice 0.5; steam 0.5; alcohol 0.6; glass 0.2; iron 0.11; copper 0.095; lead 0.031; air (at constant pressure) 0.24; hydrogen 3.4.

109. Method of Mixtures. This method of finding the specific heat of a substance is based on the principle that when heat flows from a body *A* to a colder body *B*, the loss of *A* is equal to the gain of *B*. The body whose specific heat is to be found is weighed, heated to an observed temperature, and then dropped into water the weight and temperature of which are known. After thermal equilibrium is established the common temperature is observed. From these data the specific heat of the substance may be found (p. 121).

110. Latent Heat of Fusion. When ice is melting its temperature remains at 0° C., although heat is entering the ice all the time. Melting ice when suspended in a room cools the surrounding air; if we stand under it we feel the cold currents of descending air.

Large masses of ice and snow melt slowly, even when the sun is shining directly upon them. If they were suddenly converted to water by the sun's rays, the devastation caused by spring freshets would be vastly greater than it now is.

If 1 kilogram of ice at 0° C. is mixed with 1 kilogram of water at 80° C. the result will be 2 kilograms of water at 0° C. Therefore, 80 heat units have entered the ice without raising its temperature in the least. But they have produced another effect: *they have changed the ice from a solid to a liquid.* The fact that this heat disappears when ice melts, so that it is no longer capable of affecting a thermometer, led Black, who first studied this subject (1757), to call it *latent* heat; and this name has been retained ever since.

Conversely, when water freezes the heat rendered latent on melting is set free again; it leaves the water and enters the surrounding bodies.

Every solid absorbs heat in a latent form when it melts; all this latent heat is set free again and becomes sensible when the liquid returns to the solid state.

The quantity of heat required to change 1 gram of a substance from the solid to the liquid state without change of temperature is called the *latent heat of fusion* of that substance.

The vast amount of heat latent in water serves to lessen the severity of winter cold in high latitudes; it renders freezing a much slower process than it would otherwise be. This effect, as we should expect, is most noticeable on an island in the midst of the ocean. The mean temperature in January at Iceland is about 30° F, while along the same parallel of latitude in Siberia it ranges from − 10° to − 40 F.

Latent heats of fusion. Water 80 units; zinc 28.1 units; lead 5.3 units; sulphur 9.4 units; sodium nitrate 63 units.

111. Freezing Mixtures. When a solid undergoes liquefaction without having heat supplied to it by a flame or other direct source, the heat required to produce the change of state is taken from the bodies in contact with the solid, and they fall in temperature; in common language, *cold* is produced. This is illustrated when a solid is dissolved in water, and also *by the action of a freezing mixture.*

If we stir some niter (saltpeter) with a thermometer in a glass half full of water the mercury falls several degrees.

Pounded ice and coarse salt, when mixed together, exert upon each other a molecular action which results in both solids becoming liquefied. A double absorption of heat occurs, and if enough of the mixture is placed around water or cream, the latter is frozen.

By means of a mixture of crystallized calcium chloride (3 parts) and snow (2 parts) a temperature as low as − 40° C may be obtained.

112. Latent Heat of Evaporation. When a liquid boils another remarkable disappearance of heat takes place.

The temperature of water boiling in a kettle remains at 100° C. as long as any water remains in the vessel. The heat which enters the water becomes latent so far as temperature is concerned, but *in the act of becoming latent it transforms the water into the state of a gas.*

The quantity of heat required to convert 1 gram of a liquid into vapor without change of temperature is called the *latent heat of evaporation* of the liquid, or *latent heat of its vapor.*

Heat also becomes latent when water evaporates slowly. If a saucer containing water is exposed to the sun, the temperature of the saucer remains nearly constant so long as the water is evaporating, but begins to rise rapidly as soon as the saucer is dry.

When the vapor of a liquid condenses, its latent heat is given out again. Thus, if a current of steam is carried into a vessel of cold water, the temperature of the water is quickly raised to the boiling point. The great latent heat of steam is utilized in the heating apparatus employed for heating buildings by steam.

The latent heat of steam is 536 units; that of alcohol vapor is 205 units; that of ether vapor is 90 units.

113. Cold Caused by Evaporation. When a drop of ether is placed on the skin a sensation of cold is felt. If we wet the bulb of a thermometer with water, and swing it around in a warm room, the mercury sinks. If we wet the bulb with ether the mercury sinks further. Whenever a liquid evaporates, heat is absorbed and rendered latent. This heat must come from somewhere. When not supplied by a body of high temperature, like the sun or the flame of a lamp, it is withdrawn from the liquid itself and the bodies round it, and therefore their temperature is lowered; cold is produced.

Fig. 117 represents a test tube containing water and surrounded with cotton wet with ether. When a current of air is blown through the cotton the ether evaporates so rapidly that at length the water in the tube freezes.

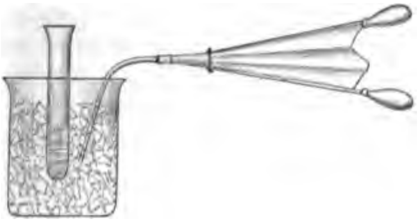


FIG. 117.

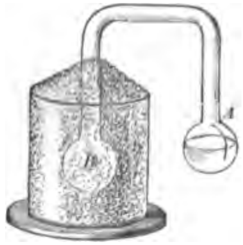


FIG. 118.

The *cryophorus* (Fig. 118) is a bent glass tube with a bulb at each end. The bulb *A* contains water, the tube and other bulb *B* contains nothing but water vapor as the tube was sealed when the water was boiling. The bulb *B* is placed in a freezing mixture; the vapor now condenses in *B* as fast as it can form in *A*. After a time the water in *A* freezes. The freezing mixture is applied to one bulb, but the freezing takes place in the other.

If a little water and above it a layer of ether are placed in a vessel under the receiver of an air pump and the air exhausted with sufficient rapidity, the ether boils and the water freezes.

Other examples are: the cooling of a room by sprinkling water on the floor; the cooling of mineral water by wrapping the bottle which contains it in wet cloths; the chilly sensations felt on coming out of a bath; a dog cooling himself by panting with his tongue exposed.

The reason why persons are liable to catch cold when sitting with wet clothes on will now be apparent; the danger is much less if the clothes are dry, even if the degree of cold is much more severe. Sometimes there is an excess of heat in the body due to exertion or hot weather. Then we *perspire* freely, and the evaporation carries off the surplus heat.

The most intense cold attainable is produced by condensing gases to liquids by means of cold and pressure, and then allowing the liquid to evaporate suddenly into a space free from air. By the evaporation of liquefied oxygen, a temperature as low as — 200° C. has been obtained.

114. Summary of Heat Effects. Let us briefly summarize the effects of heat. They fall under three principal heads:

(1) Change of volume (or change of pressure if the volume is kept constant).

(2) Change of temperature.

(3) Change of state.

If we begin with a substance in the solid state and apply heat to it continuously, effects (1) and (2) are produced at rates differing for different substances. When the temperature reaches a certain point, depending on the nature of the substance, (2) ceases to be produced, and (3) appears instead; the solid melts, and the heat which causes the melting becomes latent.

After the solid is liquefied effects (1) and (2) are produced as before; the liquid expands and grows hotter, but the rate of expansion is faster than before, and is irregular (water being a good example). At the same time vapor forms at a rate increasing with the temperature.

When the temperature reaches a point known as the boiling point, effect (2) again ceases, and (3) takes its place; the liquid boils, and a large quantity of heat becomes latent. Simultaneously the volume increases enormously.

After the change to a gas is completed, effects (1) and (2) continue to be produced so long as heat is applied. If the gas expands under constant pressure, its volume increases at a uniform rate with the temperature (Charles' law).

LABORATORY EXERCISES.

1. Find the specific heat of lead shot.

Place a known weight of lead shot in a test tube and suspend the tube in a Florence flask A containing water, as shown in Fig. 119. The tube passes through a piece of wood which serves as a support for the tube and also as a handle for pouring out the shot.

Place also a known weight of water in a metal vessel B with thin polished sides, called the *calorimeter*. Boil the water in the flask for some time till you feel sure that the shot have the same temperature as the steam. Then note the temperature of the water in the calorimeter and pour the shot into it as quickly as possible. Stir the water well, and note the resulting temperature when it becomes stationary.

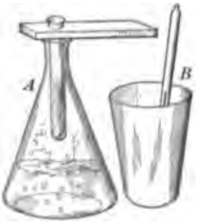


FIG. 119.

Suppose that the data are: weight of shot 200 grams, temperature of shot 100°, weight of water 100 grams, temperature of water before mixture 10°, after mixture 15°.

Solution. Let x denote the specific heat of the shot.

Loss of the shot in calories is $200\,(100 - 15)\,x$.
Gain of the water in calories is $100\,(15 - 10)$.
$\therefore 200\,(100 - 15)\,x = 100\,(15 - 10)$, whence $x = 0.029$.

This value is a little too small because the calorimeter gains heat as well as the water. Suppose that the calorimeter weighs 100 grams, and that the specific heat of its material is 0.1; then 10 calories of heat are required to raise its temperature 1°. This is the same amount of heat as would raise the temperature of 10 grams of water 1°; and it is therefore called the *water equivalent* of the calorimeter. It should be added to the weight of the water, thus making 110 grams. By making this correction we find that $x = 0.032$.

There are other smaller sources of error due to gain or loss of heat through conduction and radiation; when these are eliminated as far as possible the value of x is found to be 0.031. In order to determine the specific heat of a metal like lead or zinc a thin strip of the metal rolled up in the form of a spiral should be used.

The polished metallic surface of the calorimeter largely prevents loss of heat by radiation (see § 123); and the loss of heat may be still further diminished by surrounding the calorimeter with felt or paper.

2. Find the latent heat of fusion of ice.

Weigh a calorimeter and put into it a known weight, say 200 grams, of water. Observe the temperature t of the water. Drop into the water small pieces of ice, stir them around, and when they are just melted observe the temperature t' of the water. Then weigh the vessel to find the weight of ice added; suppose the weight of the ice is 60 grams.

Solution. Let x denote the latent heat of fusion required.

Gain of ice $= 60x$ (in melting) $+ 60t$ (in rising to t'°).
Loss of water $= 200(t - t')$.
$\therefore 60x + 60t' = 200(t - t')$, whence x can be found.

Sources of error. (1) The calorimeter cools as well as the water; (2) heat passes between the calorimeter and surrounding bodies; (3) some water is carried into the calorimeter along with the ice.

(1) is allowed for as in Ex. 1, (2) is largely neutralized by taking care to have the temperature t as much above that of the room as t' is below it, (3) is diminished by wiping the ice dry before dropping it into the water.

3. Find the latent heat of steam.

Into a weighed calorimeter put a known weight of water, and reduce the temperature of this water to about 5° C. Then conduct a current of steam into the water, taking care to note the temperature of the water *just before* the steam is admitted. When the water is warm (about 40° C.) shut off the steam, observe the temperature of the water, and weigh the calorimeter. All the necessary data are now known. The method of calculation is precisely like that in Ex. 2.

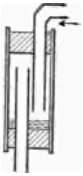


FIG. 120.

If w denote the weight of the water (including the water equivalent of the calorimeter), w' the weight of steam condensed, t the initial temperature of the water, t' the temperature after the steam has been condensed, and x the latent heat of steam, then

$$w'x + w'(100 - t') = w(t' - t).$$

A new source of error exists in the condensed water carried along with the steam into the calorimeter. To avoid this source of error a *water-trap* (Fig. 120) should be inserted just above the calorimeter. The calorimeter should be protected from the boiler by interposing a non-conducting screen.

CLASS-ROOM EXERCISES.

1. Illustrate the fact that heat is a quantity capable of measurement. Show by an example that two bodies may have the same temperature and yet contain very different quantities of heat.

2. Explain the meaning of the statement that the specific heat of water is 30 times as great as that of mercury.

3. Why is the small specific heat of mercury one of its merits as a substance to use for measuring temperatures?

4. What is meant by saying that the latent heat of water is 80 units? Mention some of the consequences that would follow if the latent heat of water were only 2 or 3 units.

5. Why do tubs of cold water protect a cellar from frost?

6. Why do persons often catch cold if they get their clothes damp?

7. On the island of Madeira in the Atlantic Ocean the mean temperature in winter is about 60° F. and in summer about 70° F. How do you account for this small variation in temperature?

8. A metal vessel with a thin bottom is placed on a wet board. The vessel is half filled with water, and a quantity of ammonic nitrate is stirred around in the water. Very soon the vessel freezes fast to the board. How is this to be explained?

9. Why does ether on the skin cause a sensation of cold?

10. Explain the cooling effect produced by watering a dusty road.

11. Describe some experiment by which a very low temperature can be produced, and explain the production of the cold.

12. Men have entered ovens heated to 250° F. where they have remained long enough to cook a steak by blowing hot air on it through a bellows. The men suffered little from the heat beyond a very profuse perspiration. Why were they not more seriously affected by the heat?

13. A vessel contains 20 kg. of water at 0°. How many calories of heat must be imparted to the water before it begins to boil?

14. How many units of heat are required to raise the temperature of 400 grams of iron from 10° to 200°? Sp. heat of iron = 0.11.

15. How many heat units are set free when a tank containing 8 tons of water cools from 30° C. to the freezing point?

16. If 7 lb. of water at 25° are mixed with 3 lb. at 65°, what will be the temperature of the mixture?

17. What is the temperature of a mixture made of 6 kg. of mercury at 20° and 4 kg. of mercury at 50°?

18. What temperature will result if we put 300 grams of copper at 100° into 200 grams of water at 10°? Sp. heat of copper = 0.095.

19. How many pounds of ice water must be poured into 40 lb. of water at 80° to reduce the temperature to 5°?

20. Two pounds of iron at 100° are placed in 3 lb. of water at 20°, and the resulting temperature is 25°. Find the specific heat of iron.

21. Find the specific heat of a substance if 250 grams of it at 78°, when immersed in 500 grams of water at 12°, give a temperature of 18°.

22. If 60 grams of iron nails at 100° are put into 136 grams of water at 16° and the final temperature is 20°, find the specific heat of the nails.

23. Five hundred grams of lead shot at 99° are poured into a calorimeter which weighs 78 grams and contains 300 grams of water at 14°. The temperature of the water rises to 18.5°. The specific heat of the calorimeter is 0.1. Find the specific heat of lead.

24. To find the temperature of a furnace, a copper ball weighing 1 lb. is put into the furnace, and after some time removed, and dropped into a bucket containing 20 lb. of water at 15° C. The temperature of the water rises to 20°. Taking the specific heat of copper as 0.1, find the temperature of the furnace.

25. How much ice at 0° will be melted by 1 kg. of boiling water?

26. Equal weights of hot water and melting ice are mixed. The result is water at 0°. What is the temperature of the hot water?

27. Equal weights of boiling water and melting ice are mixed. The result is water at 10°. Hence, find the latent heat of water.

28. How many pounds of ice must be mixed with 6 lb. of water at 95° in order to obtain water at 10°?

29. What will be the resulting temperature (1) if 2 lb. of ice at 0° are mixed with 10 lb. of water at 80°? (2) if 10 lb. of ice at 0° are mixed with 2 lb. of water at 80°?

30. How many inches of rain at 50° F. must fall in order to melt 1 inch of ice at 0°?

31. A calorimeter (sp. heat 0.1) weighing 78 grams contains 200 grams of water at 52°. When 116 grams of ice are put into the water the temperature falls to 5°. From these data find the latent heat of water.

32. A pound of coal in burning sets free 8000 units of heat. How much coal is needed to transform 1 ton of water at 0° into steam at 100°?

33. How much heat is given off when 30 lb. of steam at 100° are cooled down to water at 50°?

34. A pound of steam at 100° is blown into 10 lb. of water at 20°. Find the resulting temperature.

35. A calorimeter (sp. heat 0.1) weighing 78 grams contains 300 grams of water at 4°. In this water 23 grams of steam at 100° are condensed. The temperature of the water rises to 48°. Find the latent heat of steam.

Transmission of Heat.

115. Convection. When heat is applied underneath a mass of liquid or gas the portions first heated expand and *rise*, while the colder and heavier parts above *sink;* by means of these ascending and descending fluid currents the heat is rapidly carried throughout the entire mass. The transference of heat by the motion of heated matter carrying the heat along with it is called *convection.*

The convection currents in water when it is heated are rendered plainly visible by putting some sawdust into the water.

Three things must combine to render convection possible: (1) expansion by heat; (2) the action of gravity; (3) freedom of molecular motion. Therefore solids cannot be heated by convection.

With the apparatus shown in Fig. 121 a continuous current of hot water may be produced by convection. The vessels *A* and *B* are connected by two pipes; one *CD* extends from the top of *B* nearly to the top of *A*, the other *EF* extends from the bottom of *A* nearly to the bottom of *B*.

The whole apparatus is filled with water nearly to the top of *A*. The water should be previously boiled to expel air.

When the water in *B* is heated, a convection current circulates in the direction of the arrows. To render the current visible *A* should be filled with colored water, and sawdust may also be put into it. The apparatus illustrates how buildings are heated by hot water.

Convection currents are caused by *cooling above* as well as by heating below; as, for example, in the water of a pond after the sun sets in winter.

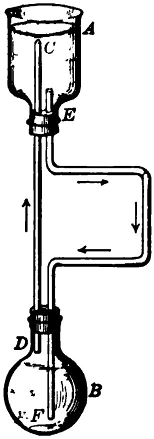


FIG. 121.

To show on a small scale currents due to cooling, nearly fill a tall beaker with cold water and carefully place above this a layer of warm colored water, by floating a thin cork on the water and then pouring the colored water on the cork. Then put a piece of ice in the colored water. Almost instantly the colored water near the ice will begin to descend through the clear water below.

116. Convection in the Air. Convection currents of enormous magnitude are caused in the atmosphere by the sun heating the surface of the earth; the heated air expands and rises mixing with the cooler air above, while currents of cooler air flow in from the sides. It is mainly by these convection currents, which we call *winds*, that the atmosphere is kept well mixed and uniform in composition.

On a hot, clear, still day, if we look over the top of a brick wall or a gentle rise of ground, the air seems to be flickering and unsteady; this appearance is an optical effect due to the motion past one another of convection currents of air.

Examples of convection currents on a small scale are the rise of hot air over a stove or through the pipes of a hot-air furnace and the draft of a chimney.

117. Draft of a Chimney. A glass chimney makes the flame of a lamp burn bright and steady because it causes a steady stream of fresh air to enter the chimney from below.

The strength of the draft depends on the difference between the weight of the heated gases inside the chimney and the weight of an equal bulk of the cooler and heavier air outside. The draft in a chimney over a fireplace is determined in the same way. Hence, the higher the chimney and the hotter the gases within it, the better the draft will be. But various other circumstances must also be taken into consideration.

The chief causes of smoky chimneys are:

(1) Insufficient height.

(2) Too large a flue for the size of the fire. In this case, currents of cold air will descend the flue and carry smoke into the room.

(3) Insufficient ventilation. The air in the room is then rarified, its pressure diminishes and may become too small to cause an up-draft; the smoke then ceases to ascend and pours out into the room.

(4) Reciprocal action of two flues. If two flues are in adjoining rooms and one has a better draft than the other, it is likely to cause a down-draft in the other.

(5) The wind.

118. Ventilation. A room is said to be "well ventilated" when a proper supply of fresh air is maintained in it. To secure good ventilation there must be one or more inlets by which the fresh air can enter, and one or more outlets by which the foul air can be expelled. Usually the apparatus for ventilation is combined with that for heating, and convection currents are utilized. Two methods are illustrated below. Both have been found to work well in practice.

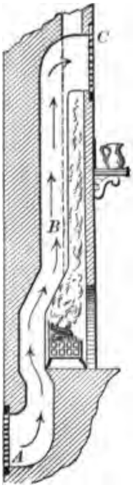


FIG. 122.

Fig. 122 gives a sectional view of a *ventilating grate* by means of which the waste heat of an open fireplace is utilized, and at the same time ventilation is secured. The cold air is taken in at *A* behind the fire, and, being warmed by the fire, ascends a chamber *B* and enters the room through a grating *C*.

This warm fresh air then spreads over the room, descending as it cools, while the foul air is drawn into the fire and passes up the chimney.

When a room is warmed by hot water or steam, heating and ventilation may be combined as shown in Fig. 123. The piping is placed near an inlet in the wall and surrounded by a metal casing provided with gratings at the bottom *C* and the top *E* through which air can pass freely. In this casing is a vertical metal diaphragm *D* separating the piping from the wall, and also two valves *A* and *B*. When the valve *A* is open, the air from outside can enter freely. By rotating *B* this air may be made to pass through the outer channel unheated, or to pass over the piping so as to become heated, or to ascend in part by one channel and in part by the other. By closing *A* the apparatus may be used entirely for heating purposes, the air of the room entering the casing at *C* and leaving it at *E*.

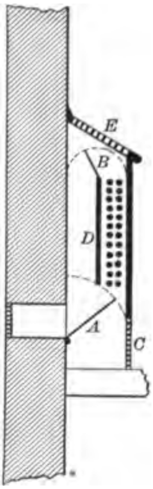


FIG. 123.

At least one outlet for the impure air of the room must be provided; and usually one is better than many. If there are several, the wind may make some of them act as inlets, thus causing objectionable down-drafts.

119. Conduction. If one end of a metal bar is heated the other end soon becomes hot; heat travels along the bar. The flow of heat towards the colder parts of a body without sensible motion of the parts themselves is called *conduction.*

Suppose we heat one end of a metal bar *AB* (Fig. 124) into which a series of thermometers have been inserted. At first the thermometers indicate that each portion of the bar is rising in temperature as well as receiving heat; this state of the bar is called the *variable* state. But after a time the mercury ceases to rise in each thermometer. Each portion of the bar is now losing as much heat as it receives, either by conduction or by convection and radiation from its surface; this is termed the *stationary* state of the bar.

Now the rate at which the temperature of the bar rises during the variable state evidently depends on the specific heat and the density of the material as well as on its power to conduct heat. The less the specific heat, or the less the density, the more rapidly will the temperature rise. Hence, the true conducting power or *conductivity* of any material can be determined only when the stationary state, as regards temperature, has been attained.

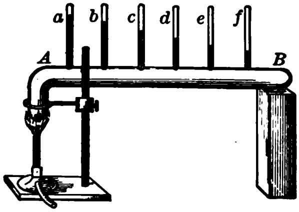


Fig. 124.

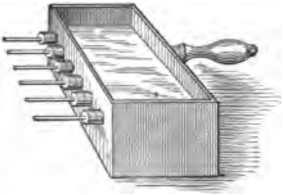


Fig. 125.

If one end of a bar is heated and thermometers are inserted along the bar, as shown in Fig. 124, then from the rate at which the temperature falls along the bar, when the stationary state has been reached, it is possible to deduce and express by a number the conductivity of the material of which the bar is made.

The conducting power of solids may be compared as shown in Fig. 125. Rods of various substances are coated with wax and inserted in the side of a metal trough, which is then filled with boiling water. The distances to which the wax melts on the rods are proportional to their conducting powers.

The best conductors of heat are the metals; and among the metals silver and copper stand first. Inferior to the metals in conducting power are stone, brick, and glass; still worse are wood, cork, sand, wool, feathers, and fur. Solids that are fibrous or loose in structure are always very bad conductors of heat. Liquids (except mercury) have very little conducting power, and gases have practically none at all. The poor conducting power of such substances as wool and fur depends mainly on the fact that they contain innumerable cavities filled with air.

The explanation of the following facts will now readily be seen:

1. A burning match can be held without discomfort till the flame almost reaches the fingers.

2. A silver spoon and an iron spoon are dipped into the same vessel of hot water. The silver spoon becomes hot much quicker than the iron spoon. How would the iron spoon compare with a wooden one in respect to temperature?

3. Flame will not pass through wire gauze (Fig. 126). The metal conducts the heat away so rapidly that the gas is cooled below the temperature at which it will burn. If the gas is turned on and a lighted match held over the gauze, the gas will burn only above the gauze.

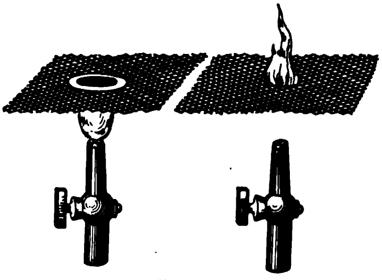


FIG. 126.

Davy's *safety lamp* to protect miners against the explosions of 'firedamp' is an application of this fact.

4. If a test tube is filled with water and a piece of ice is sunk to the bottom by winding wire around it, the water may be boiled at the top of the tube a long time without melting any of the ice at the bottom.

120. Conductivity and Sensation. Iron feels *warmer* than wood at the same temperature, if that temperature is above the temperature of the hand, but *colder* if it is below the temperature of the hand. The reason is that the sensation of heat or of cold depends on the rate at which heat is entering the hand in one case, or is being withdrawn from the hand in the other. Now iron, being a better conductor of heat than wood, imparts heat to the hand faster than wood in the first case, but withdraws it faster from the hand in the second case.

In a Turkish bath the temperature is so high that a piece of iron, if you happen to touch it, will almost burn you. In the Arctic regions iron blisters the hand because it is so cold. In both places wood can be handled without discomfort.

In common life we make various applications of good or bad conductors for the sake of comfort or of health. Thus, cooking utensils, soldering irons, etc., are provided with wooden handles, and padded holders are used in handling flatirons.

In winter we wear clothes made of wool or fur, because these materials prevent the heat of the body from escaping. The use of double windows is another example; it is not the two thin layers of glass that prevent the heat of the room from escaping, but the non-conducting layer of air between them.

"Animals," says Tyndall, "which inhabit cold climates are provided by nature with the necessary clothing." Birds are provided with feathers and down between the feathers, "the molecular constitution and mechanical texture of which render it perhaps the worst of all conductors."

Trees are protected from the effects of cold by their fibrous structure; for heat is conducted much worse in a direction perpendicular to the fibers than along their length. The effect of this is to preserve within a tree the heat acquired from the soil. "But," as Tyndall observes, "nature has gone farther and clothes the tree with a sheathing of worse conducting material than the wood itself even in its worst direction." He means, of course, the bark of the tree.

Snow is a bad conductor of heat because it contains between its feathery crystals a large quantity of air. Hunters sometimes use a covering of snow to keep from freezing, and the instinct of animals leads them to seek the same shelter.

121. Radiation. The heat which we receive from the sun must come to us by some other mode than conduction or convection, for these cannot operate except in matter, and between the earth and the sun lies a vast interval wholly destitute of matter so far as we know.

Again, if we hold our hands a little *below* a red-hot mass of iron we feel a sensation of heat. This cannot be due to conduction, for air does not conduct heat; nor to convection, for heated air always moves upwards.

This third mode of transmission of heat is called *radiation.*

The use which we make of sunshades and fire-screens shows that the radiation of heat takes place in *straight lines.*

When an eclipse of the sun occurs, his light and his heat are both cut off at the same time; whence we infer that heat travels by radiation *with enormous speed.*

The sun's rays warm the objects on which they fall, but not the air through which they pass. If in winter we light an open fire in a cold room, the frost on the window panes begins to melt before the air in the room is sensibly warmed. Facts like these show that *heat can pass by radiation through air without sensibly heating it.*

We may therefore describe radiation in general terms as the transmission of heat from place to place in straight lines with enormous speed, and without necessarily heating the medium through which it passes.

We shall not here trouble ourselves about the explanation of radiation, but confine our attention to the facts. These tell us that a hot body, whether luminous like the sun or non-luminous like a hot stove, radiates heat in all directions. We may then distinguish between *invisible* or *dark* rays that affect the nerves of the skin only, and *visible* or *luminous* rays that affect also the retina of the eye. The sun emits both kinds of rays, but a hot stove usually emits only invisible rays.

122. Laws of Radiation. By means of observation and experiment the following laws have been established:

1. *Radiant heat travels in straight lines with the velocity of light.*

2. *The intensity of radiant heat diminishes as the square of the distance from the source increases.*

3. *Radiant heat is reflected from a polished surface in the same way as light.*

4. *The rate of cooling of a body in the air varies as the excess of its temperature above that of the air.*

5. *The radiation from a body increases with the temperature.*

6. *The rate at which a body radiates heat depends on the nature of its surface. It is greater for rough or dark-colored surfaces than for smooth or light-colored ones.*

The exact statement of law 3 will be given when we come to the study of Light (Chapter VIII).

Law 4 is called Newton's law of Cooling. It is not exact, but very nearly so, if the excess in temperature is less than 20° C.

Law 6 may be illustrated by means of the *differential air thermometer* shown in Fig. 127.

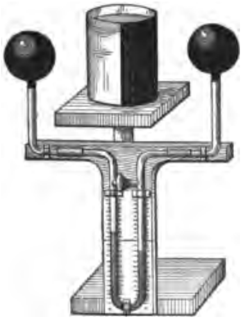


FIG. 127.

Two large glass bulbs, provided with stems, are coated with lampblack, and mounted as shown in the figure. The glass stems are bent at right angles, and are connected by short pieces of rubber tubing with the branches of a U-tube containing some colored water.

Polish as brightly as possible one half of the surface of a tin can, and coat the other half with lampblack. Fill the can with hot water (about 80° C.) and place it midway between the bulbs, so that the lampblack faces one of them, and the polished surface the other. The motion of the colored water will very soon show that the bulb opposite the lampblack is receiving heat faster than the bulb opposite the polished surface. Therefore, the lampblack radiates heat faster than the polished surface.

123. Absorption. When radiant heat falls on a body part of it is *reflected* from the surface, another part is *absorbed* by the body, and the remainder is *transmitted* through the body. The heat which is absorbed is transformed into sensible heat and raises the temperature of the body. The heat which is transmitted retains the form of radiant heat and has no effect on the temperature.

The power of a body to absorb heat, like its power to radiate heat, is greatly influenced by the nature of the surface.

Remove the lampblack from one of the glass bulbs of the apparatus shown in Fig. 127, coat the can completely with lampblack, and put into the can water at 80° C. The motion of the colored water in the U-tube shows that the bulb covered with lampblack absorbs heat faster than the other bulb.

In general,

(1) *Good radiators are good absorbers and bad reflectors.*

(2) *Bad radiators are bad absorbers and good reflectors.*

(3) *The radiating and absorbing powers of a body are equal.*

Lampblack is the best radiator and absorber of heat; *polished brass* and *polished silver* are the best reflectors.

These principles enable us to understand why water boils sooner in a kettle covered with soot than in one brightly polished, why a bright metallic teapot keeps the tea hot longer than an earthenware one, why the sun's rays will melt ice covered with ashes sooner than bare ice, and why a polished metal helmet protects the head against the burning rays of the sun.

124. Diathermacy. Bodies that transmit radiant heat are called *diathermanous,* and bodies that do not transmit it are called *athermanous.* The two words have the same relation to heat that 'transparent' and 'opaque' do to light.

The most remarkable fact about diathermanous bodies is that in general they transmit some kinds of heat rays better than other kinds. Glass is a good example. The heat of the sun will enter a room through the windows and warm the furniture and other objects in the room, but the heat radiated by these objects cannot escape through the windows. Glass is diathermanous for luminous rays of heat, but athermanous for the dark rays which radiate from bodies of low temperature.

The power of a substance to stop certain kinds of heat rays but to allow others to pass is called its power of *selective absorption*.

Dry air is very diathermanous for all kinds of heat rays. But the aqueous vapor in the atmosphere, although quite diathermanous for the sun's rays, is athermanous for the dark heat radiated by the ground and objects on the ground. Therefore, it prevents the heat of the earth from escaping into space rapidly. The value to us of this property of aqueous vapor may be gathered from the words of Professor Tyndall: "Remove for a single summer's night the aqueous vapor from the air which overspreads this country [England], and you will assuredly destroy every plant capable of being destroyed by a freezing temperature."

125. Theory of Exchanges. Every body, whatever its temperature, is constantly radiating heat at a rate depending on the nature of its surface and its temperature, but *not* on the temperature of the surrounding bodies. Thus, a red-hot iron ball radiates heat just as fast when placed in the middle of a furnace as when hung up in an ice house.

Every body is also absorbing heat from surrounding bodies. When a body absorbs more heat than it radiates, its temperature rises; when it radiates more than it absorbs, its temperature falls. If the amounts of heat radiated and absorbed are equal, its temperature does not change.

This theory explains the facts connected with the radiation and absorption of heat very simply. Thus, the ground on a clear night becomes colder than on a cloudy night. The ground loses heat by radiation just as fast on a cloudy night as on a clear night, if the temperature is the same; but the clouds radiate back again to the ground a part of the heat which they receive.

CLASS-ROOM EXERCISES.

1. How is the air of a room heated by a closed stove?

2. On the Mer de Glace at Chamouni, Switzerland, Count Rumford found holes in the ice about 4 ft. deep and 7 in. wide filled with water which deepened in summer from day to day. If water is a non-conductor of heat, how can this melting of the ice at the bottom occur?

Hint. Remember that water has its maximum density at 4° C.

3. Viscous liquids, such as molasses, oil, and thick soups, cool more slowly than water. Can you think of any explanation?

4. If you enter a cold room and touch first the fire irons, then the chimney flue, then the chairs, and then the carpet, how will they feel as regards temperature? How also, in case the room is hot instead of cold? Explain.

5. Sawdust is the same material as wood, but a poorer conductor of heat. Explain why.

6. Why are kitchen utensils often provided with wooden handles?

7. Why does the lock of a door feel colder than the wood of which the door is made?

8. Explain the use of double windows.

9. Why does woolen clothing keep us warm in winter?

10. Why do farmers like to have the ground covered with snow during the winter?

11. Tyndall, in one of his experiments, held a red-hot iron ball in his hand, first taking care to lay in his hand a sheet of asbestos paper. Why was he not burned?

12. In winter ice covered with ashes melts sooner than ice freely exposed. In summer just the reverse happens. Explain these facts.

13. Why does water boil sooner in a kettle covered with soot than in one brightly polished?

14. Why does cold, damp air chill a man more than dry air which is still colder?

15. Why does the glass covering of a conservatory make the air inside warmer than the air outside?

16. Arctic navigators have observed that in summer snow surrounding a black object melts, although the temperature of the air is below 0° C. How is this fact explained?

17. It is a well known fact that snow melts around the trunk of a tree sooner than in an open field. How do you explain this fact?

Heat in the Atmosphere.

126. The Temperature of the Air. As we rise above the earth's surface the temperature falls. The rate of fall varies greatly with place, time, and other circumstances; but the average fall is about 1° C. for 500 feet of ascent.

To explain this phenomenon, we must remember that air is not directly heated by the sun; his rays pass through it, and first warm the land and the water of the ocean, the former much more rapidly than the latter. The air is then warmed, partly by heat radiated by the land and absorbed by aqueous vapor, clouds, and dust motes in the air, but chiefly by convection currents of warm air rising from the land. These currents, however, expand as they ascend, and are thereby cooled in virtue of a general law, which will be stated in Chap. VI. Moreover, the air rapidly decreases in density as we ascend, and so offers less and less resistance to radiation into the empty space beyond; hence, mountain tops have much less effect in warming the air that surrounds them than the same amount of land surface at the level of the sea.

The distribution of temperature in the air near the surface of the earth is very unequal for reasons which are explained in works on Meteorology. The chief circumstances on which the temperature at any place depends are the following: (1) latitude, (2) the season of the year, (3) the hour of the day, (4) the position of the place with respect to land and water. The influence of (4) is much greater than commonly supposed; in some cases it is greater than a difference of many degrees in latitude. This great influence is due mainly to the more rapid heating and cooling of land than of water; and to the effect of ocean currents like the Gulf Stream.

The temperature of the air over the ocean at the equator seldom rises above 25° C., while that of the air over the Desert of Sahara often rises above 70° C.

127. Humidity. Dew Point. The higher the temperature, the greater the quantity of aqueous vapor which air is capable of holding before it is *saturated;* that is, before it contains all the vapor which it can hold at that temperature (§ 105). Air at rest above the surface of the ocean will in time become saturated with vapor. But air is seldom at rest, and is often far from saturated, especially above the land.

When air at any temperature is saturated with aqueous vapor the pressure exerted by this vapor, measured in millimeters of mercury, is called the *maximum pressure* of the vapor for that temperature. If the air is not saturated, the ratio of the pressure of the vapor actually present in the air to the maximum pressure at that temperature measures the *relative humidity* of the air. This ratio is expressed sometimes in the form of a fraction, but more usually as so many per cent. of the maximum pressure at that temperature. If this non-saturated air is cooled, its relative humidity will increase, and at last a temperature will be reached at which it becomes saturated; this temperature is called the *dew point.* If cooled still farther, the air will become incapable of holding all the vapor which it contains, and some of it will condense in the form of dew, mist, etc.

A tolerably correct determination of the dew point and relative humidity of the air at any time may be made as follows: Put some ice into a glass containing water, stir it around with a thermometer, and note the reading of the thermometer at the instant when moisture begins to condense on the outside of the glass. Suppose this reading is 12° C., while the temperature of the room is 20° C. The dew point is 12°. By reference to a table of the maximum pressures of aqueous vapor we find that the maximum pressure for 12° C. is 10.5 mm., and for 20° C. is 17.4 mm. Then the relative humidity of the air is 10.5 ÷ 17.4, or 0.603, or about 60 per cent.

Air feels damp or dry according as its temperature is near the dew point or not. In summer the air may contain much more moisture than it can hold in winter, and yet feel dry because its temperature is much above the dew point. Its relative humidity is small.

128. Dew. When warm air is cooled by contact with a colder body the moisture which it contains is often deposited on the surface of the colder body in the form of fine drops, called *dew.* If the temperature is below 0°, the dew freezes; and is then called *hoar frost.*

During the day all bodies on which the sun shines grow warmer, and moisture passes from the ground into the air by evaporation. By sunset, in well-watered countries, the air near the ground contains much moisture. During the evening the ground, and the bodies on the ground, lose heat by radiation. The consequence is that the air in contact with good radiators, like grass or leaves, soon becomes cooled below the dew point, and deposits dew upon their surfaces.

This theory explains why dew forms most copiously on bodies that radiate well but conduct badly; why it does not form in a very dry country; or on a cloudy night; or on a windy night; also why moisture collects in summer on the outside of a glass of ice water, and in winter on the inside of a cold window pane.

Dew may come from the ground and from plants as well as from the air. The under side of stones and not the upper is often covered with dew. This moisture must come from the damp ground.

129. Fogs and Clouds. Water vapor is an invisible gas. *Fogs, mists,* and *clouds,* it is now believed, are caused chiefly by the condensation of water vapor around motes of dust in the atmosphere. Such a condensation occurs very frequently at night in the damp air over a meadow or valley, and produces a fog or mist. A mist wets solid bodies; a fog does not. Clouds are mists that form at high altitudes. The minute liquid globules that compose a mist or cloud are always falling, but *very slowly* on account of the resistance of the air. Usually they do not fall far before they are evaporated, by the sun in the case of a mist, by meeting warmer and drier air in the case of a cloud.

130. Rain and Snow. When the air happens to contain fewer dust particles than usual, and becomes overcharged with vapor, each dust mote gets a thicker coating of water, and is transformed into a very small drop of water heavy enough to fall rather fast. These drops as they descend unite and form larger drops which fall still faster. If the air between them and the earth is nearly saturated, they increase in size by the condensation of more water upon their surface, and finally reach the ground as drops of *rain.*

The cooling necessary to cause rain is due sometimes to the mixing of masses of air at unequal temperatures, but more often to the ascent and consequent expansion of a mass of warm, moist air. When, for instance, a warm sea wind laden with moisture blows against the side of a mountain, the air rises, and is cooled so rapidly by expansion that the rain descends in torrents.

If condensation occurs at temperatures below 0° C., the vapor crystallizes as it condenses and reaches the ground as *snow.*

Hail consists of small pellets or balls made of compacted ice and snow. Its probable origin is by the freezing of small rain drops that form at low altitudes. These are then carried upward and downward several times by currents of air, growing larger all the time by being coated with alternate layers of snow and ice, till at last they become too heavy for further carriage and fall to the ground.

131. Winds. The unequal distribution of temperature in the atmosphere in combination with the unequal distribution of aqueous vapor causes inequalities of pressure. Wherever an inequality of pressure exists, the equilibrium of the air is destroyed, and air flows from the place of higher pressure to the place of lower pressure till the equilibrium is restored. In other words, *the wind blows* with a force proportional to the difference of pressure between the two places.

On the ocean the wind often keeps the same direction for a long period of time, sometimes for months (trade winds, monsoons, etc.); but on land, owing to various local causes, the winds are very variable in direction and intensity.

FIG. 128.

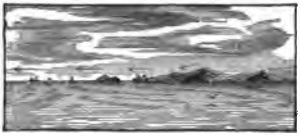


FIG. 129.

Figs. 128 and 129 illustrate the *sea and land breezes* which occur every 24 hours along most sea coasts. After what we have learned about the specific heat of water, convection and radiation, their explanation is not difficult. Fig. 128 shows the day breeze, Fig. 129 the night breeze.

132. Cyclones. This name is now given to storms occurring at irregular intervals and due to the inrush of air towards a large area of low pressure several hundred miles in diameter.

The air does not flow in straight lines to the center of the depression, but is constantly deflected in direction by the rotation of the earth on its axis. The general result is that the whole mass of inrushing air is set into a whirl about the center, the direction of the rotation in the northern hemisphere being *opposite* to that of a watch-hand; so that if you stand *with your back to the wind* the center of the cyclone will be *towards your left hand.* (See Fig. 130.) The center of a cyclone is also moving in some definite direction, carrying the whole storm along with it.

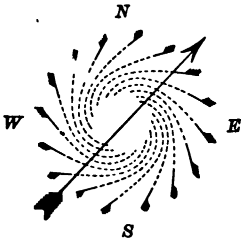


FIG. 130.

Our winter cyclones mostly originate on the cold plains of farther Canada and then move eastward to the Atlantic Ocean.

REVIEW EXERCISES ON CHAPTER III.

1. If a copper vessel tightly corked were placed over a flame, and if you very soon saw the cork driven violently from the vessel, what would you infer was in the vessel?

2. How would you proceed in order to ascertain which liquid, water or alcohol, expands at the faster rate when heated?

3. Give an example in which the expansion of a solid is put to a useful purpose.

4. Give an example in which the expansion of a solid has to be allowed for.

5. Explain what is meant by saying that the coefficient of linear expansion of iron is 0.000012.

6. Prove that the coefficient of cubical expansion of a body is equal to three times its coefficient of linear expansion (§ 96).

7. What is meant by saying that the maximum density of water is at 4° C.? How was this fact proved by Hope?

8. A bridge is built in winter, and the girders are laid end to end so that they touch one another. What will happen to them in summer?

9. What effect has the heat of summer on the rate of a clock, and why? How can you make a pendulum that will keep correct time both in summer and in winter?

10. Why is mercury commonly employed to measure temperature? Why is air better than mercury for the accurate measurement of temperature?

11. State Charles's law (1) when a gas is heated under constant pressure, and (2) when a gas is heated under constant volume.

12. Explain what is meant by the absolute zero of temperature. What is it on the Centigrade scale? What is it on the Fahrenheit scale?

13. State the laws of Fusion.

14. Why do water pipes sometimes burst in cold weather?

15. What is the meaning of regelation? Give an example.

16. On what circumstances does the rate of evaporation of a liquid depend?

17. State the laws of Ebullition.

18. Why cannot vegetables be cooked at high altitudes in open vessels? How can they be cooked in such places?

19. How would you obtain pure water from sea water?

20. Define a unit of heat, supposing a Fahrenheit thermometer is used, and that the unit of mass of water is one pound.

21. What is meant by saying that the specific heat of iron is 0.11?

22. How does the great specific heat of water benefit human beings?

23. What is meant by saying that the latent heat of water is 80 units?

24. Explain the action of a freezing mixture.

25. Give an example of the latent heat of evaporation.

26. What is meant by the statement that the latent heat of steam is 537 units?

27. Why does your hand feel cool, if you dip it in warm water and then expose it to equally warm air?

28. How can water be made to boil at a temperature less than 100° C.?

29. State briefly in what ways convection, conduction, and radiation differ from one another.

30. Give an example of convection currents usefully applied.

31. How would you compare the conducting powers of two metals?

32. How would you show that mercury conducts heat better than water?

33. Two similar bars of copper and lead are coated with wax. One end of each bar is then exposed to the same source of heat (see Fig. 125). At first more wax is melted on the lead than on the copper; after some time more wax is melted on the copper than on the lead. Explain this.

34. Which is better for the back of a fireplace, brick or polished metal?

35. How do radiation, absorption, and reflection differ from one another?

36. Which will hold heat the longer, a polished kettle or a kettle covered with rust?

37. The glass of a greenhouse has been called "a trap to catch the sunbeams." Why?

38. What are the conditions that favor the formation of dew?

39. Define the dew point, and explain how you would find it by experiment.

40. Why does dew form copiously on grass, but scarcely at all on gravel?

41. Why does dew form more copiously on a clear night than on a cloudy night?

42. A knife with a black handle is left in the open air over night. Which part of the knife, the blade or the handle, will be covered with dew in the morning? Explain the reason.

43. Why does the mercury fall, if a thermometer is carried up into the air?

44. Explain the production of land breezes and of sea breezes.

CHAPTER IV.

MATTER.

Molecular Phenomena.

133. Molecular Theory of Matter. This theory of matter may be briefly summed up in four general propositions:

(1) Every body is an aggregate of very small parts, called *molecules,* that exist in the body as separate units.

(2) The molecules of a body are in rapid motion.

(3) The molecule is the smallest portion of a body that can retain and manifest the properties of the body.

(4) Molecules are capable of division into smaller parts, called *atoms;* but when this occurs the properties of the body are changed. A chemical change takes place.

Direct positive proof of these propositions is impossible. Molecules and atoms are too small to be seen, or handled, or recognized as separate units by any of the senses. But there are many phenomena of such a nature that we cannot understand how they can occur unless the molecular theory is true. No single phenomenon is conclusive, but taken together these phenomena form a chain of evidence that seems absolutely convincing.

The evidence is of two kinds, *physical* and *chemical.* Some of the physical evidence has already appeared; for example, the properties of compressibility and porosity, the equal transmission of fluid pressure, the phenomena of fusion, evaporation, and convection. We proceed to mention certain other phenomena due to the motion of the minute parts of a body, and hence called *molecular phenomena.*

134. Solution. If we put a piece of potassium permanganate (a solid of bright purple color), no larger than a pin-head, into a quart bottle full of water, and shake the bottle for a moment, the solid disappears and the water acquires a rich red color of uniform intensity.

We say that the water has *dissolved* the solid, and we infer that the solid has been broken up into extremely minute particles which are too small to be perceived.

The same sort of change occurs every time we put a lump of sugar into a cup of tea, or common salt into water.

If we keep on adding common salt to water, at last we find that the water refuses to dissolve any more salt. The limit is reached when 100 grams of water hold in solution about 37 grams of salt. We are now said to have a *saturated solution* of common salt in water. This solution has the taste of salt. If we heat some of it in a small dish, the water will evaporate and white grains of salt will remain behind.

The quantity of a solid that a given quantity of water will dissolve depends on the nature of the solid and on the temperature of the water. Saltpeter is a good example of the effect of change of temperature: 100 grams of water at 0° C. will dissolve 13 grams of saltpeter before saturation; at 20°, 31 grams; at 40°, 64 grams; at 60°, 111 grams; at 80°, 172 grams; and at 100°, 247 grams.

The solubility of a solid in water must be found by experiment; and as a general rule the solubility of a given solid *increases with the temperature.*

There are exceptions to the rule that solubility increases with the temperature. Lime, for instance, is more soluble in cold than in hot water. Common salt is about equally soluble at all temperatures.

A solid insoluble in one liquid may be soluble in another. Camphor is insoluble in water, but soluble in alcohol. Gum-arabic is soluble in water, but insoluble in alcohol. Varnishes are made by dissolving resins in alcohol, naphtha, or oil of turpentine.

135. Crystallization. When a saturated solution of a solid falls in temperature, or evaporates at the temperature of the surrounding air, the solid gradually separates from the liquid. If the separation is allowed to take place *undisturbed,* the solid usually appears in the form of small, symmetrical bodies with flat faces and straight edges. These small, symmetrical bodies are called *crystals.*

Illustrations. 1. If a saturated solution of common salt is put into a shallow vessel and allowed to evaporate quietly for several days, crystals in the form of small *cubes* gradually collect on the bottom and sides of the vessel. A magnifying glass shows their shape very plainly, and also shows that their faces and edges are perfectly formed only where the crystals are freely exposed to the liquid.

2. A *hot* saturated solution of saltpeter is put into a beaker glass and allowed to cool. Crystals form so rapidly that one can see them forming as fine points, growing larger, and then sinking. In this case the crystals are *prisms,* resembling that shown in Fig. 131, but usually much longer.

3. A solution of blue vitriol, when allowed to crystallize slowly, yields large blue crystals, called from their shape *rhombic prisms* (Fig. 132).

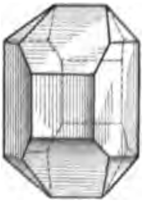


FIG. 131.

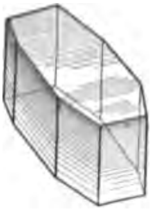


FIG. 132.

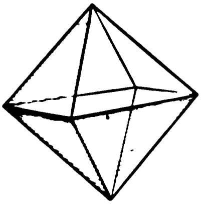


FIG. 133.

4. Hot saturated solutions of common alum (colorless) and chrome alum (dark purple) yield crystals of the same form, that known as the *octahedron* (Fig. 133). If threads are suspended in the solutions they become soon covered with crystals. If a crystal of chrome alum is suspended in a strong solution of common alum, it grows larger by the deposition of common alum upon it; it is possible to obtain in this way a pink octahedron enclosed by a colorless casing of common alum.

In all cases, the *slower* the process of crystallization the *larger* are the crystals which are obtained.

By countless experiments like those just described the following truths have been established:

1. Solids, when dissolved in liquids (or when melted), on returning to the solid state, *show a tendency to assume the form of crystals*, and always assume this form if they can.
2. Each kind of solid *has a crystalline form peculiar to itself*, and usually unlike those of other solids.
3. Crystals cannot form unless the particles of the solid *have perfect freedom of motion in the liquid.*

Large perfect crystals are never formed unless

(1) the liquid is kept in a state of perfect rest;
(2) the liquid cools or evaporates very slowly.

Even if both these conditions are fulfilled, the crystals, although large, are almost always imperfect in form. The reason may be gathered from the preceding illustrations. Perfect crystals can form only when they are surrounded by the liquid on all sides. But this very seldom is the case. Usually they are in contact with the sides of the vessel or with one another. In the first case, the surfaces in contact have no chance to develop; in the second case, the crystals grow into one another, so to speak.

The process of crystallization is constantly going on in nature. Most minerals are crystalline in form. They must originally have been in the liquid state, and have acquired a crystalline form by slow cooling and evaporation in past ages. The diamond is carbon crystallized under natural conditions, which we cannot successfully imitate in our laboratories. Snow and hoar frost consist of water solidified in various symmetrical crystalline forms, all of which are starlike in appearance; we often see them ornamenting a window pane on a frosty morning.

Uses of crystallization. (1) When the chemist wishes to obtain a substance in a perfectly pure state he tries to crystallize it; for only particles of the same kind can unite to form a crystal, nor can bits of dirt or other impurities enter into it.

The sugar refiner obtains pure sugar in the same way.

(2) Crystalline form is one of the characteristic marks by means of which the mineralogist is able to distinguish one mineral from another. Mere traces of a substance too minute to be otherwise detected are often recognized by examining the forms of their crystals under a powerful microscope.

136. Crystalline Structure. If we break a lump of alum and examine the fractured surface, we find that the particles of alum appear to be arranged in a definite way along certain lines and planes. It is true we cannot distinguish any well-defined crystals in the shape of the octahedron. But the lump splits up most readily along planes which can be proved to be always parallel to the faces of an octahedron; and under a microscope a small bit of well formed alum presents the appearance represented in Fig. 134. The fact is, a lump of alum forms under conditions that prevent the formation of good crystals. They try to form, but cannot succeed because they crowd against one another; the result is that they become matted together into a single mass. A solid built up in this way is said to have a *crystalline structure.*

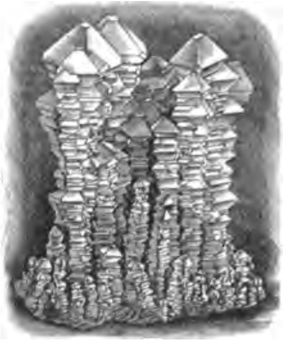


FIG. 134.

Mineralogists distinguish different kinds of crystalline structure, and give them special names, such as the following:

(1) *Columnar* (passing into *fibrous*). Alum, gypsum, asbestos.
(2) *Laminated* (splitting into thin leaves). Slate, mica.
(3) *Granular* (dividing into grains). Loaf sugar, marble.

When a solid appears to have no structure of any kind it is termed *amorphous* (without form). Glass, gum, albumen.

137. Absorption. Solids that are porous, or that have a fibrous structure, are capable of *absorbing* in greater or less degree both liquids and gases.

1. A piece of freshly burnt charcoal gains some 15% or 20% in weight after a few days by absorbing gases and moisture from the atmosphere. The presence of air in wood charcoal is shown by simply dropping a lump of it into a vessel of hot water; instantly a stream of air bubbles issues from the charcoal and rises to the surface of the water. The same effect in less degree is seen when a lump of alum or blue vitriol is placed in warm water.

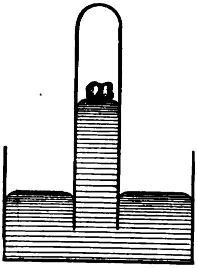


FIG. 135.

2. If a test tube is filled with dry ammonia gas by displacement over mercury, and a piece of charcoal, after being strongly heated in order to expel the air, is put inside the tube at its mouth, the mercury rises in the tube. This shows that the charcoal is absorbing the ammonia gas (Fig. 135).

3. When fibrous solids like wood absorb moisture they expand with great force. Granite can be split by drilling holes along the desired line of division and then plugging up the holes with dry wood. The wood is then moistened, or left exposed to damp weather. After a time the wood expands and splits the rock.

Liquids can absorb gases and hold them in solution, often in large quantities. Gases that are easily liquefied by pressure are most readily absorbed. The power of a liquid to absorb a gas increases as the pressure of the gas increases, but diminishes as the temperature increases.

Soda water and mineral waters are water charged under great pressure with carbonic acid gas. When the pressure is removed (as in drawing a glass of soda water from a fountain) the gas escapes, causing an effervescence and an agreeable taste.

Ordinary drinking water always contains air. When the water is heated, this air collects in bubbles and escapes (§ 104). If a test tube full of cold water is allowed to stand in a warm room, its sides become covered with bubbles of air which separate from the water.

138. Liquid Diffusion. If we pour a strong solution of blue vitriol into a glass jar, we may fill the jar by gently pouring water down a tube upon a floating cork, and yet not disturb the solution. If the jar is then left undisturbed for some weeks, the blue color will rise gradually into the upper part of the jar and become gradually weaker in the lower part. Finally the color will become sensibly uniform throughout the contents of the jar.

This process of intermixture is called *diffusion.*

If instead of blue vitriol we use a solution of common salt, we find that the liquid at the top of the jar gradually acquires the well known taste of salt.

Water and alcohol behave in a similar way when placed in contact and left to their own action: the two liquids gradually intermix until the composition of the whole is the same in every part.

Consider the conditions under which the process occurs. No external agency such as stirring is applied. Convection currents are impossible, for the temperature is everywhere the same. The mixture takes place in opposition to the force of gravity; for the heavier liquid moves upward through the lighter, and the lighter liquid moves downward into the heavier. Forces are certainly at work in the liquid, and motion of some kind is going on. But with our most powerful microscopes, we cannot see anything that is moving.

The only way to explain the phenomenon is to assume that the liquids are composed of exceedingly small parts (molecules) which exert mutual action upon one another and are in motion.

The diffusion of liquids has been carefully studied by Professor Graham, of London, and by other investigators. One of the general facts established is that the rate of diffusion increases with the temperature. The application of heat makes the molecules of a liquid move more rapidly than before.

139. Osmosis. Two liquids which will diffuse into each other will also intermix when separated by a porous wall (*e.g.*, a wall of unglazed clay) or by a membrane (*e.g.*, bladder or parchment paper). This phenomenon is called *osmosis.*

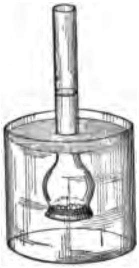


FIG. 136.

Close the wide end of a thistle tube (Fig. 136) by tying wet parchment paper across it; pour into the tube a saturated solution of blue vitriol, and then suspend it in a vessel of water so that both liquids shall stand at the same level. The liquid column in the tube will slowly ascend, while the water in the vessel will gradually acquire a blue tint. This shows not only that the two liquids are traveling in opposite directions through the membrane, but also that the water travels faster than the liquid in the tube. The rise of the liquid up the tube takes place contrary to the action of gravity.

In osmosis the two liquids, as a rule, travel into each other through the partition walls *at unequal rates;* so that the volume of liquid on one side of the wall increases and the volume on the other side diminishes. This change of volume occurs although the force of gravity is opposing it.

Substances that diffuse rapidly through an organic membrane are mostly crystalline in structure, and are called *crystalloids.* Substances that diffuse slowly or not at all are amorphous, or glue-like, and are called *colloids.* By taking advantage of this difference in diffusibility, crystalloids can be separated from colloids. The process is called *dialysis.*

Dialysis is often employed for the purpose of testing the contents of a stomach for a crystalloid poison, like arsenic or strychnine. A shallow tray, called the *dialyzer*, having for a bottom a sheet of parchment paper or an animal membrane, is floated on water. The contents of the stomach are then placed in the tray. The crystalloid poison passes into the water, and can then be easily detected by chemical tests. The colloidal matter remains in the tray.

Osmosis plays a very important part in the circulation of blood in animals and sap in plants.

140. Gaseous Diffusion. The power of gases to intermix, even in opposition to gravity, is very remarkable. When two gases are placed in contact they begin at once to diffuse into each other, and the rate of diffusion is much more rapid than in the case of liquids.

Illustrations. 1. When a bottle of ammonia water (water holding ammonia gas in solution) is opened in a room where the air is still and the temperature constant, the smell of the gas quickly pervades every part of the room, finally becoming equally strong in every part.

2. If a lighted match is thrust into a jar full of carbon dioxide it immediately goes out. But if a lighted match is put in after the jar has been standing one or two hours, it continues to burn. The gas (which is 1½ times as heavy as air) has left the jar, and air has taken its place.

3. A mixture of hydrogen gas and air explodes when a lighted match is applied; the hydrogen unites with the oxygen of the air to form water. If we fill an inverted bottle with hydrogen (which is the lightest of all gases), and after a few minutes apply a match, a violent explosion occurs, showing that air, in spite of its greater weight, has ascended into the bottle and mixed with the hydrogen.

4. Other examples are the diffusion of perfume in a room, the ventilation of a room by opening a window on a calm day, the discovery of a leak in a drain pipe by the use of oil of peppermint vapor. The spreading of carbon dioxide and aqueous vapor in the atmosphere is due partly to diffusion, but mostly to air currents.

All these phenomena seem natural enough if we picture to ourselves a gas as composed of a very great number of molecules capable of moving freely in obedience to their mutual action; otherwise they cannot be accounted for.

There are other phenomena manifested by gases which, though unlike simple diffusion in certain respects, indicate molecular constitution very plainly. Thus, gases pass through porous walls and membranes with great ease (osmosis); and light gases effect the passage more rapidly than heavier ones. A rubber balloon filled with hydrogen quickly collapses. Hydrogen passes freely through thin sheets of very hot metal. Carbon monoxide, a very poisonous gas, will pass through red-hot iron; it is often formed in a stove when the supply of air is imperfect, and its passage through the lining of the stove into the room is very dangerous to persons in the room.

141. Molecular Motion. We have seen that molecules, if they exist, must under certain conditions be in a state of motion. The molecular theory claims that the molecules of every body are always in a state of very rapid motion. The evidence in favor of this claim involves an inquiry into the nature of *heat*, and will be presented in Chapter VI.

CLASS-ROOM EXERCISES.

1. Why does stirring hasten the solution of a solid in water?

2. What would you do in order to make a pint of water dissolve as much of a solid as possible? And what in order to make the water dissolve the solid as quickly as possible?

3. Why does a solid dissolve more readily in the form of powder than in the form of a solid lump?

4. Does the fact that we can thrust our finger into water, or drive a nail into wood, conflict with the statement that matter is impenetrable?

5. What is the reason that water will not run out of a tightly fitting funnel into an empty bottle?

6. Why is it necessary, in making good barometers, to boil the mercury contained in the tube?

7. Doors that open freely in winter are sometimes found to stick in summer. What is the explanation?

8. When a vessel has sunk to the bottom of the sea and been there for some time, the wooden parts when set free will not rise to the surface. What is the explanation?

9. How do you explain the crackling of wood when it burns?

10. How do you explain the coloring of a meerschaum pipe?

11. What in general is the effect of heat on the capacity of water (1) to dissolve a solid, (2) to dissolve a gas?

12. How would you separate common salt and sand if they were mixed together?

13. Persons who camp out in the woods sometimes find their supply of salt mixed with dirt. How can they free it from the dirt?

14. Mention facts or phenomena which show that water is capable of holding gases in solution.

15. Which will diffuse the more rapidly through a membrane by osmotic action, common salt or gelatine?

Molecular Forces.

142. Cohesion. The mutual attraction of the molecules of a body for one another is called *cohesion.*

The effects of cohesion are seen in solids and liquids, and in solids far more strongly than in liquids.

Cohesion holds together the molecules of a solid in fixed relative positions, thus giving to the solid a definite shape; whereas exceedingly slight forces are sufficient to overcome the cohesion of liquid molecules and set them in motion.

The following are some general facts about cohesion:

(1) Cohesion acts only through extremely small distances.

(2) Cohesion varies enormously for different solids.

(3) In solids of crystalline or fibrous structure cohesion is much greater in some directions than in others.

(4) Cohesion diminishes as the temperature of a body increases.

We infer (1) to be true because, if we cut almost any body in two and then press the parts together just as they were before, they will not unite. But two bright, smooth surfaces of lead, if firmly pressed together, will cohere with considerable force, and two smooth plates of glass or steel will cling together with such tenacity that it is not easy to separate them.

If pure powdered graphite is subjected to enormous pressure in a hydraulic press, it is converted into a coherent mass, which can be sawed into strips and used for lead pencils.

Wood furnishes a good example of the truth of (3); it has much greater strength in a direction parallel to the fibers than in a direction perpendicular to them.

The effect of heat on cohesion is well shown when we hold by the ends a piece of glass tubing over a hot flame. Very soon the glass softens and can be pulled apart with the greatest ease. If we now gently press together the soft, hot ends in the flame and then remove them from the flame, they reunite as firmly as before. The heat causes the molecules of the glass to intermingle so closely that, on cooling, the force of cohesion again comes into full play. In the process of *welding* two pieces of white-hot iron together a similar phenomenon occurs.

143. Properties Depending on Cohesion. By virtue of cohesion every solid has more or less *rigidity*, or power to resist stress of any kind. But no solid is perfectly rigid; all solids yield somewhat to stress, or can be *strained*. If the strain disappears entirely when the stress is removed, the solid is said to be perfectly *elastic* (§ 24). If, however, the solid undergoes a permanent alteration of form, it is said to manifest the property of *plasticity*. If the stress is increased far enough, the solid breaks.

In the case of building materials, the tensile stress that will break a bar of any material one square inch in cross section is called the *breaking strength* or tenacity of that material.

A solid is said to be *brittle*, if it breaks before showing any plasticity at all; *tough*, if it combines great plasticity with great strength; *malleable*, if it shows great plasticity under the blows of a hammer; *ductile*, if it shows great plasticity when drawn into wire through the holes of a drawplate.

The relative *hardness* of two solids (for example, two minerals) is tested by seeing which will indent or scratch the other when they are rubbed together.

We find great strength and perfect elasticity combined in steel better than in any other solid. The breaking strength of the best steel wire is over 100 tons; that of good bar steel varies from 30 to 60 tons; that of wrought iron from 15 to 35 tons; that of cast iron from 5 to 10 tons.

The strength of a metal is increased by such processes as rolling, hammering, and wire-drawing.

Slow cooling from a red heat makes a solid less brittle, softer, and yet tougher. Glass vessels would be too brittle for ordinary use, if they were not *annealed* (as it is termed) by passing them slowly through a long chamber, hot at one end and gradually growing cooler towards the other end.

Sudden cooling from a red heat makes a solid harder and more brittle. Edge tools are made in the proper shape while the steel is soft, then heated red hot and suddenly cooled, then *tempered* by heating to the proper shade of color and cooling slowly.

144. Cohesion in Liquids. The formation of drops proves that cohesion exists in liquids. Closer examination shows that the sensible effects of cohesion are confined practically to the *surface* of the liquid. The following are some of the phenomena which lead to this conclusion :

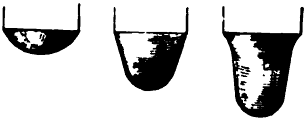


FIG. 137.

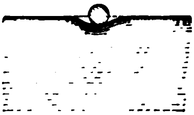


FIG. 138.

1. If we watch a drop of water, or still better a drop of oil, as it forms, we observe that before becoming heavy enough to fall, it elongates in a way that suggests an elastic bag made of thin rubber and filled with water. The film of liquid on the outside of the drop behaves like a stretched membrane that holds together by virtue of cohesion up to a certain limit, and then breaks.

2. A needle can be made to float by placing it carefully on the surface of water. On examining the surface around the needle we find a depression, indicating that the surface is like a delicate elastic membrane which is stretched by the weight of the needle, but supports it by the cohesion of its molecules. If the needle breaks through this liquid skin, it instantly falls to the bottom of the vessel.

3. Fill with water a glass tumbler with dry edges. Then drop carefully into the water a number of coins. In this way the water can be raised above the edge of the tumbler as much as an eighth of an inch without running over. All around the edge the water rises in a smoothly curved surface which must be under tension, for the water presses upon it, but which is kept by cohesion from tearing open.

In short, the superficial film of a liquid is in a condition similar to that of a sheet of rubber stretched over a sphere ; it exerts pressure on the liquid underneath, and is also in a state of tension, but is kept entire by the cohesion of its molecules. It can be proved that this condition must exist if liquids are composed of molecules attracting one another, but the proof cannot be given here.

145. Surface Tension. This term is used to denote the tension of the superficial film of a liquid. Surface tension varies in amount for different liquids, and diminishes as the temperature of the liquid rises. It is greater for pure water than for any other liquid except mercury.

Surface tension enables us to understand some very curious phenomena, of which the following are illustrations:

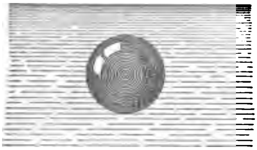


FIG. 139.

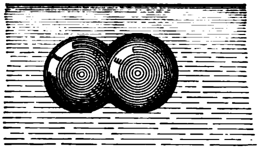


FIG. 140.

1. If we mix water and alcohol in such proportions that the mixture has the same density as olive oil, and then carefully introduce into the mixture a little oil, we obtain a body perfectly free to obey molecular forces. Under these circumstances the oil assumes the form of a sphere (Fig. 139). If two spheres of oil are brought in contact, they unite instantly and form a larger sphere. The reason is that the elastic skin of the oil tries to contract, and the sphere is that solid which for a given volume has the least surface.

2. Put a thin layer of water on glass, and let a drop of alcohol fall on it. Owing to the weaker tension of the alcohol, the water will draw away from it in all directions, leaving it surrounded by a dry area. For the same reason, a bit of wood floating on water, if met on one side with alcohol, will move off in the opposite direction with great velocity.

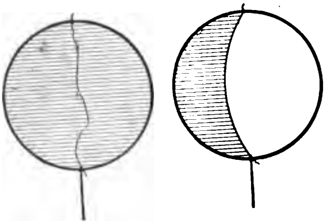


FIG. 141.

3. Soap bubbles and films afford many very interesting illustrations of surface tension. In Fig. 141 a ring of iron wire is shown, covered with a soapy film, on which lies a silk thread tied to the ring as shown in the figure. If the film on one side of the thread is broken (by means of a hot needle), the thread is instantly drawn to the other side so as to form an arc of a circle.

146. Adhesion. The attraction between the molecules of two different substances is called *adhesion.* It may act between (1) two solids, (2) a solid and a gas, (3) a solid and a liquid, (4) two liquids, (5) a liquid and a gas, (6) two gases. The last three cases are illustrated by the processes of diffusion and absorption already considered. It remains to notice briefly some phenomena that fall under cases (1), (2), and (3).

(1) *Two solids.* Familiar examples of adhesion acting between solids are: chalk sticking to a blackboard, the marks of a lead pencil on paper, dust adhering to a ceiling in opposition to gravity, glue holding together two broken surfaces.

(2) *Solids and gases.* The layer of air in contact with the surface of a solid usually adheres so firmly that it cannot be removed except by heating the solid, or rubbing it with a liquid or with a porous substance like charcoal.

When water is heated, very soon we see air bubbles separate from the water and collect on the sides of the vessel. By applying more heat these bubbles are finally dislodged, and rise to the surface of the water.

(3) *A solid and a liquid.* A glass rod, if dipped into water and withdrawn, is found to be *wetted* by the water; if dipped into mercury and withdrawn, it is not wetted. In the first case adhesion between the solid and the liquid overpowers cohesion in the liquid; in the second case the reverse is true.

A liquid which wets the sides of the containing vessel is *raised* near the sides, and its surface is *concave* (Fig. 142). But a liquid which does not wet the sides is depressed near the sides, and its surface is *convex* (Fig. 143).

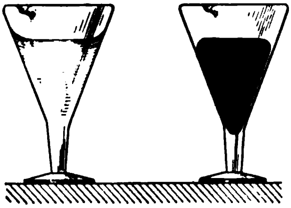


FIG. 142. FIG. 143.

If adhesion between the solid and the liquid overpowers cohesion in the *solid,* then the solid is *dissolved* in the liquid.

147. Capillary Action. Tubes of fine bore are called *capillary* tubes (from the Latin word *capillus*, a "hair").

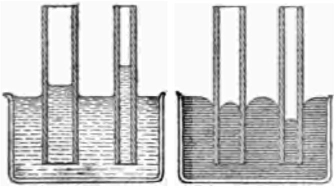


FIG. 144.

If a glass capillary tube is dipped into water, the water within the tube rises above the general level outside, and its free surface is *concave.* If the tube is dipped into mercury, the mercury within the tube is depressed below the general level, and its free surface is convex (Fig. 144). The smaller the tube the greater is the elevation or depression of the liquid.

These phenomena are called *capillary* phenomena, and they are found to obey the following laws:

1. *A liquid in a capillary tube is raised or depressed according as it wets or does not wet the tube.*
2. *The elevation or the depression varies inversely as the diameter of the tube.*
3. *The elevation or the depression diminishes as the temperature increases.*

Capillary phenomena are caused by the combined action of cohesion in the liquid and adhesion between the liquid and the tube. If the liquid wets the tube, the surface tension pulls the liquid up the tube till the weight of the raised liquid balances the upward pull. If the liquid does not wet the tube, the surface tension exerts a downward pull, and causes a depression.

Capillary action explains why water rises in a piece of cloth or a lump of sugar that is touching water, why oil rises in a lamp wick, and why blotting paper absorbs ink. It also explains why bits of floating wood rush together when they get near enough; a liquid column which is exerting less than atmospheric pressure is raised between them, and the pressure of the air on their outer sides drives them together.

148. Viscosity. If we stir water in a tumbler with a spoon round and round till it is in a state of rapid rotation, and then leave it to itself, it soon comes to rest. Evidently the water molecules, as they glide past one another, meet with a kind of resistance analogous to friction, which opposes the motion and finally stops it altogether. This kind of molecular friction is called *viscosity.*

All liquids have more or less viscosity, but they differ greatly from one another in the amount which they possess. Honey and molasses, for example, flow very sluggishly; they are very viscous liquids. Water is much less viscous than honey, alcohol less than water, and ether less than alcohol. Such liquids as alcohol and ether, from the ease with which they flow, are called *mobile* liquids.

Gases also possess viscosity. When a stream of gas flows along an iron pipe, its progress is impeded by the action of the sides of the pipe; when it flows through air, its motion is retarded by contact with the air, and by the intermixing of the rapidly moving molecules of the gas and the slowly moving molecules of the air.

Heat diminishes viscosity and cold increases it. Thus, very cold molasses will hardly flow at all; and if poured into a flat dish, it will form a heap which flattens out very slowly; while hot molasses will run almost as freely as water, and spread over the dish almost as fast.

The boundary line between solids and liquids is the same as that between plasticity and viscosity, and is drawn by considering not only the *amount* of the force applied but also its *time of action.* We may illustrate this point by comparing beeswax with pitch. Beeswax will retain its form for any length of time unless a *sufficient force* acts upon it, and then the form changes at once. Therefore, beeswax is a soft or plastic solid. Pitch is much harder than beeswax, but it will suffer a change of form under the action of the very smallest force, if *sufficient time* is allowed; thus, a lump of pitch, if placed on a table, will slowly flatten out under its own weight; and, if put into a funnel, it will in time flow down through the funnel in a slow, continuous stream. Therefore, pitch is a liquid though a very viscous one.

LABORATORY EXERCISES.

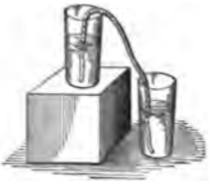


FIG. 145.

1. Find the breaking strength of brass wire, using size No. 30, diameter 0.255 cm., Brown & Sharpe's gauge, and a 24-lb. spring balance.

2. Place two tumblers, one higher than the other. Connect them by a cotton wick. Pour some water into the upper glass. What will happen? Write out an explanation of this effect.

3. Experiments with soap bubbles and films.

A good mixture for soap bubbles is as follows: Fill a clean stoppered bottle three quarters full of water. Add of oleate of soda one fortieth of the weight of the water. Wait 24 hours for the soda to dissolve, then fill up the bottle with Price's glycerine and shake well. Leave the bottle stoppered for one week in a dark place. Then siphon away the clear liquid from the scum which will have collected on top. Add one or two drops of strong liquid ammonia to every pint of the liquid. Keep the liquid in a stoppered bottle in a dark place. Do not warm or filter the liquid while making it, and do not allow the liquid to be exposed to the air more than is necessary. (See C. V. Boys on "Soap Bubbles," p. 142.)

4. Experiments on capillary elevation and depression in glass tubes of different diameters.

CLASS-ROOM EXERCISES.

1. How is the tenacity or breaking strength of a substance measured? Name a substance having very great tenacity.

2. Explain the meaning of the terms: ductile, malleable, brittle.

3. Describe annealing and tempering. What purpose do they serve?

4. Explain the rise of oil in the wick of a lamp.

5. Explain why two pieces of wood floating near each other in water are attracted to each other.

6. Explain the action of blotting paper in removing ink spots.

7. Give examples of capillary phenomena.

8. Two capillary tubes dip into water. The diameter of one is half that of the other. The water rises in the larger tube 1 cm. How far will it rise in the smaller tube?

9. Which of the following liquids are viscous and which mobile: castor oil, water, glycerine, ether, alcohol, molasses, tar?

Chemical Changes.

149. Mechanical Mixture. If we mix together table salt and sugar by grinding them up in a mortar, we find that the mixture has the saline taste of salt and also the sweet taste of sugar. Each ingredient retains its own properties in the mixture. We may vary the proportions of salt and sugar as much as we please. If we use much salt and little sugar, the saline taste will predominate; if we use much sugar and little salt, the sweet taste will be especially strong.

A mixture of two or more substances, in which each substance retains its own properties, and in which the weights of the substances may be varied in any proportion, is called a *mechanical mixture.*

A solution of salt in water is another example of a mechanical mixture; only in this case the mixture is on a much finer scale of subdivision than in the case of two solids. In fact, there is good reason to believe that in this case the molecules of the solid become uniformly intermixed with those of the liquid. A solution of sugar in water is another example of the same kind.

By means of water or other liquids the ingredients of a mixture may often be separated from one another. Thus, if we throw a mixture of sand, salt, and sawdust into water, the sand will sink, the sawdust float, and the salt dissolve.

Gunpowder is a mixture of sulphur, charcoal, and saltpeter (niter). These are first ground separately, then moistened with water and ground together for several hours to mix them as intimately as possible. The mixture is then subjected to great pressure and finally broken up into grains. But each grain still remains a mechanical mixture of the three substances. The saltpeter may be washed out with water, the sulphur dissolved out of the remainder by adding a liquid called carbon disulphide, and the charcoal will be left. If we evaporate the water, the saltpeter will be recovered unchanged; and if we allow the disulphide solution to volatilize or escape as vapor, the sulphur will remain behind.

150. Chemical Combination. If we mix fine iron filings with powdered sulphur, we obtain a dark gray powder. Under a magnifying glass we can see the particles of iron lying side by side with those of sulphur. If we stir some of the mixture in water, the lighter particles of sulphur will collect above the heavier particles of iron, and a partial separation takes place. If we stir the mixture with a magnet, the iron particles will stick to the magnet, but those of sulphur will not. Evidently we have a mechanical mixture of iron and sulphur. But now let us put into a test tube a mixture of 21 grams of iron filings and 12 grams of powdered sulphur, and heat the tube over the flame of a Bunsen lamp. As soon as the mass turns red, remove the lamp. The glowing continues for some time. Something is evidently taking place in the mixture. If we break the tube after it has cooled, we find a compact mass of matter which weighs exactly 33 grams. If we break up this mass and examine it through a glass, we can no longer distinguish the particles of iron from those of sulphur; neither can we separate them by the use of water or a magnet. A *new substance* has been formed which contains, indeed, both iron and sulphur, but which has a set of properties peculiar to itself, unlike those of either iron or sulphur. We say that the iron and the sulphur have *entered into chemical union or combination.* The name of the new substance is *iron sulphide.*

Mix some powdered sulphur with twice its weight of powdered zinc (zinc dust), put the mixture on a brick, and apply a lighted match; there is a flash of light, and in place of the mixture we have a grayish powder formed by the union of the zinc and the sulphur, called *zinc sulphide.*

Mix a little powdered sulphur with six times its weight of mercury, put the mixture into a test tube, and apply heat; a vapor rises from the mixture and condenses in the solid state in the cooler parts of the tube. If we break the tube and collect this solid, we obtain a bright red powder. It is *mercury sulphide* (vermilion), formed by the union of the mercury with the sulphur.

Caution: Avoid inhaling the vapor of the mercury sulphide.

151. Chemical Separation. When an electric current is allowed to pass through water acidified with sulphuric acid, a remarkable change occurs. In Fig. 146 the water is contained in a small vessel closed below by a wide cork, through which pass the wires from the battery. The wires terminate in small platinum plates, called *electrodes*, placed a little distance apart. Over the electrodes stand two test tubes, each full of the acidified water. When the current is turned on, bubbles of gas begin to form upon each electrode and to ascend to the top of the tube. The tubes gradually fill with the gases. It will be noticed that the volume of gas in one tube is *twice as great* as that in the other tube; as soon as the first tube is full of gas remove it, and apply quickly to its mouth a lighted match; a mild report is heard and a pale blue flame is seen, showing that the gas in the tube is burning. This gas is called *hydrogen.*

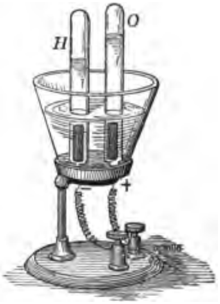


FIG. 146.

When the other tube is treated in the same way, the gas in it will not burn, but it gives intense brilliancy to the flame of the match. This gas is called *oxygen.*

In this experiment water has been *analyzed* or *decomposed,* and we have obtained from it two colorless gases, oxygen and hydrogen, and in the exact proportion of one part by volume of oxygen to two parts by volume of hydrogen. The two gases have properties quite unlike those of water, and the properties of one are quite different from those of the other.

If 1 volume of oxygen and 2 volumes of hydrogen are placed in a dry vessel and an electric spark applied, the two gases unite with a loud report, and the sides of the vessel are found to be covered with moisture. This experiment confirms the result of the analysis of water, and proves that water is a compound of oxygen and hydrogen alone.

As another example of chemical decomposition, heat a few grains of red mercury oxide in a tube of hard glass, from which a delivery tube leads to a glass jar full of water and inverted in a pneumatic trough (Fig. 147). On strongly heating the tube, gas begins to collect in the jar, and at the same time globules of mercury begin to collect on the upper cooler parts of the tube. When the jar is full of gas, blow out a lighted candle, and hold the still glowing wick at the mouth of the jar. The wick bursts into flame. The gas is oxygen gas. The mercury oxide has been decomposed by the heat into mercury, which volatilizes and then condenses on the cooler parts of the tube, and oxygen gas, which collects in the glass jar.

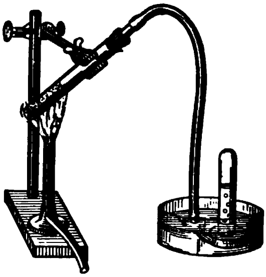


FIG. 147.

152. Elements and Compounds. Water has been decomposed into oxygen and hydrogen gases, but here the process of analysis comes to an end. No method yet tried has been successful in breaking up either oxygen or hydrogen into other kinds of matter. They are therefore called *simple* substances or *elements.* On the other hand, every substance that can be decomposed into two or more substances is called a *compound* substance or a *compound.*

At the present time 72 substances are recognized as elements. The number of compounds is innumerable.

All the metals are elements. Some of them are not in common use as elements, but form with other elements very important compounds. Potassium, sodium, and calcium are examples. Common salt, for instance, is sodium chloride.

About 50% of the solid crust of the earth consists of oxygen (in combination with other elements), 25% is silicon, and 10% aluminium. The water on the earth consists eight ninths of oxygen and one ninth of hydrogen. Somewhat more than one fifth of the atmosphere is oxygen, and nearly all the remainder is nitrogen.

The four principal elements that enter into the structure of living things are oxygen, hydrogen, nitrogen, and carbon.

153. Chemical Exchange. The most common kind of chemical change is neither simple combination nor simple separation, but the simultaneous occurrence of both. We bring into intimate contact (usually by means of solution) two substances, A and B, and apply heat if necessary. Then a certain element in A *changes places* with a certain element in B. The result is that both A and B lose their properties, and new substances having new properties appear instead. Some illustrations will make this clear.

Illustrations. 1. Put into one test tube a clear solution of copper sulphate (blue) and into another a clear solution of barium chloride (colorless). Slowly pour the contents of the first tube into the second. Instantly a dense white solid is formed in the midst of the liquid. A solid which is formed under these conditions is called by chemists a *precipitate.* Here the precipitate is barium sulphate. The chemical change consists in the copper of the copper sulphate changing places with the barium of the barium chloride. The copper sulphate is thus transformed into barium sulphate, which is a white, insoluble substance. The barium chloride is changed to copper chloride, which is soluble and remains in solution; by filtering and evaporating the liquid, the copper chloride can be obtained in a solid state.

2. If we perform a similar experiment, using as substances potassium chromate (yellow) and lead acetate (white), a bright yellow precipitate of lead chromate is formed. In this case the potassium and the lead change places. The potassium acetate which is also formed is soluble, and therefore is not seen.

3. Zinc is an element, and sulphuric acid is a compound containing three elements: hydrogen, oxygen, and sulphur. When a strip of zinc is dipped into dilute sulphuric acid, bubbles of gas are given off in great numbers and the zinc slowly wastes away. What really happens is that the zinc displaces the hydrogen from the acid, and enters into chemical union with the oxygen and sulphur to form zinc sulphate, which being soluble remains in solution.

4. Tie some pieces of granulated zinc to a thread and suspend them in a solution of lead acetate. Very soon the zinc becomes covered with a brilliant coating of metallic lead. This "lead tree," as it is called, continues to grow until all the lead in the lead acetate has been deposited. Lead acetate and zinc have disappeared, and in their places we have zinc acetate and lead.

154. Laws of Chemical Combination. All chemical changes are found by experiment to obey the following general laws:

1. *The sum of the weights of the substances before the change is exactly equal to the sum of the weights of the new substances after the change.*

This is the Law of the *Indestructibility of Matter.* Matter is neither created nor destroyed by a chemical change.

2. *When two elements unite with each other, they always unite in a fixed definite ratio by weight.*

This is called the Law of *Definite Proportions by Weight.* Thus (§ 150) 21 grams of iron unite with just 12 grams of sulphur; that is, in this case the ratio is 7:4. If we try to make 8 grams of iron unite with 4 of sulphur we shall have 1 gram of iron left in a free state; and if we try to make 7 grams of iron unite with 5 of sulphur we shall have 1 gram of sulphur left in a free state.

3. *If two elements unite to form more than one compound, the weights of one which unite with a given weight of the other are proportional to the whole numbers 1, 2, 3, etc.*

This is the Law of *Multiple Proportions.* Take as an example carbon and oxygen. In carbon monoxide 12 parts of carbon are united to 16 parts of oxygen. In carbon dioxide 12 parts of carbon are united to 32 parts of oxygen, or just double as much as in carbon monoxide.

4. *Two gaseous elements always unite in a simple ratio by volume, and the volume of the compound is always equal to twice that of the unit of volume employed, if all the volumes are estimated at the same pressure and temperature.*

This is called the Law of *Definite Proportions by Volume.* It is illustrated by the analysis of water (§ 151). It appears that 1 volume of oxygen unites with just 2 volumes of hydrogen to form water. Moreover, if the oxygen and hydrogen unite under such conditions that the resulting water is in the state of vapor, having the same temperature and pressure as those of the combining gases, it is found to be exactly equal to 2 volumes, or to twice the volume of the oxygen gas.

155. The Atomic Theory. Each element is assumed to consist of indivisible particles called *atoms.* The atoms of the same element have the same weight, but the atoms of different elements have different weights. Every chemical change is a re-arrangement of atoms. When one or more atoms of one element unite with one or more atoms of another element, a *molecule* of a new substance is formed. This act, repeated many millions of times, produces a sensible amount of a new substance.

Since only entire atoms can unite to form a molecule, this theory gives a simple explanation of the laws of definite and multiple proportions.

Thus, carbon and oxygen are united in the ratio by weight of 12 to 16 in carbon monoxide, and in the ratio of 12 to 32 in carbon dioxide. If we assume that the monoxide molecule contains one atom of each element, then the dioxide molecule must contain one atom of carbon and two atoms of oxygen.

Hydrogen is the lightest of all the elements. Assuming the weight of the hydrogen atom to be 1, chemists have found numbers that express with a high degree of probability the *relative* weights of the atoms of the other elements. These numbers are called the *atomic weights* of the elements.

156. Chemical Symbols. The first letter (or the first two letters) of the English or the Latin name of an element is employed to represent one atom of the element. These letters are called *chemical symbols.* The following are examples, with the atomic weights added:

Hydrogen, H,	1	Carbon, C,	12	Zinc, Zn,	65	Silver, Ag,	108
Oxygen, O,	16	Sulphur, S,	32	Copper, Cu,	63.2	Potassium, K,	39
Nitrogen, N,	14	Iron, Fe,	56	Lead, Pb,	208	Sodium, Na,	23

A molecule of a compound is represented by writing one after another the symbols of the elements that it contains. Subscript figures show the number of atoms of each element.

Thus, H_2O stands for one molecule of water, FeS for one molecule of iron sulphide, CO_2 for one molecule of carbon dioxide, H_2SO_4 for one molecule of sulphuric acid.

157. Composition of Air. Lavoisier, a French chemist, performed in 1777 an experiment which proved that the air we breathe is not a simple substance, but contains two perfectly distinct gases, oxygen and nitrogen.

He put some mercury into a glass retort, so arranged that the air above the mercury communicated with the air in a receiver which was inverted over a vessel of mercury (Fig. 148). He then kept this mercury heated nearly to the boiling point for 12 days. After a time red specks began to appear on the surface of the mercury, and increased in number for several days, but at last ceased to form. After the apparatus had cooled he noticed that the volume of the confined air, which at first was about 820 ccm., had diminished to 672 ccm., and he also found that this residual air had lost all the chief characteristics of ordinary air. A lighted match plunged in it was at once extinguished, and a mouse placed in it soon died. This gas was nitrogen.

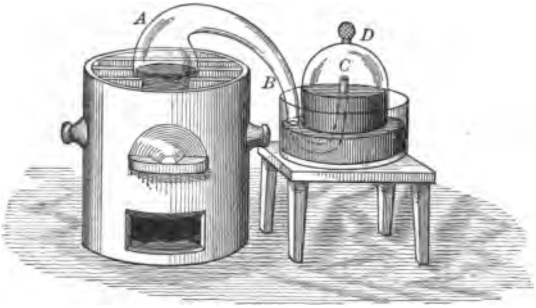


FIG. 148.

On heating the red film that had formed on the mercury, he obtained metallic mercury, and 148 ccm. of a gas that possessed in an exaggerated degree all the properties which the air had lost. This gas was oxygen.

Air is not a chemical compound, but a mechanical mixture of oxygen and nitrogen in the ratio of 23% oxygen to 77% nitrogen by weight, and 21% to 79% by volume.

Air also contains, on the average, 4 parts of carbon dioxide in 10,000, and water vapor from a mere trace to 3 per cent.

158. Chemical Combination and Heat. Chemical combination is usually attended with an evolution of heat. Very often the act of combining will not begin till the temperature is raised to a certain point; but when it does begin the action itself in some way generates heat, and sometimes in very large quantities.

On the other hand, chemical separation is usually accompanied with a loss of heat; in other words, heat must be expended in order to cause a chemical separation.

Thus, the heat generated by the union of iron with sulphur (§ 150) keeps the mass red-hot as long as the action lasts, while in order to break up mercury oxide into mercury and oxygen heat must be continually applied to the compound (§ 151).

In chemical exchanges (§ 153) there is both absorption and evolution of heat; hence, in many cases the general change of temperature is small.

The combination of oxygen with another element is called *oxidation*, and the compound formed by the union is called an *oxide*.

Oxidation attended with a decided evolution of heat and light is generally called *burning* or *combustion*.

The combustion of the two elements hydrogen and carbon far outweighs in importance all other cases put together. The various substances which we employ for the artificial production of heat and light, such as wood, coal, petroleum, and coal gas, consist largely of hydrogen and carbon, but more especially carbon. In the act of burning the substance breaks up into its elements; the hydrogen and the carbon then unite with oxygen from the air, the hydrogen forming water vapor and the carbon forming carbon dioxide.

If a dry test tube is inverted over a candle flame, moisture soon collects on the inside; this proves that water vapor is being formed. If the gases from the flame are conducted into lime water (a solution of slaked lime in water), the latter soon becomes opaque and milky. This is because carbon dioxide from the flame unites with the lime to form an insoluble compound, calcium carbonate.

159. Respiration. By the act of breathing or *respiration* the oxygen of the air enters the lungs, and passes by diffusion through the air cells into the blood vessels, whence it is carried by the blood stream through the arteries to every part of the body. It is thus, through chemical changes, brought into contact with carbon contained in the food we eat. Oxidation takes place, resulting in the production of carbon dioxide, and maintaining by the evolution of heat the warmth of the body. The carbon dioxide thus formed is carried back to the lungs through the veins and exhaled into the atmosphere.

The change in the color of the blood from bright red in the arteries to dark purple in the veins is brought about by the chemical changes, principally oxidation, which occur in the tissues of the body.

Animal heat, therefore, is kept up by the slow oxidation of a portion of the food that we eat, and respiration is the process by which the necessary oxygen is supplied and the resulting carbon dioxide removed.

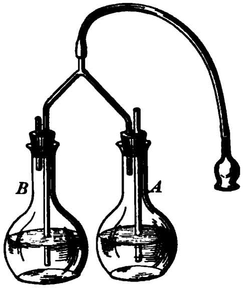


FIG. 149.

The difference between inhaled and exhaled air may be illustrated by arranging two flasks, each half full of lime water, as shown in Fig. 149. The flasks are closed by rubber stoppers, through which pass two tubes, one very short, the other extending below the surface of the liquid; and a T tube is joined by one branch to the short tube of flask *A* and by another branch to the long tube of flask *B*. The third branch is connected with the mouth. Under these circumstances, the air inhaled is obliged to pass through the lime water in flask *A*, and the exhaled air through the lime water in flask *B*. It is found that the air which enters the lungs contains so little carbon dioxide that the lime water in flask *A* suffers no change, while the air expelled from the lungs soon causes the lime water in flask *B* to become opaque, owing to the formation of calcium carbonate.

160. Reduction. The removal of oxygen from a compound that contains it is called *reduction.* Heat is necessary to effect reduction; and usually some element or compound, called a *reducing agent,* must be supplied for the oxygen to unite with as soon as it is set free. Carbon and hydrogen are powerful reducing agents.

Illustrations. 1. If a mixture of lead oxide and sawdust is heated to full redness in a porcelain crucible, first the wood chars or is converted to charcoal, which is carbon in a nearly pure state. Then this carbon unites with the oxygen of the lead oxide forming carbon dioxide, which escapes into the air. Metallic lead is left behind in the crucible. Metals are often extracted from their ores by a process of this kind.

2. When a stream of dry hydrogen gas is allowed to flow over copper oxide heated to redness, vapor of water is formed, and the copper oxide is reduced to metallic copper.

3. Place some fresh green leaves in an apparatus like that shown in Fig. 150. Fill the apparatus completely with water saturated with carbon dioxide, and then expose the leaves to sunlight for an hour or two. Bubbles of gas form on the leaves and can be collected in the tube at the top. When the tube is full of gas, remove it, and insert a glowing splinter. It bursts into flame, proving the gas to be oxygen.

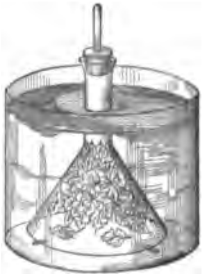


FIG. 150.

The last example illustrates the reduction of carbon dioxide, which the plant world is constantly effecting on a gigantic scale. The green parts of plants, under the influence of the sun's rays, are able to decompose carbon dioxide into carbon and oxygen. The carbon contributes to the growth of the plant, the oxygen is restored free to the atmosphere. The carbon stored up by the plant becomes food for animals or fuel for fires; in either case it reunites with oxygen, and again enters the atmosphere in the form of carbon dioxide. The cycle of changes is now completed, and begins over again.

REVIEW EXERCISES ON CHAPTER IV.

1. Sum up the molecular theory of matter in four propositions.

2. What is meant by a *saturated* solution of a substance?

3. What is the usual effect of temperature on solubility?

4. What conditions favor the formation of large, perfect crystals?

5. Illustrate the uses of crystallization.

6. Give an instance of a solid having a crystalline structure, and name the kind of structure.

7. Give an example of the power of a solid to absorb a gas.

8. Give an example of the power of a liquid to absorb a gas.

9. Give an example of liquid diffusion, and show how it tends to establish the molecular theory of matter.

10. Define with illustrations the meaning of the terms *osmosis*, *crystalloid*, *colloid*, *dialysis*.

11. Give examples of the power of gases to mix with one another.

12. What is the effect of heat on cohesion? Give an example.

13. Explain with examples the meaning of the terms *elasticity*, *plasticity*, *brittleness*, *toughness*, *malleability*, *ductility*, *hardness*.

14. Give examples of cohesion in liquids.

15. How can a liquid be shown to have a surface tension?

16. Give examples of adhesion (1) between two solids, (2) between a solid and a gas, (3) between a solid and a liquid.

17. What are *capillary* phenomena and their chief laws?

18. Give examples of *viscous* and of *mobile* liquids. How do we distinguish between solids and liquids?

19. Illustrate by examples the difference between a mechanical mixture and a chemical compound.

20. Explain how water can be decomposed into oxygen and hydrogen.

21. Distinguish between an *element* and a *compound*, and give examples.

22. Give an example of chemical exchange.

23. State the general laws of chemical combination.

24. Give a brief account of the atomic theory.

25. How did Lavoisier show that the air we breathe is a mixture of oxygen and nitrogen gases?

26. What is the general effect of chemical combination and chemical separation as regards heat? Give an example of oxidation, and also one of combustion.

27. Describe how animal heat is maintained.

28. Describe how the reduction of carbon dioxide is effected by the plant world.

CHAPTER V.

MOTION.

Elementary Ideas about Motion.

161. Uniform Motion. A point is said to be in *motion* when its position is changing; and the motion is said to be *uniform* if the point passes over equal lengths in equal intervals of time, however small these intervals may be.

The *velocity* of a moving point is the rate at which its position is changing. A velocity possesses direction as well as magnitude. Both the magnitude and the direction must be known before the velocity is fully known.

In uniform motion the velocity is constant, and is measured by the distance passed over in one unit of time, usually one second. If we denote the constant velocity by v, then in 2 seconds the point will pass over the distance $2v$, in 3 seconds the distance $3v$, and so on. Hence, if s denote the distance passed over in t seconds,

$$s = vt.$$

If two points are moving with different velocities v and v', then the distances s and s' which they pass over in the same time t will be

$$s = vt, \text{ and } s' = v't.$$

Therefore, $$\frac{s}{s'} = \frac{vt}{v't} = \frac{v}{v'}.$$ That is,

The distances passed over in the same time by two points, moving with different velocities, are proportional to the velocities.

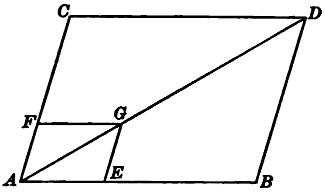


FIG. 151.

162. Composition of Motion. Let a ring move with uniform velocity along a straight rod from *A* to *B* (Fig. 151), while the rod itself moves with uniform velocity in the direction *AC* from the position *AB* to the position *CD*, keeping parallel to its first position. Then the ring, by the combined effect of these two independent but simultaneous motions, must arrive at the fourth corner *D* of the parallelogram *ABDC*. For any part of the whole time we obtain a parallelogram *AEGF*, similar to *ABDC* and similarly placed.

Therefore, the ring moves along the *diagonal AD*; and moves with a uniform velocity along the diagonal, because each of the component motions is uniform.

If the time of motion is one second, *AB* and *AC* represent the two independent velocities, and *AD* the actual velocity of the ring. *AB* and *AC* are called the *component* velocities, *AD* the *resultant* velocity.

Velocities, therefore, are compounded in the same way as forces; that is, by means of the parallelogram law (§ 43).

If a and b denote two component velocities at right angles to each other, and c their resultant, then $c^2 = a^2 + b^2$.

Example. Find the velocity in magnitude and direction of a ship sailing east at the rate of 10 miles per hour, and carried by a current north at the same rate.

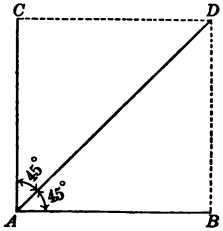


FIG. 152.

Represent the velocities by *AB* and *AC* (Fig. 152), complete the parallelogram *ABDC* (here a square), and draw *AD*, which bisects the right angle at *A*. Then

$$\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2 = 100 + 100 = 200.$$

$$AD = \sqrt{200} = 10\sqrt{2} = 14.14.$$

Ans. 14.14 miles per hour, northeast.

163. Resolution of Motion. Just as a resultant force is equivalent to its components and can replace them, so the resultant of two simultaneous velocities is equivalent to its components and can replace them. And just as a force can be resolved into two components in given directions, so a velocity can be resolved into two component velocities in given directions. It is only necessary to construct a parallelogram such that the given velocity shall be the diagonal, and the adjacent sides of which shall have the given directions of the two components.

Resolution into rectangular components is most often useful. If the direction of one component is given, the direction of the other is known, and a simple construction determines the magnitudes of both.

Examples. 1. A cannon ball is fired at an angle of 45° with the horizon. The velocity of the ball is 1200 ft. per second. What are its horizontal and vertical components?

Let *AB* represent the velocity of the ball. Draw *AM* horizontal, *AN* vertical, *BD* horizontal, and *BC* vertical. *AC* and *AD* are the two components, and are equal.

If x denote the value of either component,

$$x^2 + x^2 = 1200^2, \text{ or } 2x^2 = 1200 \times 1200,$$

whence $x = \sqrt{720000} = 600\sqrt{2} = 848.4.$

Each component = 848.4 ft. per sec.

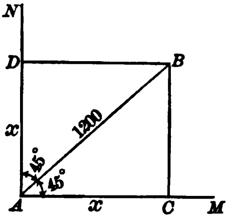


FIG. 153.

2. A straight track is inclined 30° to the horizon, and a train is ascending the incline at the rate of 10 miles per hour. Through what distance is the train lifted against the force of gravity in one hour?

In the right triangle *ABC* (Fig. 154), if *AB* represents 10 miles, *BC* represents the required distance.

Since in a right triangle the side opposite an angle of 30° is equal to ½ the hypotenuse, *BC* = 5 miles.

How can the value of the other component *AC* be found?

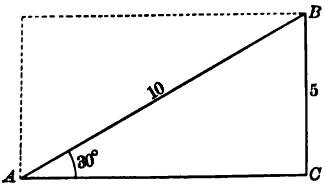


FIG. 154.

164. Relative Motion. We say that a steamboat going down a river is in motion, and that the bank of the river is at rest. We treat the earth as if it were a fixed body, and for all practical purposes this may be assumed as true.

The earth, however, is *not* a fixed body; it turns on its axis every day, and it also revolves round the sun once every year. We may indeed regard the sun as a fixed body so long as we confine our attention to the solar system. But this again will not do, when we extend our observations to the fixed stars. No body is in a state of *absolute* rest.

Both rest and motion are purely *relative* terms. Two points A and B are at rest with respect to each other if the line AB preserves both its length and its direction unchanged; otherwise A and B are in motion with respect to each other.

Thus, two men seated in a railway car are at rest with respect to each other, whether the car is stopping at a station or moving swiftly along a straight track. But if one man walks through the car while the other keeps his seat, then they are in a state of relative motion.

Suppose that two points A and B are both in motion with respect to surrounding objects. How can we find the velocity of A with respect to B? Imagine a velocity given to each point *equal but opposite* to that which B already has, and combine each with the velocity of each point. This will bring B to rest, and the new velocity of A will be its velocity with respect to B.

Examples. 1. Suppose two bodies A and B are moving east along the same straight line, A with the velocity 15, B with the velocity 10.

Give to each the velocity 10 due *west;* then B is brought to rest, and the velocity of A with respect to B is seen to be $15 - 10$, or 5.

2. But suppose that A is moving east with the velocity 15, and B is moving *west* with the velocity 10.

Give to each a velocity of 10 due *east;* then B is brought to rest, while the velocity of A becomes $15 + 10$, or 25 due east; this, then, is the velocity of A relative to B.

3. Why does rain falling vertically appear to move in a slanting direction to a man who is moving rapidly in a horizontal direction?

Let AB (Fig. 155) represent the vertical velocity of a raindrop and AC the horizontal velocity of the man. Apply to each a horizontal velocity equal but opposite to that of the man. This brings the man to rest, and the new velocity of the raindrop is AE. To the man, therefore, the effect is the same as if he were at rest and the rain moved with a velocity represented in magnitude and direction by AE.

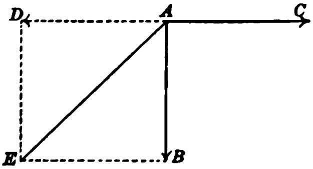


FIG. 155.

165. Translation and Rotation. An extended body can move in two different ways:

(1) It may have a motion of *pure translation*, like that of an up and down saw. In this case every point in the body moves with the same velocity, and every straight line in the body preserves its direction unchanged. In this case, therefore, it is sufficient to consider the motion of any one point, since all points of the body move exactly alike.

(2) It may have a motion of *pure rotation*, like that of a circular saw. In this case every point in the body describes a circle round some fixed straight line as an axis of rotation (Fig. 156).

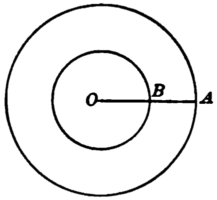


FIG. 156.

Since every point of a rotating body describes a circle in the same time, and since the circumferences of circles are proportional to their radii, the velocity of a point varies directly as its distance from the axis. Thus, a point A which is twice as far from the axis as a point B moves just twice as fast as B.

Very often the motion of a body is a combination of translation with rotation. The motion of a wheel on a track, and the motion of a screw in a nut are examples.

CLASS-ROOM EXERCISES.

1. How far will an eagle fly in 1 hour at the rate of 100 ft. per sec.?

2. A velocity of 60 miles an hour is how many feet per second? 88

3. A train leaves Boston at 2 P.M., and reaches Chicago the next day at 5 P.M. The distance is 1025 miles. What is its average rate? 37.9 per hr.

4. A man looking out of the window of a car observes that the time between passing one milestone and the next is 80 seconds. What is the velocity of the train in miles per hour? 45

5. A cannon is fired on the water one mile from a cliff, and the sound of the echo is heard after 9½ seconds. Find the velocity of sound. 558.

6. The Danish astronomer, Roemer, found that it took light 16 min. 26 sec. to cross the earth's orbit, a distance of 186 millions of miles. Find the velocity of light in miles per second. 188640.9

7. A steamer can go down a river at the rate of 16 miles an hour, and up the river at the rate of 4 miles an hour. What is the rate of the current, and the rate of the steamer in still water? 6, 10

8. A sledge party is traveling on the ice towards the North Pole at the rate of 16 miles a day. The ice is drifting southwards at the rate of 22 yards a minute. In what direction, and at what rate, is the party really moving?

9. How great is the change of velocity from 28 east to 20 west?

10. Find the resultant of the following velocities: 12 north, 2 east, 4 south, 8 west, and show its direction by a figure.

11. A steamer is driven east by the steam at the rate of 15 miles an hour and north by a current at the rate of 8 miles an hour. What is its actual velocity relative to the earth?

12. A ship is making 12 miles an hour on a northeast course. How fast is she moving north? How fast is she moving east?

13. A man rows directly across a river 2 miles wide. He rows at the rate of 6 miles an hour, and the current flows at the rate of 3 miles an hour. How long will he be in crossing, and where will he land?

14. A ship goes 12 miles an hour, and a man walks straight across the deck at the rate of 5 miles an hour. What is his velocity relative to the water? Illustrate by a figure.

15. A balloon ascends along a line inclined 60° to the horizon with a velocity of 20 miles an hour. What is its velocity estimated in a horizontal direction?

16. If the minute hand of a clock is 6 in. long, what is the linear velocity of the end of the hand? What is its *angular* velocity, that is, the angle it will describe in one second of time?

Falling Bodies and Projectiles.

166. Variable Velocity. A velocity may change either in magnitude or in direction.

The velocity of a point varies in magnitude if the point passes over unequal distances in equal intervals of time, however small these intervals may be; and it is measured at any instant by the distance that would be passed over in a unit of time, if the velocity at the instant considered were to remain unchanged for a unit of time.

When we say that a bullet leaves a rifle with a velocity of 1500 ft. a second we mean that if the velocity remain unchanged for one second, the bullet during this second will describe a path 1500 ft. in length. The bullet may strike a target, and be brought to rest after it has passed over only a few feet, but this will not alter the fact that it left the rifle with a velocity of 1500 ft. per second.

167. Mean Velocity. When a point moves with variable velocity for any interval of time, the uniform velocity with which it would describe the same space in the same interval of time is called the *mean* velocity of the point during that interval. The mean velocity of a point during any interval of time may be found by dividing the whole space described during the interval by the time taken to describe it. (§ 161).

If a point travel with a variable velocity 100 ft. in 5 seconds, its mean velocity is 20 ft. per second; its actual velocity must have been greater than this at some instants and less at others. The shorter the interval of time considered, the more nearly will the mean velocity of a point during the interval approach the actual velocity of the point at any instant of the interval.

Thus, a more accurate value of the actual velocity (in ft. per sec.) of a train at any instant is obtained by dividing by 5 the number of feet described by the train in 5 seconds than by dividing by 60 the number of feet described in 1 minute.

168. Acceleration. Rate of change of velocity is called *acceleration.* When constant, it is measured by the change of velocity produced in a unit of time; when variable, by the change that would be produced in a unit of time if the acceleration were uniform during the unit and equal to its actual value at the instant under consideration.

If the motion is in a straight line, the acceleration is *positive* or *negative* according as the velocity is *increasing* or *diminishing.* Negative acceleration, in common language, is called *retardation.*

Thus, if a point whose velocity is 20 ft. per second receive a uniform acceleration of 4 ft. per second for 3 seconds in the direction of the motion, the velocity will become equal to $20 + 12$, or 32 ft. per second; but if this acceleration be in the direction opposite to the motion, then after 3 seconds the velocity of the point will become equal to $20 - 12$, or 8 ft. per second.

169. Laws of Falling Bodies. The motion of a falling body is plainly an accelerated one; the farther a body falls, the harder it is to follow the body with the eye, and the heavier is the blow which it will give to the hand. But the exact laws of the motion are not at all obvious. They were discovered by Galileo (1584–1642).

He began by asserting that all bodies, if unimpeded, fall at the same rate, and that the reason why a piece of paper, for example, falls more slowly when spread out than when rolled up into a ball is on account of the resistance of the air. In support of this assertion, he caused various heavy bodies to be dropped at the same instant from the top of the Leaning Tower of Pisa, and he found that they struck the ground at almost the same instant. When the air pump was invented, the truth of Galileo's view was confirmed by dropping such bodies as a guinea and a feather in a space void of air.

We have, therefore, as the first law of falling bodies,

1. *The acceleration of gravity is the same for all bodies.*

Galileo then investigated in what ways the time of falling, the velocity acquired, and the space described must be related to one another, *provided the acceleration is constant;* that is, the same for each second of the time of falling.

Let g denote the numerical value of the acceleration, and v the velocity acquired in t seconds; then in 1 second the velocity acquired is g, in 2 seconds $2g$, in 3 seconds $3g$, etc.

Therefore, in general,

$$v = gt. \qquad [1]$$

Since g is constant, v varies as t. That is,

2. *The velocity acquired is directly proportional to the time of falling.*

Let s denote the space described in t seconds. The final velocity, as we have just seen, is gt. Since the rate of increase is uniform, the *mean* velocity must be equal to the actual velocity at the end of half the time, or $\frac{1}{2}v$. Hence, the space described must be the same as if the body moved *uniformly* during the whole time with this mean velocity; in other words, $s =$ mean velocity × time $= \frac{1}{2}vt$.

Putting in place of v its equal, gt, we have

$$s = \tfrac{1}{2}gt^2. \qquad [2]$$

Since $\frac{1}{2}g$ is constant, s varies as t^2. That is,

3. *The space described varies as the square of the time.*

We can obtain the relation between v and s by eliminating t from equations [1] and [2].

From equation [1], $t = \frac{v}{g}\cdot$

Substituting this value of t in equation [2], and reducing, we obtain

$$v^2 = 2gs.$$ Therefore,

4. *The space described varies as the square of the velocity.*

170. Verification of the Laws. After obtaining the preceding laws by reasoning, Galileo found by experiment that falling bodies actually obey them; whence he concluded that the acceleration of gravity is constant, as he had assumed it to be in obtaining the laws.

In attempting to verify the laws experimental difficulties are encountered. The velocity acquired cannot be directly observed, because gravity acts all the time and the velocity is constantly changing. The space described in a given time can be accurately measured only when the rate of the motion is by some device diminished. Galileo's device consisted in allowing a body to descend a smooth inclined plane; in this case only a part of the weight of the body causes the motion, instead of the whole weight, as in the case of free fall (see § 46). The motion follows the same law as in the case of free fall, but takes place more slowly. Galileo found that the space described by the body varied as the *square* of the time. This verified law 3 directly, and laws 2 and 4 indirectly, because the three laws are so connected that the truth of one implies the truth of the others.

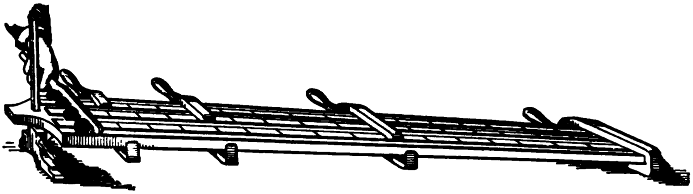


FIG. 157.

Galileo's experiment may be performed as shown in Fig. 157. The four balls (marbles) move along grooves in the inclined plane and are started at the same instant. Stops are placed at the distances 1, 4, 9, and 16 dm., respectively. The balls are heard to strike the stops at *equal* intervals of time. This shows that the times of falling are as the numbers 1, 2, 3, 4; while the distances are as the *squares* of these numbers.

171. Value of g. The most direct way to find the value of g is to observe how far a body will fall in a given time, say one second. If we put $t=1$ in the formula $s=\frac{1}{2}gt^2$, we obtain $g=2s$; that is, the acceleration of gravity is equal to *twice* the space described in the first second by a body falling from rest.

A much more accurate value of g can be obtained by observing the motion of a pendulum.

With the foot and the second as units, the value of g varies from about 32.091 at the equator to about 32.252 at the poles. In the latitude of New York its value is about 32.16; that is, the velocity of a body falling in a vacuum is increased by 32.16 ft. each second. With the centimeter and the second as units, the extreme limits are about 978 and 983; and in the latitude of New York the value is about 980.

172. Motion down an Inclined Plane. Acceleration, like velocity, is subject to the parallelogram law, and this explains why motion down an inclined plane is slower than free fall, even if the plane be perfectly smooth. Only that component of g which acts down the plane causes motion, the other component being destroyed by the reaction of the plane.

In Fig. 158 the length AD represents g, and the lengths AE, AF its components. AE alone causes motion. From the similar triangles AED, ABC,

$$AE:AD=AB:AC.$$

Therefore, AE is less than AD in the same ratio that AB, the height of the plane, is less than AC, its length. Thus, if $AB=\frac{1}{4}AC$, then $AE=\frac{1}{4}AD$, or about 8 ft. per second.

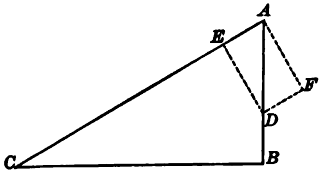


FIG. 158.

Hence, the formulas on page 181 apply to motion down an inclined plane, if in place of g we substitute the number obtained by multiplying g by the ratio of the height of the plane to its length.

173. Projectiles. Galileo appears to have been the first to see clearly that the force of gravity has the same effect in changing the motion of a body whether the body is initially at rest, or already in motion. To state the principle more precisely, *gravity always impresses upon a body, free to move, a downward velocity of g units each second, whether the body starts from a state of rest, or is moving already with any velocity in any direction.*

By means of this principle, it is easy to explain the motion of a projectile or body thrown into the air in any direction.

We will consider three cases:

(1) Let a body be thrown vertically downwards with the velocity u, and let v denote the velocity after t seconds. The velocity will receive an increase of g units each second; therefore, after t seconds

$$v = u + gt.$$

In this time the body will describe the space ut on account of the throw, and $\frac{1}{2}gt^2$ on account of gravity; therefore, if s denote the whole space described,

$$s = ut + \tfrac{1}{2}gt^2.$$

(2) Let the body be thrown vertically upwards with the velocity u. The only difference between this case and the preceding is that gravity now acts *against* the motion; therefore, g is *negative*, and we have

$$v = u - gt,$$
$$s = ut - \tfrac{1}{2}gt^2.$$

Since the body loses g units of velocity each second,

$$\textit{time of rising to the highest point,} = \frac{u}{g}.$$

Substituting this value of t in the value of s, we have,

$$\textit{height ascended} = \frac{u^2}{2g}.$$

(3) Let a body be projected horizontally from the point A (Fig. 159) with the velocity u. If the force of gravity did not exist, the body would describe each second a horizontal distance equal to u. But gravity is constantly acting on the body, and will cause it to descend at the same rate as if there were no horizontal motion at all. The actual position of the body after any interval of time is found by compounding these two independent motions.

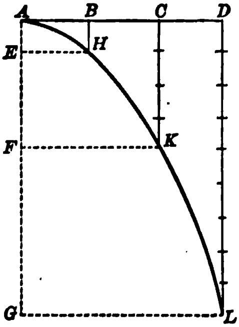


Fig. 159.

From A lay off horizontal distances, $AB = u$, $AC = 2u$, $AD = 3u$, etc.; also vertical distances equal to the spaces described by a falling body in 1, 2, 3, etc., seconds, namely: $AE = \frac{1}{2}g$, $AF = \frac{4}{2}g$, $AG = \frac{9}{2}g$, etc. Complete the parallelograms $ABHE$, $ACKF$, $ADLG$. After 1 second the body will be found at H, after 2 seconds at K, and after 3 seconds at L.

By taking smaller intervals of time we can determine intermediate positions of the body. A smooth curve drawn through all these positions represents the actual path of the body. It is a curve called by mathematicians a *parabola*.

We have supposed that the motion takes place in a vacuum. Practically, the resistance of the air prevents the path of a projectile from being a true parabola.

Example. A cannon ball is fired horizontally from the top of a cliff on the seashore. The velocity of the ball is 1500 ft. per second, and the cliff is 400 ft. high. What is the time of flight, and the range (the horizontal distance described by the ball)?

Solution. The time of flight is that required for falling 400 ft. By substituting 400 for s in the formula $s = \frac{1}{2}gt^2$, and reducing, we find that the time of flight is equal to 5 seconds.

During each second the ball travels a horizontal distance of 1500 ft. Therefore, the range $= 1500 \times 5 = 7500$ ft.

13/ 2

11 (1)

2) s = −16

s = −144

s = 400

CLASS-ROOM EXERCISES.

Values of g: 32 ft. per sec. each second, 9.8 m. per sec. each second.

1. What velocity will a body acquire in falling for 10 seconds, and through what distance will it fall?

2. How far will a body fall in 4 seconds? How far during the *fifth* second? What is its mean velocity during the 5 seconds?

3. How long will it take a body to fall 400 ft., and what velocity will the body acquire? What is its mean velocity?

4. How far must a body fall to acquire a velocity of 96 ft. a second?

5. A stone is dropped into a mine and reaches the bottom in 6 seconds. How deep is the mine?

6. A stone is thrown down a well with a velocity of 50 ft. per second, and reaches the bottom in 2 seconds. How deep is the well?

7. Through what height must a body fall to acquire a velocity of 1000 ft. per second?

8. A body is dropped from a certain point, and a second later another body is dropped from the same point. How far apart are the two bodies when the first body has been falling for 7 seconds?

9. A stone is thrown vertically upwards with a velocity of 192 ft. per second. How high will it rise? How long will it be in the air?

10. With what velocity must a stone be thrown up in order that it may rise 1600 ft.?

11. A stone thrown vertically upwards returns to the ground after 10 seconds. Find (1) velocity of projection, (2) height to which it rises, (3) height from the ground after 1, 2, 3, 4, and 5 seconds, respectively.

12. A stone dropped from a balloon reaches the ground in 20 seconds. How high is the balloon, (1) if at rest in the air, (2) if ascending with a velocity of 160 ft. per second when the stone is dropped?

13. A bullet is fired vertically upwards with a velocity of 800 ft. per second. How high will it rise? How high will it be after 30 seconds, and what will be its velocity at this instant?

14. A stone is dropped into a mine, and the sound, when it strikes the bottom, is heard after 10 seconds. If the velocity of sound is 1120 ft. per second, find the depth of the mine.

15. A stone is dropped down a well 400 ft. deep. If the sound of the splash is heard after $5\frac{4}{11}$ seconds, find the velocity of sound.

16. A body is thrown vertically upwards with a velocity of 49 meters per second. With what velocity will it pass a point 100 meters from the ground (1) when ascending, (2) when descending?

17. Two balloons start upwards together, one with a uniform velocity of 8 ft. per second, the other with a uniform acceleration of 8 ft. per second. How far apart will they be after 1 min.?

18. How long will it take a body to slide down a smooth inclined plane 100 ft. long and 25 ft. high, and what velocity will it acquire?

19. One body is allowed to slide down a smooth inclined plane 800 ft. long and 100 ft. high. Another body is allowed to fall vertically through the height of the plane. Find (1) the velocity of each body on reaching the base of the plane, (2) the time required for each body to fall.

20. Same as No. 19, only the length of the plane is l, and the height h.

21. A body starts with the velocity 4, and has a uniform acceleration of 2. Find (1) the velocity after 10 seconds, (2) the mean velocity for this time, (3) the space described.

22. A body has the velocity 30 and a uniform retardation of 3. Find (1) the velocity after 8 seconds, (2) the mean velocity for this time, (3) the space described, (4) the whole space described before coming to rest.

23. In 3 minutes after starting from a station a train is traveling at the rate of 40 miles an hour. Find (1) the mean acceleration for this interval in feet per second, and (2) the space described.

24. A train is moving at the rate of 45 miles per hour. On rounding a curve the engineer sees another train a quarter of a mile ahead at rest on the track. He reverses his lever and puts on the brakes, thus causing a retardation of 3 ft. per second each second. Will the train stop in season to avoid a collision?

25. A car is started down an incline of 8 per cent grade. Friction causes a retardation of 2 ft. per second each second. How far will the car move in 2 minutes?

26. Show by a figure how to find the magnitude and direction of the velocity of the projectile in Fig. 159 after it has been moving 1 second.

27. With what velocity must a ball be fired horizontally from a point 160 ft. above the ground to have a horizontal range of 2000 ft., the resistance of the air being left out of account?

28. A cannon ball is fired at an angle of elevation of 30° with a velocity of 1600 ft. per second. Find (1) the time of flight, (2) the maximum height, (3) the horizontal range.

Hints. Resolve the velocity into vertical and horizontal components. The vertical component = 800, the horizontal component = $800\sqrt{3}$. The time of ascent, and the height ascended are the same as if the ball had been projected vertically with the velocity 800. During this time the ball also has a horizontal velocity of $800\sqrt{3}$.

First Law of Motion.

174. Inertia of Matter. The fundamental laws of motion were stated by Sir Isaac Newton (1642–1727) with a clearness and precision that cannot be improved.

The First Law is as follows :

Every body continues in a state of rest, or of uniform motion in a straight line, unless compelled by external forces to change that state.

That property of matter in virtue of which a body tends to maintain its state of rest, or of uniform motion in a straight line, is called *inertia.*

The First Law of Motion asserts that matter has inertia, and that on account of inertia force is required to change a body's state of rest or motion.

Examples of inertia are very numerous.

(1) When a car starts suddenly the feet of a person standing in the car are made for an instant to move faster than his head, and so he is in danger of falling *backwards.* If the car stop suddenly, he is in danger of falling *forwards.*

(2) When a circus rider leaps through a hoop he leaps simply *upwards.* He retains the forward motion which he has in common with the horse, and by reason of both motions combined he passes through the hoop.

(3) Every boy knows how to make a stone "skip" along the surface of water. The inertia of the water prevents the swift-moving stone from sinking when it strikes the water, and causes it to rebound into the air. If the stone moves slowly, the water yields and the stone sinks.

(4) If you pile up a column of wooden blocks on a table, and then give the lowest one a sharp horizontal blow, it will fly out without disturbing the others. A steady push would make the whole column move.

(5) If you throw a grain of wheat into the air and strike it a very sharp blow, the grain will break into fragments, although free to move.

(6) The tendency of a body to preserve its state of motion is manifested by the difficulty of stopping a car on a slippery track, or of pulling up a horse at full gallop, or of turning a sharp curve when we are running or skating very rapidly.

175. Useful Applications of Inertia. The inertia of matter is useful in a great variety of ways.

(1) When a carpet is beaten the carpet moves forward while the dust remains behind. In this way the dust is removed from the carpet.

(2) In grain warehouses the grain is distributed from the top floor over the lower floors by a process which includes an ingenious application of Newton's First Law. The grain, after descending an incline called a shoot, falls upon a flat, broad band which is in rapid motion, and which runs over two rollers, as shown in Fig. 160. It is then carried forward to the top of the roller *B*, where the band suffers a sudden change in direction. The stream of grain, however, retaining the velocity given to it by the band, shoots forward in a jet over the top of *B* as if it were a stream of water, and falls into the spout *C*, whence it slides down to a second band on the floor below. The edges of the band are turned up to keep the grain from spilling over them.

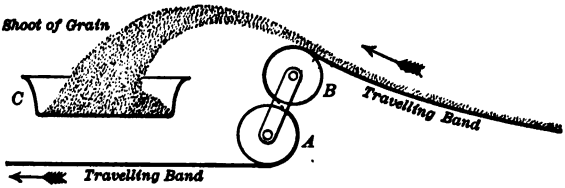


FIG. 160.

(3) In the manufacture of lead shot, the melted lead is allowed to fall in drops from a high tower into a cistern of water. The perfectly spherical shot are then separated from the imperfect by allowing them all to roll down a smooth inclined plane. The perfectly round shot acquire a velocity sufficient to carry them over certain pitfalls along the way, while the imperfect ones move more slowly and drop into the pitfalls.

(4) In order to prevent sudden changes of velocity and the consequent wear and tear of the machinery, stationary steam engines are provided with *fly wheels*. They are large heavy wheels, with most of the material (iron) collected around the rim so that it may have as great a velocity as possible. The inertia of this rapidly rotating mass is so great as to compel all the moving parts of the engine and the machinery connected with it to maintain a nearly uniform speed, in spite of frequent variations in the force of the steam, and the resistances which have to be overcome.

176. Inertia Manifested in Rotation. When you whirl a stone attached to a string round in a circle at uniform speed, you feel that you are constantly pulling on the string. Yet the stone does not approach your hand. What, then, is the effect of the pull? The answer is that it causes a continual change in the *direction* of the motion. The stone, if left to itself, would move in a straight line. It will not move in a circle unless compelled to do so by the constant action of a sufficient force directed towards the center of the circle. The pull of the hand is this needed force. It overcomes the inertia of the stone, not by altering the rate of the stone's motion, but by changing the direction of the motion.

At any instant the stone is moving in the direction of the tangent *ST* (Fig. 161), and would continue to move in that line were it not prevented by the pull of the hand. Action and reaction here appear in a new light. The pull of the hand is the action, and the resistance which the stone offers to a change in the direction of its motion is the equal opposite reaction. The string is in a state of stress, and the more rapid the motion the greater the stress becomes. If the string breaks, both action and reaction vanish instantly; and the stone, in obedience to the First Law of Motion, flies off along the tangent.

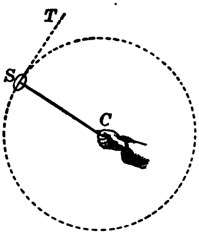


FIG. 161.

The particles of a fly wheel are forced by cohesion to move in circles, and the motion of each particle is in all respects like that of the stone just mentioned. If the velocity of the fly wheel is made so great as to overcome the force of cohesion, the wheel will burst, and each piece will at once begin to move with great velocity in a straight line.

A sling is a device by which a great velocity of rotation is suddenly transformed into a velocity of translation.

177. How the First Law is Proved. No direct experimental proof of the First Law of Motion can be given, because a body cannot be wholly removed from the action of such external forces as friction and the resistance of the air.

We find, however, that the more these retarding forces are diminished, the more nearly does the character of the motion resemble that described in the statement of the law. For example, a pendulum, if set to swinging in the air, soon stops ; but in a vacuum it will vibrate for more than a day.

Our belief in the truth of Newton's Laws of Motion is really based on the fact that the conclusions drawn from them are always found to agree with experience.

CLASS-ROOM EXERCISES.

1. How is the head of a hammer often fastened when it gets loose, and how is the action which takes place explained ?

2. How do you explain the fact that a circus rider can leap through a hoop by simply springing directly upwards ?

3. Why does a pendulum continue its swing against the action of gravity after the bob reaches the lowest point ?

4. Suspend a weight by a string (Fig. 162), and attach a piece of the same string to the weight underneath. If you pull the lower string with a sudden jerk, it breaks. But if you pull steadily, the upper string breaks. Explain.

5. A bullet may be fired through a pane of glass and do no further damage than to leave a small round hole. But the bullet, if thrown by the hand against the glass, would shatter it in pieces. Account for this difference.

6. Why cannot a plate full of soup be quickly pushed across a table without spilling the soup ?

7. A charge of dynamite placed on the top of a rock and exploded will shatter the rock. Can you account for this ?

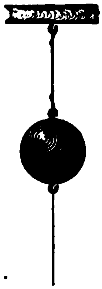


FIG. 162.

8. How does a hare, when pursued by a hound, make use of the law of inertia in order to escape from the hound ?

9. Explain why you can throw a stone a greater distance by means of a sling than by your hand alone.

Second Law of Motion.

178. Momentum. A new idea must now be introduced. The quantity of motion, or *momentum*, of a moving body is measured by multiplying the number of units in the mass of the body by the number of units in its velocity.

Momentum = mass × velocity.

Two bodies may have equal momenta, although their masses and velocities differ greatly. Thus, a 64-lb. cannon ball, moving with a velocity of 5 ft. per second, has the same momentum as a bullet weighing $\frac{1}{4}$ lb., and moving with a velocity of 1280 ft. per second; for $64 \times 5 = \frac{1}{4} \times 1280 = 320$. The meaning of this equality is that it would take the same force the same interval of time to bring either body to rest, the force being supposed in each case to act directly against the motion.

179. Dynamical Measure of Force. Newton's Second Law tells us how a force, which causes a change in the motion of a body, is to be measured.

Change of momentum is proportional to the impressed force, and takes place in the direction of the force.

This law asserts that if a force act on a body, it will overcome the inertia of the body, and change the momentum of the body (unless prevented by the action of other forces). Hence the proper measure of the force is the *rate* at which the momentum changes; in other words, the change of momentum produced in one second. And the simplest measure is obtained by making the force and the rate of change of momentum numerically equal.

Force = rate of change of momentum.

The rate of change of momentum is found by multiplying the mass of the body by the change of velocity in one second or acceleration caused by the force; therefore,

Force = mass × acceleration.

180. Units of Force. It follows from the equation

$$Force = mass \times acceleration,$$

that if the factors, mass and acceleration, are each equal to 1, then the force will also be equal to 1. Therefore, the *dynamical unit of force* is that force which will impress a unit of velocity upon a unit of mass in a unit of time.

If the unit of mass is one pound, and the unit of velocity is one foot per second, the unit of force is called a *poundal.*

If the unit of mass is one gram, and the unit of velocity is one centimeter per second, the unit of force is called a *dyne.*

The poundal and the dyne are called *absolute* units, because their values are invariable. The statical units of force, the pound weight and the gram weight, are called *gravitation* units, because their values depend on the force of gravity, and are therefore variable.

A simple relation exists between absolute and gravitation units. A mass of one pound, if allowed to fall freely at the latitude of New York, will acquire in one second a velocity of 32.16 ft. per second; hence, a pound weight at New York is equal to 32.16 poundals. Similarly, a gram weight at New York is equal to 980 dynes. In general, one gravitation unit of force is equal to g absolute units.

Therefore, to change a force from gravitation to absolute measure, *multiply* the number of pounds (or grams) by g.

Conversely, to change a force from absolute to gravitation measure, *divide* the number of pounds (or grams) by g.

Examples. 1. If a force, acting on a mass of 200 lbs., gives to it in one second a velocity of 8 ft. per second, what is the value of the force in poundals and also in pounds? Take $g = 32$.

Answer. Force $= 200 \times 8 = 1600$ poundals $= 1600 \div 32 = 50$ lb.

2. If a force of 5 grams act on a mass of 35 grams, what velocity will it impress on this mass in every second? Take $g = 980$.

Answer. Force $= 5 \times 980 = 4900$ dynes. Mass $= 35$ grams. Acceleration $= 4900 \div 35 = 140$ cm. per second each second.

181. Mass Proportional to Weight. Mass, or quantity of matter, is measured by inertia; and quantity of inertia is precisely defined by the equation

Force = mass × acceleration.

For, suppose that equal forces act upon two different bodies and impress upon them equal accelerations. It follows from the above equation that the masses of the bodies must also be equal. We thus obtain a definition of equal masses which is independent of gravity or any particular force, and dependent solely on the inertia of matter.

Two bodies have equal masses if the same force will generate in them the same velocity in the same time.

Let us now suppose that two bodies are in equilibrium when placed on the pans of a balance. Then we know that the pull of gravity on each body is the same; that is, the two bodies are under the action of equal forces. If we allow the bodies to fall so that the resistance of the air does not interfere with the motion, we find that gravity will impress upon each body the same acceleration, namely, that denoted by g. Therefore, the bodies must have equal masses.

If we put both bodies in one pan of the balance, and place in the other pan a third body such that equilibrium is again secured, it is clear that this third body must have twice the weight, and also twice the mass, of either of the others. In general, *the mass of a body is directly proportional to its weight.*

Thus, the method of comparing masses by weighing them is consistent with the Second Law of Motion.

The masses of two bodies might be compared, though much less conveniently and accurately, by making use of some other force than gravity. We might, for example, arrange two springs so that they should act on the two bodies with equal force at the same instant. If the bodies moved off with equal velocities, the inference would be that their masses were equal. If one of them moved twice as fast as the other, what would the inference be?

182. Effect of a Constant Force. If a body is acted upon by a constant force so that no rotation takes place (§ 165), the motion of the body will be uniformly accelerated.

Since force is equal to mass × acceleration, the value of the acceleration is given by the equation

$$Acceleration = \frac{moving\ force}{mass\ moved}.$$

Let a denote this acceleration (or retardation), v the velocity acquired (or destroyed) in the time t, s the space described in the same time. Since the acceleration is uniform, the motion is in all respects like that of a body acted upon by the force of gravity, except that the acceleration may have any value whatever. Therefore, the formulas given in § 169 will hold good if we substitute a in place of g; that is,

$$v = at. \qquad [1]$$

$$s = \tfrac{1}{2}vt = \tfrac{1}{2}at^2. \qquad [2]$$

$$v^2 = 2as. \qquad [3]$$

Example. Masses of 17 lb. and 15 lb. are connected by a string passing over a fixed pulley (Fig. 163). Find (1) the moving force, (2) the acceleration, (3) the velocity acquired in 3 sec., (4) the space described in 3 sec., (5) the tension of the string. ($g = 32$.)

(1) Moving force $= 17 - 15 = 2$ lb. $= 64$ poundals.

(2) The mass moved is $17 + 15$, or 32 lb.; therefore, acceleration $= 2$ ft. per second each second.

(3) Velocity after 3 seconds $= 6$ ft. per second.

(4) Space described $= \frac{1}{2} \times 2 \times 9 = 9$ ft.

(5) Let T denote the tension of the string in pounds. The forces which act on the mass of 17 lb. are 17 lb. downwards and T upwards. The value of their resultant is $17 - T$ pounds, or $32(17 - T)$ poundals. This force gives to the mass of 17 lb. an acceleration of 2 ft. per second each second; therefore, its value is 34 poundals.

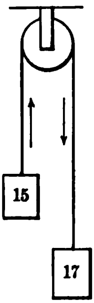


FIG. 163.

Therefore, $34 = 32(17 - T)$, whence $T = 15\frac{15}{16}$ lb.

The same value of T will be obtained by considering the forces which act on the mass of 15 lb. Their resultant is $32(T - 15)$ poundals.

183. Impulse of a Force. The product of a force and the time during which it acts is called the *impulse* of the force. If we multiply both sides of the equation

$$\textit{Force} = \textit{mass} \times \textit{acceleration}$$

by the time during which the force acts, we obtain

$$\textit{Force} \times \textit{time} = \textit{mass} \times \textit{acceleration} \times \textit{time}.$$

But velocity produced = acceleration × time; therefore,

$$\textit{Force} \times \textit{time} = \textit{mass} \times \textit{velocity}.$$

That is to say, the impulse of a force is equal to the total momentum produced by the force.

In the case of *blows*, like that of a bat upon a ball, or a hammer upon a nail, the time of action is too small to be measured. Such forces are called *impulsive* forces, and are measured by their impulse, or the total change of momentum which they produce.

184. Physical Independence of Forces. The Second Law of Motion states that the change of momentum produced by a force takes place in the direction of the force. This is a very concise way of saying that a force always produces its full effect in its own direction, whether it act alone on a body at rest, or with other forces on a body already in motion.

If the body is already moving when the force begins to act, then the actual motion of the body is found by compounding, by means of the parallelogram law, the motion already existing with the new motion impressed by the force. In studying the motion of a projectile we have seen how this is done.

If several forces act simultaneously on a body, the actual motion of the body will be the resultant of the component motions due to the several forces. The motion of a boat which is rowed directly across a river, and at the same time carried down stream by the current, is an example.

CLASS-ROOM EXERCISES.

Values of g: 32 with the foot as unit, and 980 with the cm. as unit.

1. What must be the velocity of 1 ton that it may have the same momentum as a mass of 2 lb. moving with a velocity of 1200 ft. per second? 2 ft.

2. The mass of an iceberg is 50,000 tons, and that of a steamboat is 200 tons. Their respective velocities are 6 ft. a minute and 15 miles an hour. Which has the greater momentum?

3. What is the momentum of a mass of 1 lb. after falling 1 second? 32

4. What is the momentum of a mass of 1 lb. after falling 400 ft.? 160

5. Find the force in dynamical measure with which gravity pulls (1) a mass of 4 lb., (2) a mass of 4 grams, (3) a mass of 4 kg. 3920 392000

6. What acceleration will a force equal to a weight of 6 lb. impart to a mass of 24 lb.? How far will the mass move in 3 seconds? a=8; 36 ft.

7. What force in grams weight will give to a mass of 200 grams a velocity of 6 m. in 1 second? ~~120000~~ 122

8. A mass of 1 ton under the action of a constant force describes 160 ft. from rest in 8 seconds. Find the force in pounds weight.

Hints. Find the acceleration by the formula $s = \frac{1}{2} at^2$. Force = mass × acceleration. Then reduce poundals to pounds (§ 180). 312.5

Sep 9' **9.** Masses of 7 lb. and 9 lb. are connected by a light string passing over a smooth pulley. Find (1) the velocity after 2 seconds, (2) the distance either mass describes, (3) the tension of the string. 5 ... 7 7/8

Sep 9' **10.** Masses of 30 grams and 40 grams hang by a cord over a pulley. Find the space described in 1 second by each mass. 70

Sep 9' **11.** Masses of 14 lb. and 18 lb. are joined by a string over a pulley. How far will they move in 3 seconds? What is the tension of the string? 18; 63/4

12. Two masses of 48 grams and 50 grams are attached by a cord passing over a pulley. Starting from rest, each mass describes 10 cm. in 1 second. Find the value of *g*. 980

13. How far on a smooth horizontal plane will a mass of 3 lb. move in 2 seconds, if the moving force is a mass of 1 lb. connected with the mass of 3 lb. by a cord and hanging vertically? 21

14. How far will a mass of 48 lb. move on a horizontal plane in 1 minute, if acted on by a force equal to the weight of 12 lb., (1) supposing no friction, (2) supposing the coefficient of friction to be $\frac{1}{6}$?

14400 — 4800

Hint. In case (2) the friction = 8 lb; therefore, the moving force = 12 − 8, or 4 lb. = 4 × 32 poundals.

15. In what time will 4 lb. hanging vertically draw 12 lb. through 45 feet on a smooth horizontal plane (1) supposing no friction, and (2) supposing the coefficient of friction to be $\frac{1}{4}$? 2.9 5.8

16. How far will a mass of 60 kg. move on a horizontal plane in 10 seconds if acted on by a force equal to the weight of 30 kg., (1) supposing no friction, (2) supposing the coefficient of friction to be $\frac{1}{5}$? $\frac{1}{2}$

17. A train whose mass is 200 tons, moving at the rate of 30 miles an hour, is brought to rest in 20 seconds. What is the average value of the brake power? How far does the train move before coming to rest?

Hints. 30 miles an hour = 44 ft. per second.
Momentum = 200 × 2000 × 44. Let x = force of the brakes.
Impulse of this force = the momentum destroyed.
Or, $20x = 200 \times 2000 \times 44$, whence x = 880,000 poundals = 27,500 lb.
Retardation $= \frac{\text{force}}{\text{mass}} = \frac{880{,}000}{200 \times 2000} = \frac{11}{5}$ ft. per second each second.
Space described $= \frac{1}{2}at^2 = \frac{1}{2} \times \frac{11}{5} \times 20^2 = 440$ ft. 2.15 a

18. A train of 640 tons is moving at the rate of 30 miles an hour when the steam is shut off, and a brake power equal to the weight of 20 tons is applied. In what time will the train be brought to rest, and how far will it move before stopping? T = 44 sec.

19. A 1000-lb. shot strikes a target with a velocity of 1600 ft. a second. How far will the shot penetrate if the target exert upon the shot an average pressure of 12,000 tons? 5/3 ft.

Hint. Time of stopping = momentum ÷ force = $\frac{1}{480}$ second.

20. A weight of 2 tons drops on the head of a pile from a height of 32 ft., and drives the pile through a distance of 1 ft. What is the average resistance of the ground, the weight of the pile being neglected?

21. A train whose mass is 200 tons is drawn up an incline of 4 per cent grade at uniform speed. The resistance of friction amounts to 8 lb. per ton. Find the tractive force of the engine.

22. What force will stop in half a mile a train of 300 tons moving at the rate of 45 miles an hour?

23. What pressure will a man weighing 160 lb. exert on the floor of an elevator which is ascending with an acceleration of 4 ft. per second?

Hint. The force required to give the man an acceleration of 4 ft. per second = 160 × 4 poundals = 20 lb. The pressure = his weight + 20 lb.

24. A balloon descends with a uniform acceleration of 8 ft. per sec. What pressure will a man weighing 200 lb. exert on the floor of the car?

Third Law of Motion.

185. Law of Action and Reaction. Newton's Third Law of Motion is as follows:

To every action there is always an equal and contrary reaction; or, the mutual actions of two bodies are equal and in opposite directions.

This law expresses the fact that the action of force is always a two-sided phenomenon. If a body A acts upon a body B, then B always reacts with equal force upon A. When we confine our attention to one of the bodies, we see only one side of the phenomenon, and call the action an external force. When we take both bodies into account, we call the whole phenomenon a *stress*, and call its two sides or aspects the *action* and the *reaction*.

In the case of a balanced force the word 'action' means *pressure* or *tension*. If you press with your hand upon a wall, the hand exerts an action upon the wall, and the reaction is the equal opposite pressure which you feel when the hand touches the wall. If a body is suspended by a cord, the action is the weight of the body, and the reaction is the equal upward pull of the cord.

In the case of an unbalanced force the word 'action' means *change of momentum*. If two boats are side by side, and a man in one of them pushes the other, both boats move, but in opposite directions. Here the action is measured by the momentum acquired by one boat, and the reaction by the momentum acquired by the other boat.

When a gun is fired, the powder is instantly converted into gas at a very high pressure. The force exerted by this gas against the bullet is equal and opposite to that exerted against the gun. Hence, the momentum acquired by the bullet is equal and opposite to that acquired by the gun, if the latter is free to move.

186. Impact. The term *impact* denotes the action which lasts a very short time when two bodies come into collision.

If a body A strike a body B, then, by the third law, the action of A on B at each instant while they are in contact is equal and opposite to that of B on A. By the second law, the change in the momentum of B is equal and opposite to the change in the momentum of A. Hence, the sum of these changes, one being considered positive and the other negative because they are in opposite directions, is zero. Therefore, *the total momentum of the bodies after impact is equal to their total momentum before impact.*

Suppose that the bodies move along the same straight line, and let m and m' denote their masses, u and u' their velocities before impact, v and v' their velocities after impact; then

the total momentum before impact $= mu + m'u'$;
the total momentum after impact $= mv + m'v'$.

Therefore,

$$mv + m'v' = mu + m'u'. \qquad [1]$$

If the bodies are perfectly inelastic, mutual action will cease as soon as their velocities become equal, and they will move on together as a single body.

Therefore, in this case, $v' = v$, and equation [1] becomes

$$(m + m')\, v = mu + m'u';$$

whence,

$$v = \frac{mu + m'u'}{m + m'}.$$

If the two bodies before impact are moving in *opposite* directions, and u is considered positive, then u' must be considered *negative.* Suppose, for example, that an inelastic mass of 4 lb., moving with a velocity of 12 ft. per second, strikes an inelastic mass of 2 lb. moving with a velocity of 9 ft. per second in the same direction; then

$$(4 + 2)\, v = 4 \times 12 + 2 \times 9; \text{ whence } v = 11 \text{ ft. per second.}$$

If, however, the second mass is moving in the opposite direction,

$$(4 + 2)\, v = 4 \times 12 - 2 \times 9; \text{ whence } v = 5 \text{ ft. per second.}$$

Let us now suppose that the two bodies which collide are perfectly elastic (§ 24). In this case the bodies are first compressed by their mutual action till they have the same velocity, and then the elastic force developed by the compression causes each body to recover its original form. During this period of recovery the mutual action continues, and causes the same change of velocity in each body as during the period of compression.

Therefore, if c denote the common velocity at the instant of greatest compression:

$$c - v = u - c,$$

and $$v' - c = c - u';$$

whence, by addition, $$v' - v = u - u'. \qquad [2]$$

or, *the difference of the velocities is unaltered by the impact.*

Equations [1] and [2] enable us to compute the values of v and v' when those of m, m', u, and u' are given.

If $m = m'$, equation [1] reduces to

$$v + v' = u + u'.$$

Equation [2] is $$v' - v = u - u'.$$

Adding, $$2v' = 2u, \text{ or } v' = u.$$

Subtracting, $$2v = 2u', \text{ or } v = u'.$$

That is, if the bodies have equal masses, they *exchange velocities.* This truth is illustrated in Fig. 164. Two ivory balls, A and B, are suspended side by side. A is drawn aside to C, and then allowed to fall and strike B. A will be brought to rest by the collision, while B will move off with the velocity acquired by A. In this case the entire momentum of one body is transferred to the other. The equation $v' - v = u - u'$ applies to the case of a perfectly elastic ball striking a perfectly hard plane at right angles if we suppose that $u' = 0$ and $v' = 0$. In this case $v = -u$; that is, the ball will *rebound* with a velocity equal to that with which it struck the plane.

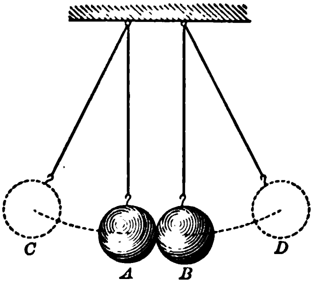


FIG. 164.

187. Action at a Distance. In many cases there is action between two bodies, although the bodies are not in contact, and no third body, like a string or rod, capable of transmitting the action, can be detected. The attraction between the earth and a body above its surface, which causes the body to fall to the ground, is a familiar example. The mutual action of two magnets or of two electrified bodies, which attract or repel each other, is a similar case. This mutual action is called *attraction*, if it tends to bring the bodies together, and *repulsion* if it tends to separate them.

The third law applies to every case of this kind. If the bodies are free to move, each body undergoes the same change of momentum in the same time, but the changes are opposite in direction. If the bodies are prevented from moving, the mutual action shows itself in the form of a pair of equal opposite pressures or tensions.

The ancients observed that a magnet attracts iron, but they overlooked the fact that the iron attracts the magnet. Newton showed by a simple experiment that these two forces are equal and opposite. His experiment may be repeated by fastening a magnet and a piece of iron to wooden blocks, and then floating the blocks near each other in water (Fig. 165). The blocks will rush together, and then remain at rest. Since neither block is able to move the other block together with itself through the water, the attraction of the iron on the magnet must be equal and opposite to that of the magnet on the iron.

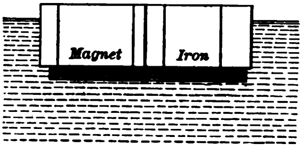


FIG. 165.

If a large magnet and a small piece of iron are suspended near each other by means of strings, the iron will rush up to the magnet, while the magnet hardly moves at all. The reason is this: the momenta acquired by the two bodies are equal; consequently the body with the greater mass must have the smaller velocity.

If the mass of the magnet is 100 times that of the iron, then the velocity with which the iron moves will be 100 times as great as that with which the magnet moves.

188. Internal Forces. If we look upon two or more bodies as forming one material system, then all forces that act between parts of this system are called *internal* forces. Thus, we may regard a vessel with all it contains as a material system. In this case, the forces exerted by the sailors in the discharge of their duties would be internal forces. On the other hand, the force exerted by the wind upon the sails would be an external force.

The application of Newton's Third Law to internal forces leads to the following general propositions :

1. *The total momentum of a material system is unaltered by the action of internal forces.*
2. *The motion of the center of gravity of a material system is unaltered by the action of internal forces.*

The truth of the first principle is illustrated in the cases of impact already considered (§ 186). In applying the principle, oppositely directed velocities must be considered to have opposite signs.

To show the truth of the second principle in a case where only two bodies are involved, imagine two masses, m and $2m$, placed at A and B, respectively (Fig. 166). Their center of gravity will be at a point C, so situated that $AC = 2\,BC$ (§ 57). Now suppose that these masses move towards each other in obedience to mutual attraction, and that after any interval of time they arrive at the positions D and E.

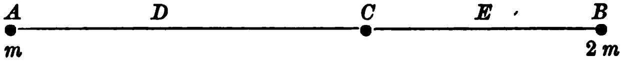


FIG. 166.

By the third law, the masses in the same time acquire equal momenta. It follows that at each instant the velocity of the mass m must be twice that of the mass $2m$. Hence, the distance described by m must be twice that described by $2m$, or $AD = 2\,BE$. Therefore, $AC - AD = 2\,(BC - BE)$, or $DC = 2\,CE$; so that the point C will still remain the center of gravity.

When a bombshell explodes in the air, the fragments may fly in any direction, but their center of gravity continues to describe the parabolic path which it was describing when the explosion occurred.

CLASS-ROOM EXERCISES.

Note. In the exercises on collision the motion is supposed to be in a straight line passing through the centers of gravity of both bodies.

1. A 24-lb. ball leaves a gun whose mass is 5 tons with a velocity of 1200 ft. per second. Find the velocity of recoil of the gun.

2. A 64-lb. shot is fired from a gun whose mass is 8 tons; the velocity of recoil of the gun is 6 ft. per second. Find the velocity of the shot.

3. A mass of 10 lb., moving with a velocity of 50 ft. per second, strikes a mass of 50 lb. at rest. If the masses keep together after impact, what is their common velocity?

4. Two inelastic masses, $A = 8$ lb., $B = 12$ lb., collide. Find the common velocity after collision, if the velocities before collision are 7

(1) *A* 10 ft. per second, *B* 5 ft. per second in the same direction.
(2) *A* 10 ft. per second, *B* 5 ft. per second in opposite directions.
(3) *A* 10 ft. per second, *B* 20 ft. per second in opposite directions.
(4) *A* 10 ft. per second, *B* at rest.

5. Two inelastic bodies, moving in opposite directions, collide, and are brought to rest. The velocities of the bodies before collision were 48 m. and 100 m. per second. The first body has a mass of 150 kg. What is the mass of the other body?

6. A body *A* with mass 8 and velocity 10 overtakes a body *B* with mass 12 and velocity 5. After impact, *B*'s velocity is 9 in the same direction as before. What is *A*'s velocity?

7. A perfectly elastic mass of 120 kg., moving at the rate of 286 m. per second, collides with a similar mass of 45 kg. which is at rest. Find the velocities after collision.

8. A fishing boat weighing 4 tons is 40 yards from the shore. A man in it hauls a cask weighing 200 lb. from the shore. How far will the cask be from the shore when it reaches the boat? 117 3/4 ft.

9. A hand-car weighing 720 lb. and running 10 ft. per second collides with another weighing 640 lb. and running in the opposite direction at the rate of 15 ft. per second. What will be the result?

10. A shell weighing 200 lb. and moving at the rate of 1000 ft. per second explodes into two parts, one of which weighs 50 lb. and is just brought to rest. What is the velocity of the other part? 1333 1/3

11. An ounce bullet is fired into a block of wood at rest, which weighs 4 pounds. If the block by the impact receives a velocity of 16 ft. per second, find the velocity of the bullet before impact. 1040

The Pendulum

189. The Simple Pendulum. A body supported so that it can move to and fro about a fixed point under the action of gravity is called a *pendulum.* In order to study the motion of a pendulum, the body is reduced in thought to a material particle, and the string or rod by which it is supported is assumed to have no weight; this ideal pendulum is called a *simple pendulum.*

A small bullet suspended by a fine silk thread is a close approximation to a simple pendulum.

If a simple pendulum is drawn aside from its vertical position AB (Fig. 167) to any other position AC and then let go, there is an unbalanced component of gravity CE which causes the bullet or *bob* to descend from C to B, in the arc of a circle, with an accelerated motion. After reaching B the bob continues in motion by reason of the momentum which it has acquired, and rises against the action of gravity almost to the point D in the same horizontal line as C. Then gravity, which continues to act, reverses the motion, and the pendulum retraces its path. Thus the pendulum will move back and forth on both sides of the line AB, till it is brought to rest by the resistance of the air and the friction at the point of support.

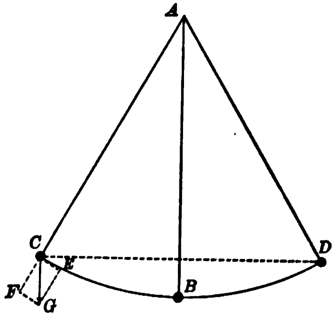


FIG. 167.

The motion of the pendulum from C to D, or from D to C, is called one *oscillation* or *vibration.* The *time of vibration* is the time required for the pendulum to swing from C to D.

Half the angle CAD, or the arc BC (measured in degrees) is called the *amplitude* of the vibration.

190. The Pendulum Formula. If the amplitude of vibration is very small, it can be proved that the time of vibration depends only on the length of the pendulum and the acceleration of gravity, and is given by the formula

$$t = \pi \sqrt{\frac{l}{g}},$$

where $t =$ the time of vibration, $l =$ length of the pendulum, $g =$ the acceleration of gravity, and $\pi = 3.1416$ (the ratio of the circumference of a circle to its diameter).

191. Laws of the Pendulum. If we examine the pendulum formula, we find in it neither the amplitude of vibration nor the mass of the bob. In this formula, t increases as l increases, but diminishes as g increases. The value of t, however, changes more slowly than those of l and g, because the latter are under the square root sign.

The formula, therefore, contains the following laws:

1. *Small vibrations are isochronous (made in equal times).*
2. *The time of vibration at any place is independent of the mass of the bob or the nature of the material.*
3. *The time of vibration varies directly as the square root of the length.*
4. *The time of vibration varies inversely as the square root of the acceleration of gravity.*

The meaning of law 1 may be illustrated by supposing four pendulums to be made to vibrate so that the respective amplitudes of vibration are 1°, 2°, 3°, and 4°. Then these pendulums will be found to vibrate in the same time.

Newton proved law 2 experimentally by causing two pendulums, equal in length, to swing side by side. The bobs were round equal wooden boxes, so that the resistance of the air affected them equally. He placed in the boxes at different times lead, glass, sand, salt, water, and other materials, and found that the times of vibration were always the same. From this result Newton inferred that the value of g is the same for all bodies, and that the mass of a body varies directly as its weight.

192. The Compound Pendulum. Every actual pendulum may be termed a *compound* pendulum; for it may be regarded as consisting of an indefinite number of simple pendulums, one for each particle of matter in the compound pendulum. All these particles are rigidly connected with the axis of the pendulum and also with one another, so that they are compelled to vibrate in the same time. If the particles were free, each would vibrate in the time determined by its distance from the axis; hence those near the axis would vibrate faster than those remote from the axis (law 3). The rigid connection effects, so to speak, a compromise in the times of vibration; it retards the motion of the nearer particles and accelerates the motion of those more remote. There must be somewhere a particle *C* (Fig. 168) which is neither accelerated nor retarded, but which vibrates with the others in exactly the same time that it would if free.

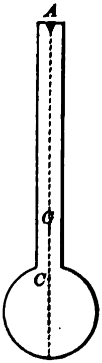


FIG. 168.

The position of *C* is called the *center of oscillation* of the pendulum. The distance *AC*, from the axis to the center of oscillation, is called the *length* of the pendulum; it is this value of l which must be used in the pendulum formula to find the time of vibration of the pendulum.

The position of the center of oscillation may be found either by calculation or by experiment. The latter method is based on a remarkable truth discovered by the famous Dutch physicist, Huyghens (1629–1695). He showed that, if a pendulum be inverted and suspended by an axis passing through the center of oscillation, the time of vibration will be unchanged. This truth is often expressed by saying that *the centers of suspension and oscillation are reversible.*

The center of oscillation is also called the *center of percussion*, because a blow delivered at this point will set the pendulum into vibration without causing any jar at the point of support.

193. Uses of the Pendulum. The chief practical use of the pendulum is to regulate the motion of a clock, and the chief scientific use is to determine the value of g, the acceleration of gravity, at different places.

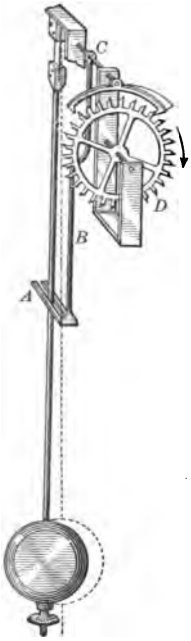


FIG. 169.

Fig. 169 will serve to illustrate how the motion of a clock is regulated by a pendulum. The rod of the pendulum, as it swings, communicates motion to the metal piece AB, which turns about a horizontal axis C. To this axis is attached a curved piece of metal called an *escapement*, terminating in two projections, called *pallets*, which work alternately against the *escapement wheel* D. This wheel is urged in the direction of the arrow by a weight and wheel work not shown in the figure, and its motion is alternately permitted and arrested by the action of the pallets as the pendulum swings to and fro. At the same time the wheel gives to the pendulum the small pushes required to keep up its motion. We owe to Huyghens this ingenious application of the pendulum as a means of measuring time.

The methods which have been employed to measure the value of g by means of a pendulum are based on the pendulum formula, whence we have

$$g = \frac{\pi^2 l}{t^2}.$$

In Captain Kater's method, the length of a compound pendulum of peculiar construction is determined with great accuracy by applying the principle that the centers of suspension and oscillation are interchangeable; then the time of vibration is very accurately measured by counting the number of vibrations made in a given period of time, such as an hour, and then dividing the period of time, expressed in seconds, by the number of vibrations; and finally the values of l and t thus found are substituted in the above formula.

If L denotes the length of a pendulum vibrating once a second at any locality, the value of g at that locality is given by the formula

$$g = \pi^2 L.$$

CLASS-ROOM EXERCISES.

1. Find the time of vibration of a simple pendulum 49 cm. long at a place where $g = 980$ cm. per second each second. .7024 7 sec.

2. Find the length in centimeters of a pendulum vibrating once a second at a place where $g = 980$. 99, [illegible] 7

3. Find the length of a seconds pendulum (a pendulum that vibrates in one second) at New York where $g = 32.16$. 3.2

4. Compare the lengths of four pendulums that oscillate in 1, 2, 3, and 4 seconds, respectively.

5. Assuming a seconds pendulum to be 1 meter long, what will be the length of a pendulum that vibrates in 10 seconds, and the length of a pendulum that vibrates in half a second?

6. If a certain pendulum vibrate once a second, what is the time of vibration of a pendulum 9 times as long? Also, what is the time of vibration of a pendulum half as long?

7. If an iron ball suspended by a fine wire from the cupola of St. Paul's in London makes 176 vibrations in half an hour, what is the height of the cupola above the floor?

8. How will the time of vibration of a pendulum be affected by taking it to the top of a high mountain?

9. If a seconds pendulum were taken to a place where the force of gravity is only one fourth of that at the surface of the earth, in what time would the pendulum make one vibration? If the pendulum were attached to a clock, how much would the clock appear to lose in 24 hr.?

10. What is meant by the *length* of a compound pendulum? How is the length found by experiment?

11. A clock keeping correct time at Greenwich, where $g = 32.19$, gains 16 sec. a day at another place. Find the value of g at this place.

Solution. One day contains 24 × 60 × 60, or 86,400 seconds.

Time of one vibration at the other place $= \frac{86{,}400}{86{,}416}$ seconds.

From the formula $t = \pi\sqrt{\frac{l}{g}}$, we have $l = \frac{t^2 g}{\pi^2}$.

Since at Greenwich $t = 1$, and $g = 32.19$, therefore $l = \frac{32.19}{\pi^2}$.

From the same formula $g = \frac{\pi^2 l}{t^2}$. Substituting for l and t their values,

$$g = \frac{32.19 \times 86{,}416 \times 86{,}416}{86{,}400 \times 86{,}400} = 32.204 \text{ nearly.}$$

Circular Motion.

194. Acceleration in Circular Motion. Let a body revolve in a circle of radius r with the uniform velocity v. Some force must act on it; otherwise it would move in a straight line. Since the rate of motion is constant, this force must be perpendicular to the direction of the motion, and, therefore, must always be directed towards the center of the circle; for this reason the force is called a *centripetal* force (center-seeking force). Again, since the direction of the motion changes by equal amounts in equal times, the acceleration of the force must be constant.

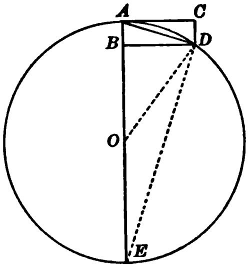


FIG. 170.

Let a denote the value of the acceleration, and let AB (Fig. 170) be the distance through which the body would fall from a state of rest towards the center in a small interval of time t. Then (§ 182)

$$AB = \frac{1}{2} at^2.$$

During this time t, the body actually describes the arc AD of the circle with the uniform velocity v. Therefore (§ 161), arc $AD = vt$.

If t is made smaller and smaller, ultimately the chord AD and the arc AD will coincide. Therefore, ultimately, the chord $AD = vt$.

Now the chord AD is a mean proportional between AB and the diameter AE of the circle, or

$$\overline{AD}^2 = AB \times AE.$$

Substituting the values of AD, AB, and AE, we have

$$v^2t^2 = \frac{1}{2} at^2 \times 2r;$$

$$\text{whence } a = \frac{v^2}{r}.$$

195. Force in Circular Motion. Let m denote the mass of the revolving body in Fig. 170, and F the centripetal force *measured in dynamical units* (poundals or dynes); then, since force = mass × acceleration (§ 179),

$$F=\frac{mv^2}{r}.$$

Let t denote the time required for the body to make one revolution; then, since the circumference of a circle of radius r is equal to $2\pi r$,

$$vt=2\pi r.$$

If the value of v, given by this equation, is substituted in the equation at the bottom of the preceding page, we obtain

$$a=\frac{4\pi^2 r}{t^2}.$$

We may, therefore, express the value of F as follows:

$$F=\frac{4\pi^2 mr}{t^2}.$$

If the value of F, as obtained by either of these equations, is divided by g, we have the force in statical units (pounds or grams).

If the velocity is given in angular measure (degrees per second), the linear velocity v is found by the proportion

$$v:2\pi r=\text{angular velocity}:360^\circ.$$

Circular motion implies, therefore, the constant action of a force directed towards the center of the circle. But there is no action without an equal opposite reaction. In this case the reaction is an *outward* pull on the center, and it is often called the *centrifugal* force. Centrifugal force is the resistance which the inertia of the revolving mass offers to change in the direction of its motion, and tends to make the center move towards the revolving mass.

196. Centrifugal Force. Centrifugal force shows itself by a tendency on the part of the revolving body to increase its distance from the axis. If the pull or push towards the axis is less than that required by the formula for F in the last section, the body will move away from the axis.

Illustrations. (1) A horizontal rod fixed in a wooden frame is made to rotate rapidly around a vertical axis (Fig. 171). A metal ball can slide along the rod. If the rotation is rapid enough the ball will slide to the end of the frame, where it meets with an inward push equal to the centripetal force required by the formula for F. By attaching the ball to a spiral spring, the value of F for different velocities can be read from a scale graduated to pounds or grams.

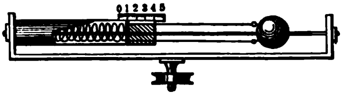


FIG. 171.

(2) Fig. 172 represents an iron rod mounted so as to revolve in a vertical position. Four elastic strips of steel are fastened to the bottom of the rod, and are connected at the top to a ring which can slide up and down the rod. When the apparatus is set into rapid rotation the ring slides down the rod, and the faster the motion the farther down it will move. At a high rate of speed the strips run together in appearance, and assume the shape of a solid called a spheroid.

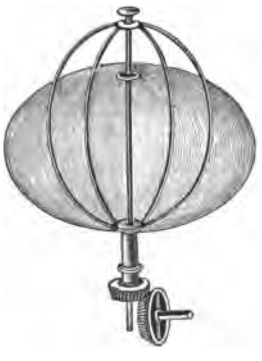


FIG. 172.

The earth has such a shape, its polar diameter being about 26 miles less than the equatorial. The earth also rotates on its axis every 24 hours, so that all parts of its surface are subject to centrifugal force, increasing in amount from the pole to the equator. It is considered probable by men of science that the earth was once a hot fluid sphere. If so, its present shape is explained by centrifugal force operating at a time when the materials were plastic enough to obey its action. The effect of this centrifugal force is to diminish the value of g, as we go from the pole to the equator, by an amount equal to $\frac{1}{289}$ of its value at the pole.

(3) In the manufacture of window glass a lump of melted glass is attached to the end of an iron tube and blown into a hollow globe *A* (Fig. 173). This, while soft, is opened into a cup *B*, which is held in front of a furnace and rapidly whirled round on the tube as an axis. The edges soon widen out and assume the shape *C*; and finally the whole spreads out into a thin uniform flat sheet *D*.

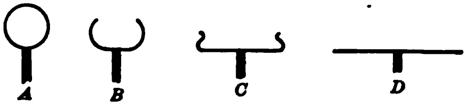


FIG. 173.

CLASS-ROOM EXERCISES.

1. Give a clear explanation of the meaning of the terms "centripetal force" and "centrifugal force," and illustrate by an example.

2. A stone weighing 10 lb. is attached to a cord 3 ft. long and whirled around twice a second. Find the tension of the string.

Solution. Circumference of the circle = 6π ft.

Velocity of the stone = 12π ft. per second.

Tension of string $= \dfrac{mv^2}{r} = \dfrac{10 \times 144\pi^2}{3} = 4800$ poundals $= 150$ lb.

3. A mass of 5 lb. is compelled to move in a circle of radius 16 ft. at the rate of 900 ft. per minute. Find the centripetal force. 2.19 lb

4. A mass of 10 lb. at the end of a wire 4 ft. long revolves 100 times a minute. Find the tension of the wire. 137.07 lbs

5. A car whose mass is 4 tons and velocity 20 miles an hour goes round a curve of 1100 ft. radius. Find the pressure on the rails. 633

6. How many times a minute must a mass of 1 lb. revolve horizontally at the end of a string 1 ft. long in order that the tension of the string may be equal to the weight of 1 lb.?

7. Taking the earth's radius as 4000 miles, and $\pi^2 = 10$, find by how much the weight of a body at the equator is diminished by the rotation of the earth on its axis.

Hints. From the formula $a = \dfrac{4\pi^2 r}{t^2}$, we obtain by substituting the value of r in feet and t in seconds, $a = \dfrac{55}{486}$; hence $a : g = \dfrac{55}{486} : 32$.

Whence $a = \dfrac{1}{282}\, g$ nearly. A more accurate value is $\dfrac{1}{289}$.

Universal Gravitation.

197. The Law Stated. Newton, in his *Principia* (Principles of Natural Philosophy), gives as examples of centripetal force: "gravity by which bodies tend to the center of the earth," and "that force, whatever it is, by which the planets are perpetually drawn aside from the rectilinear motions which otherwise they would pursue, and made to revolve in curvilinear orbits."

By the discovery of the Law of Universal Gravitation Newton proved that these two forces are one and the same.

The law may be stated as follows:

Every particle of matter in the universe attracts every other particle with a force which varies directly as the product of their masses, and inversely as the square of the distance between them.

The mathematical statement of the law is as follows:

$$F = \frac{kmm'}{d^2}.$$

Where m and m' denote the attracting masses, d their distance apart, F the force of attraction, and k the value of F when mm' and d are each unity (k is a very small fraction).

The meaning of the law is best seen by assuming that the quantities m, m' and d undergo variations.

Thus, if m is doubled, F is doubled; if both m and m' are doubled, F is quadrupled; if m is increased 6 times, and m' 4 times, F is increased 6×4 or 24 times.

Again, if d is doubled, F is made one fourth as great; if d is trebled, F is made one ninth as great, etc.

If mm' and d are each equal to unity, then $F = k$. But suppose that $m = 6$, $m' = 8$, $d = 4$; then

$$F = \frac{k \cdot 6 \cdot 8}{4 \cdot 4} = 3k.$$

If mm' and d are each doubled, F is evidently unchanged.

198. What the Law Explains. Newton concluded that the Law of Gravitation is true, because he found that it would explain the motion of the moon around the earth. He then proceeded with marvelous mathematical power to apply the law to the explanation of various important phenomena:

(1) He proved that the moon is attracted by the sun as well as by the earth; and he calculated the chief of the lunar perturbations due to the sun's attraction.

(2) He applied the idea of centrifugal force to the earth, considered as a rotating body originally in a fluid state. He showed that the earth could not be a true sphere, and calculated that the polar diameter must be about 28 miles less than the equatorial diameter.

(3) He found that the force of gravity varies at different places on the earth, partly by reason of the shape of the earth, partly by reason of centrifugal force; and he determined the amount of variation due to each of these causes.

He found that gravity at the pole exceeds gravity at the equator by $\frac{1}{590}$ of the whole on account of the shape of the earth, and by $\frac{1}{289}$ on account of centrifugal force. Both causes combined make the amount $\frac{1}{195}$ of the whole; that is, a body weighing 195 lb. at the pole will weigh only 194 lb. at the equator.

(4) He proved that a uniform sphere attracts external bodies as if its mass were all concentrated at its center; and also that the attraction at an internal point varies directly as its distance from the center.

Thus, the force of gravity is a maximum at the surface of the earth, and decreases as you go in either direction, up or down.

(5) He showed that the attraction of the sun, moon, and planets upon the earth's equatorial protuberance causes the phenomena known as precession and nutation.

(6) He proved that the tides are caused by the attraction of the moon and the sun upon the waters of the earth.

REVIEW EXERCISES ON CHAPTER V.

1. Define *uniform motion* and *velocity.*

2. How is the resultant of two component velocities at right angles to each other found?

3. Show that rest and motion are relative terms.

4. Define motion of *translation* and motion of *rotation.* What sort of motion has a point on the tire of a carriage wheel?

5. How is variable velocity measured? How is the mean velocity of a moving point for any interval of time found?

6. What is meant by *acceleration?* How is it measured?

7. Explain the meaning of the formulas, $v = gt$, $s = \frac{1}{2} gt^2$, $v^2 = 2gs$; and show how they are obtained.

8. Prove that acceleration down an inclined plane is found by multiplying g by the ratio of the height of the plane to its length.

9. Prove that a body thrown vertically upwards with the velocity u will rise to the height of $\frac{u^2}{2g}$.

10. Show by a diagram that the path of a projectile is a curve.

11. State Newton's First Law of Motion, and give examples.

12. Apply the First Law to the motion of a body in a circle.

13. State Newton's Second Law. What is the measure of a force?

14. Define the *poundal* and the *dyne.* How are forces reduced from gravitation measure to absolute measure, and *vice versa?*

15. What is the dynamical definition of equal masses?

16. Define the *impulse* of a force, and show that it is equal to the total momentum produced by the force.

17. Give an example of the physical independence of forces.

18. State and illustrate Newton's Third Law.

19. Prove the formula for the collision of two inelastic bodies.

20. Prove that when two perfectly elastic bodies collide the difference of the velocities is unaltered by the collision.

21. What laws result from the application of Newton's Third Law to internal forces?

22. State the laws of the motion of a pendulum.

23. What is meant by the *center of oscillation* of a compound pendulum?

24. Prove the formula for acceleration in circular motion.

25. State the Law of Universal Gravitation. Mention some of the phenomena which Newton explained by means of it.

CHAPTER VI.

ENERGY.

Mechanical Work.

199. Definition of Work. *Work* is the act of changing the position of a body by overcoming resistance to the change.

It involves three elements: a resistance to be overcome, a force that overcomes it, and a distance through which the body moves. The work is done *by* the force and *against* the resistance.

A man does work when he lifts a weight, or turns a grindstone, or drives a nail into wood. A horse does work when he moves a loaded cart. A steam engine does work when it is used to pump water, or saw wood, or propel a railway train.

A man may work with his mind as well as with his body. He then performs intellectual labor, while, if he works with his body, he performs physical labor or mechanical work. The only kind of work which we consider is mechanical work.

200. Work done against Gravity. One of the most common forms of work is the raising of a weight against the action of gravity. The work done in raising a weight of one pound through a distance of one foot is chosen by engineers as the unit, and is called a *foot-pound* (ft.-lb.).

The corresponding metric unit is the *kilogram-meter* (kg.-m.).

If a pound weight is raised 2 ft., or two pounds are raised 1 ft., in either case 2 units of work are done; if a body weighing 8 lb. is raised 6 ft., then 8 × 6, or 48 units of work are done.

Work done against gravity = weight × height.

The foot-pound varies in value with the latitude (§ 171), but the variations are so small that for engineering purposes they may be neglected.

201. Work done by Gravity. Suppose that, instead of raising a body against the action of gravity, we allow it to fall from an elevated position to the ground. In this case work is done by gravity against the inertia of the body. The work is measured by the product of the weight of the body and the distance through which it falls.

Thus, when a piece of slate weighing 2 lb. falls to the ground from the edge of a roof 60 ft. high, the force of gravity does 2 × 60 or 120 ft.-lb. of work.

202. Work done by Other Forces. Forces may be expressed in pounds or in kilograms; moreover, the same amount of work is done, whether a pressure of 1 lb. is exerted through a distance of 1 ft. in a vertical direction, or in any other direction. Therefore, the foot-pound or the kilogram-meter may be used to measure every kind of mechanical work.

If the direction of the applied force coincides with the direction of the motion, and the word *distance* is understood to mean the distance through which the body moves, then the work done by the force is given by the formula

$$Work = force \times distance.$$

This product is also the work done against the resistance.

Thus, if a man walk a mile against a gale of wind, the mean pressure of which against his body is 100 lb., he will do 100 × 5280 or 52,800 ft.-lb. of work; or as much work as he would do in raising 200 lb. of coal out of a mine half a mile deep.

To do work requires the combination of force and motion.

If a man holds a heavy weight in his hands, he will, no doubt, suffer fatigue; but there is no motion, and no work is done on the weight.

If a body slides along the surface of a table, work is done in overcoming friction. If, however, the table were *perfectly* smooth, no work would be done; there would be motion, but no resistance to be overcome. Cases like this may be imagined, but they do not actually exist.

203. Work of Oblique Forces. Very often a force acts on a body in one direction and the body moves in some other direction. Let us examine two cases.

Case 1. Let the direction of the force be at *right angles* to the direction of the motion. In this case no work is done, either *by* the force or *against* it. Thus, no work is done, either by gravity or against gravity, when a railway train is moving along a perfectly level track.

Case 2. Let the direction of the force be *oblique* to the direction of the motion. As a typical case of this kind, consider a body of weight W sliding without friction by the action of gravity down an inclined plane ABC (Fig. 174).

Here the force W acts in the direction AB, while that of the motion is AC. Let AD represent W, and resolve it into the components AE and AF. AE does no work (Case 1), and the work done by AF is $AF \times AC$ (§ 202). Since the triangles ABC, AFD are similar, $AF : AB = AD : AC$.

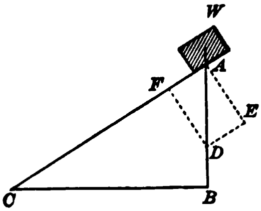


FIG. 174.

Therefore, $AF \times AC = AD \times AB = W \times AB$.

Hence, the product $W \times AB$ is the work done by gravity in moving the body down the plane from A to C.

Now, AB is the distance through which the body moves *in the direction of gravity*, and, as a like result is obtained in the case of any force, we have the following general conclusion:

The work done by a force acting obliquely on a moving body is measured by the product of the force and the distance through which the body moves in the direction of the force.

If the body (Fig. 174) were made by some force to move up the plane from C to A, then it would move through the distance BA *opposite* in direction to that of gravity, and work equal in amount to $W \times AB$ would be done by the force *against* the action of gravity.

204. Rate of Working. A workman who carries a hod full of bricks weighing, say, 60 lb., up to a staging 20 ft. high, does 1200 ft.-lb. of work, whether he ascends the ladder in one minute or in one hour, or in any other time.

But, from a practical point of view, the *time* taken to do the work is very important; thus, if one man can do the same amount of work in less time than another man, we say he has greater working power, and is on that account to be preferred. Hence, in measuring the working power of agents and machines, we must consider not only the amount of work done, but also the *rate* at which it is done.

The rate at which work is done is called *power*. A *horse power* (H. P.) is the power to do 33,000 foot-pounds of work per minute, or 550 foot-pounds per second.

James Watt, the inventor of the steam engine, considered that a horse could yield this amount of work per minute; but probably the power of an average horse is only about 26,000 foot-pounds per minute.

205. Dynamical Units of Work. In the scientific measurement of work, where strict accuracy is required, dynamical or *absolute* units of work, derived from the dynamical units of force, must be employed.

A *foot-poundal* is the work done by a poundal in moving its point of application one foot in the direction in which it acts.

An *erg* is the work done by a dyne in moving its point of application one centimeter in the direction in which it acts.

Quantities of work are changed from gravitation measure to absolute measure, and *vice versa*, by the following rules:

To reduce foot-pounds to foot-poundals *multiply* by g.

To reduce foot-poundals to foot-pounds *divide* by g.

To reduce kilogram-meters to ergs *multiply* by 100,000 g.

To reduce ergs to kilogram-meters *divide* by 100,000 g.

The factor 100,000 is required, because 1 kilogram-meter is equal to 1000 × 100, or 100,000 gram-centimeters.

CLASS-ROOM EXERCISES.

1. How much work is done in raising 6 cubic feet of water to a height of 8 ft.?

2. How high must 7 lb. be raised in order to do 84 ft.-lb. of work?

3. The ram of a pile driver weighs 784 lb., and has a fall of 23 ft. How much work must be expended in raising the ram?

4. In what time will a man pump 400 cubic feet of water to the mean height of 30 ft., allowing that he can perform 2600 ft-lb. of work per minute, and neglecting friction?

5. If a pit 20 ft. deep and 8 sq. ft. in section is excavated, how much work is done, a cubic foot of earth weighing 100 lb.?

Solution. Weight of earth excavated = 20 × 8 × 100 = 16,000 lb. The C. G. of this earth is raised 10 ft. ∴ Work done = 160,000 ft.-lb.

6. By pumping 3 tons of water out of a well, the distance to the water level is increased from 20 ft. to 26 ft. Find how much work is done?

7. A mass of 200 lb. is pushed 40 ft. up an incline of 10 per cent grade. The friction amounts to a force of 16 lb. How much work is done against gravity? How much against friction?

8. A man weighing 140 lb. puts a load of 140 lb. on his back, and carries it up a ladder to the height of 50 ft. What amount of work does he do in all, and how much of this work is done usefully?

9. The area of the piston of a steam engine is 2000 sq. in., the mean effective pressure of the steam is 30 lb. per sq. in., the length of the stroke is 4 ft., and the number of complete strokes per minute is 40. How much work does the engine do in 1 minute?

10. A bicyclist makes 15 miles an hour on a level road. He exerts a downward pressure of 20 lb. with each foot during the down stroke, and the length of the stroke is 1 ft. The diameter of the driving wheel is 28 inches. Find the work he does per minute.

11. What is the horse power of an engine that raises 60 cubic feet of water per minute from a depth of 242 ft.?

Here, the work done = $60 \times 62\frac{1}{2} \times 242$ ft.-lb.

$$\text{Therefore, the horse power} = \frac{60 \times 125 \times 242}{2 \times 33{,}000} = 27.5.$$

12. Find the horse power of an engine that will raise 14 tons of coal (2240 lb. each) in 1 hour from a pit whose depth is 480 ft.

13. A forge hammer weighs 484 lb. and rises once a second, each lift being 3 ft. What is the horse power of the engine that works it?

14. Find the horse power of an engine capable of raising 30 tons through a height of 44 ft. in 5 minutes.

15. A windmill raises by means of a pump 15 tons of water per hour to a height of 66 ft. Calculate its horse power.

16. How many tons of coal (2240 lb. each) will an engine of 3 H. P. raise per hour from a coal pit whose depth is 720 ft.?

17. From what depth will an engine of 20 H. P. pump 40 cubic feet of water in 1 minute?

18. In what time will an engine of 40 effective H. P. pump 4000 cubic feet of water to a mean height of 360 ft.?

19. If a man does 1,000,000 ft.-lb. of work in a working day of 8 hr., with what fraction of a horse power does he work on the average?

20. A vertical shaft 440 ft. deep and 10 sq. ft. in cross-section is full of water. What is the horse power of an engine that will empty it in 6 hours, if friction consumes one third of the power of the engine?

21. What is the horse power of an engine that draws a train of 132 tons at the uniform rate of 45 miles an hour against a resistance amounting to 900 lb.?

22. At what rate is an engine working when it drives a train of 300 tons at the rate of 15 miles an hour, the resistance to motion being equal to 22 lb. per ton?

23. A steam engine supplies 1000 houses with 22 cubic feet of water daily, the water being raised to a mean height of 100 ft. If the engine works 12 hours a day, what is its rate of working?

24. Find the amount of horse power transmitted by a belt passing over a wheel 7 ft. in diameter which makes 3 revolutions a second, the tension of the belt being equal to 88 lb.

25. The area of the piston of a steam engine is 500 sq. in., the mean pressure of the steam is 33 lb. per square inch, the length of the stroke is 4 ft., and the number of full strokes per minute is 32. Find the horse power of the engine.

26. A shaft 560 ft. deep and 5 ft. in diameter is full of water. How long would it take a 3½ H. P. engine to pump out the water, friction being neglected?

27. A water-power engine of 10 H. P. is supplied from a tank 12 ft. high, 8 ft. long, 6 ft. wide, at a height of 120 ft. Supposing the tank to be full, but no supply, find how long the engine could run.

28. A steam crane, working with 6 H. P., raises 20 tons to a height of 150 ft. in an hour. How much of the work is done against friction?

29. If it requires a 2 H. P. engine to raise 200 cubic feet of water per hour through 80 ft., what per cent of the work is waste work?

Machines.

206. Use of a Machine. Suppose that we wish to draw a nail out of a piece of wood (Fig. 175), and that a force of 500 lb. is required to start the nail. A direct pull by the hand is clearly of no use. But if we apply to the nail a lever in the form of a claw hammer, which grasps the nail at the distance of an inch from the fulcrum *F*, and if we exert by the hand at the distance of 10 in. from *F* a pull of 50 lb., then by the law of the lever a pull of 500 lb. is exerted upon the nail, and this pull draws it from the wood.

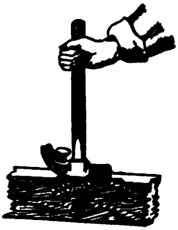


FIG. 175.

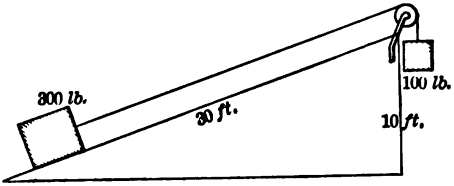


FIG. 176.

Suppose that a body weighing 300 lb. is at the bottom of an inclined plane, the height of which is 10 ft. and length 30 ft. (Fig. 176). If we apply to this body a force of 100 lb., acting up the plane, as shown in the figure, the effect of gravity is neutralized (§ 46), and, if friction did not exist, the slightest push would set the body in motion up the plane.

The claw hammer and the inclined plane are examples of *simple machines.* A machine is a contrivance by means of which a force may be applied so as to perform a definite kind of work. It enables us to perform work to better advantage by making a suitable change in the magnitude, direction, or point of application of the force employed.

In most cases the advantage which we desire consists in diminishing the magnitude of the force which must be applied.

207. Principle of Work. If we disregard friction, only two forces in a machine have to be considered: P, the applied force, and Q, the resistance overcome. Let the reactions of the fixed parts of the machine be such that P just balances Q statically, and let the machine be set in motion; then the motion will be uniform, and work will be done by P against Q.

Let a and b denote the distances traveled in the same time by the points of application of P and Q, respectively, each measured in the direction of the force; then Pa is the amount of work done by P or the *applied work*, and Qb is the amount of work done against Q or the *useful work* done by the machine. The principle of work asserts that in every machine, if we ignore friction, *these amounts of work are equal.*

Work applied = useful work done.

$$P \times a = Q \times b.$$

Thus, in the case of the claw hammer (Fig. 175), for every inch the nail moves, the hand, being 10 times as far from the fulcrum, moves 10 inches; and the products, 50 × 10 and 500 × 1, are equal.

In the case of the inclined plane (Fig. 176), the work applied is 100 × 30, or 3000 ft.-lb., and the work done, 300 × 10, or 3000 ft.-lb.

Therefore, a machine cannot *create* work. All that a machine can do is to vary the ratio of the factors, force and distance, whose product measures the work, the product itself remaining unchanged; just as the number 24 may be resolved into various pairs of factors, 24 × 1, 12 × 2, 8 × 3, etc.

If P is made *less* than Q by means of a machine, then the factor a becomes in the same ratio *greater* than the factor b; also the point of application of Q will move in the same ratio *slower* than that of P. This is commonly expressed by saying that *what is gained in power is lost in speed.*

Attempts to construct a machine able to create work are called the search after perpetual motion, and always result in failure.

208. Waste Work. In every actual machine there exist certain resistances that have to be overcome, and a portion of the work applied to the machine must be expended in overcoming them. Chief among them stands friction, sometimes sliding friction, as on the inclined plane, but more often the friction of axles on their bearings, as in wheelwork. The stiffness of cords and belts is another source of waste, and sometimes even the resistance of the air is a serious drawback to useful work, as when a railway train is proceeding against a gale of wind. Let us call the work done in overcoming these resistances *waste* work; then the complete statement of the Principle of Work for a machine whose parts are in a state of uniform motion is as follows:

$$\textit{Work applied} = \textit{useful work} + \textit{waste work}.$$

For example, suppose that the friction of the body on the inclined plane already mentioned (Fig. 176) amounts to a force of 50 lb. Then the applied force P must be 150 lb. instead of 100 lb.; and by making the proper substitutions for the quantities in the above formula, it becomes

$$150 \times 30 = 300 \times 10 + 50 \times 30.$$

In some cases the useful work performed by a machine is less than half of the whole amount of work applied to the machine.

209. Efficiency and Mechanical Advantage. These terms are defined as follows:

$$\textit{Efficiency} = \frac{\textit{useful work}}{\textit{whole work applied}} = \frac{Qb}{Pa}.$$

$$\textit{Mechanical Advantage} = \frac{Q}{P}.$$

If there were no waste work, the efficiency of a machine would be equal to unity; and the mechanical advantage would be equal to the ratio of a to b (P's velocity to Q's velocity), for in this case $Qb = Pa$, and therefore $Q : P = a : b$.

In the inclined plane above mentioned, the mechanical advantage is 3 without friction and 2 with friction; the efficiency is $\frac{2}{3}$.

210. Simple Machines. There are six machines which are termed *simple machines* or *mechanical powers*, namely: the lever, the inclined plane, the wheel and axle, the pulley, the wedge, and the screw.

The lever and the inclined plane have been considered. We will now apply the principle of work to the others, assuming the motion to be uniform and disregarding friction.

Let P denote the force applied, and let Q denote the resistance overcome in the performance of the useful work.

211. Wheel and Axle (Fig. 177). In this machine it is clear that while P descends a distance equal to the circumference of the wheel, Q ascends a distance equal to the circumference of the axle. Hence, by the principle of work,

$$P \times \textit{circumference of wheel} = Q \times \textit{circumference of axle}.$$

Let r and r' denote the radii of the wheel and the axle, respectively.

Then $$P \times 2\pi r = Q \times 2\pi r',$$
or, $$Pr = Qr'.$$

Therefore, the mechanical advantage $= \frac{Q}{P} = \frac{r}{r'}$.

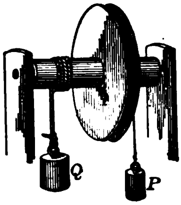


FIG. 177.

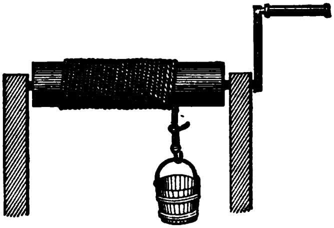


FIG. 178.

We obtain the same equation $Pr = Qr'$ by applying the law of moments (§ 51); and in this way we verify the principle of work.

In the *windlass* (Fig. 178) the wheel is replaced by a handle, but the principle of work applies precisely as before.

In the *capstan*, used on board ships for raising the anchor, the axle or barrel is vertical. The power is applied by the sailors pushing against the bars. The rope is usually too long to be completely wound round the barrel; so after passing two or three times round the barrel the free end of the rope is held by a man, who draws it taut enough to keep the rope from slipping. The same method is sometimes applied for moving heavy weights (Fig. 179).

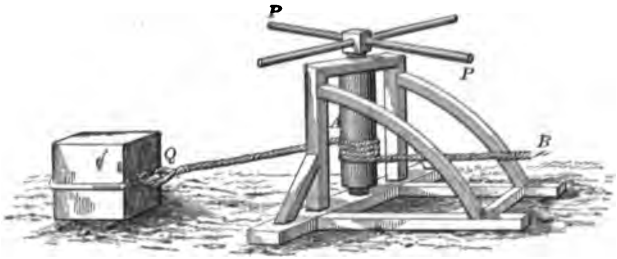


FIG. 179.

212. The Pulley. A single movable pulley is represented in Fig. 180. The weight Q is apparently supported by a force P only half as great as itself, the reason being that the other half of the necessary force is supplied by the hook which is fixed in the beam and to which one end of the cord is attached. The principle of work leads to the same result. For if Q rises one foot, each portion of the cord must be shortened by one foot, and therefore the free end of the cord where P acts must fall two feet. Therefore,

$$2P = Q, \text{ or } P = \tfrac{1}{2}Q.$$

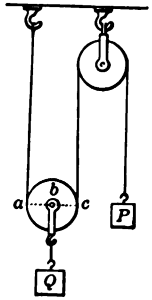


FIG. 180.

Thus, the mechanical advantage of a simple movable pulley is 2. But Q will move only half as fast as P; so that what is gained in power is lost in speed.

We may regard the pulley in Fig. 180 as a movable lever of the second class with the fulcrum at *a*, the weight acting at *b*, and the power applied at *c*. It is easy to see that if we apply the law of moments (§ 51), we are again led to the result, $P = \frac{1}{2} Q$.

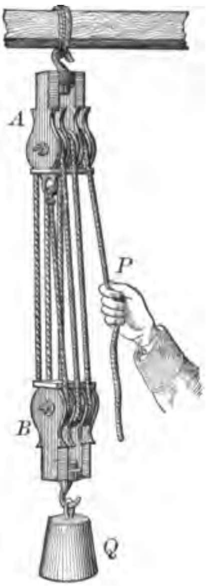


FIG. 181.

By using more pulleys, the mechanical advantage can be increased. Fig. 181 shows the common arrangement. *A* and *B* are the *pulley blocks*, each having three pulleys. The weight *Q* is attached to *B*. The force *P* is applied to one end of the cord, which, after passing round all the pulleys, is tied to the block *A*. Now suppose that *Q* is raised 1 ft.; then each one of the six portions of the cord between *A* and *B* is shortened by 1 ft.; and, therefore, the point of application of *P* will descend 6 ft. Therefore, by the Principle of Work,

$$6\,P = Q, \text{ or } P = \tfrac{1}{6}\,Q.$$

In all cases the mechanical advantage is equal to the number of parts of the cord at the lower block *B*.

This system of pulleys is in common use for raising heavy weights in quarries, in house-building, in machine shops, in dockyards, and on board ships.

The *differential wheel and axle* (Fig. 182) is a combination of the ordinary wheel and axle and the single pulley, by means of which almost any mechanical advantage may be secured. The axle is composed of two parts or drums having different radii. The rope is coiled in opposite directions on the two drums, so that as they revolve it winds up on one drum but unwinds from the other. Therefore, the length of rope below the drums is by one revolution diminished by the difference between the circumferences of the drums; and, since the rope passes round a pulley to which the weight is attached, the weight rises by just *half* this difference. By making the drums nearly equal in size, we can make this difference as small as we please.

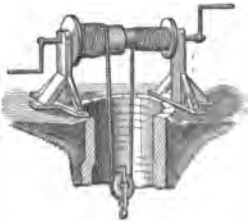


FIG. 182.

213. The Wedge. The wedge is an instrument designed to overcome a very great resistance through a very small space. It is employed for splitting wood and raising large blocks of stone in order to put under them chains or rollers. Axes, knives, and chisels are thin wedges with sharp edges.

Suppose that a wedge ABC (Fig. 184) has been driven into wood by its whole length DC; then the work done by the applied force P is equal to $P \times DC$. Q, the resistance overcome, acts at right angles to the slant side AC, and by the motion of the wedge has been overcome through the distance DE; and an equal amount of work has been done against Q, acting on the other slant side BC. That is, the total work done against Q is $Q \times 2\,DE$. Therefore,

$$P \times DC = Q \times 2\,DE,$$

or,

$$\frac{P}{Q} = \frac{2\,DE}{DC}.$$

FIG. 183.

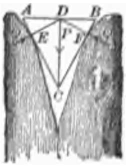


FIG. 184.

Since the triangles DEC, ADC are similar,

$$\frac{DE}{DC} = \frac{AD}{AC}.$$

Therefore, by substitution we obtain

$$\frac{P}{Q} = \frac{2\,AD}{AC} = \frac{AB}{AC} = \frac{\textit{thickness of wedge}}{\textit{slant side of wedge}}.$$

This is the theoretical condition of equilibrium in the wedge. This condition, however, has little or no practical value, owing to two circumstances: the great amount of friction, and the fact that the force P is not applied as a pressure, but as a blow.

214. The Screw. The *screw* is a cylinder having on its surface a uniform projection in the form of a spiral curve, called the *thread.*

The screw may be regarded as a spiral inclined plane, as will appear if we cut out a piece of paper in the shape of an inclined plane and then wind it around a pencil.

In order that a screw may overcome resistance, it must work against a corresponding hollow screw called the *nut,* the thread of the screw fitting into the hollows of the nut.

The *screw press* (Figs. 185, 186) and the *vice* (Fig. 10, page 14) are familiar examples of the application of the screw.

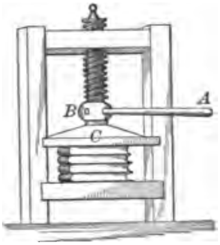


FIG. 185.

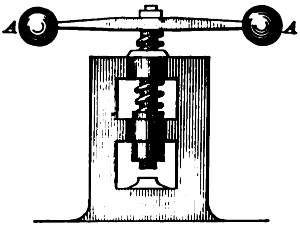


FIG. 186.

In the screw press the nut is fixed in a strong frame, and the screw is turned around by applying the force P at the end of a lever AB. During one revolution P describes a circle whose radius is AB, and the screw moves downward, pressing the plate C against the books, through a distance equal to the interval between two successive threads, and termed the *pitch* of the screw. Therefore, by the principle of work,

$$P \times \textit{circumference described by } P = Q \times \textit{pitch of screw};$$

$$\textit{mechanical advantage} = \frac{Q}{P} = \frac{\textit{circumference described by } P}{\textit{pitch of screw}}.$$

Practically, the mechanical advantage is much less than this on account of the great amount of friction.

215. Compound Machines. However complicated a machine may appear to be, when we come to analyze it we find that it is merely a combination of the simple arrangements which have just been explained. In order to determine the mechanical advantage of the machine, this analysis need be carried no farther than is required to discover the relative distances described in the same time by the applied force P and the resistance overcome Q. For, by the principle of work,

$$P \times \textit{distance } P \textit{ moves} = Q \times \textit{distance } Q \textit{ moves}.$$

Therefore,

$$\textit{Mechanical advantage} = \frac{Q}{P} = \frac{\textit{distance } P \textit{ moves}}{\textit{distance } Q \textit{ moves}}.$$

This is the mechanical advantage in theory; practically, friction always diminishes its value, sometimes more than 50 per cent.

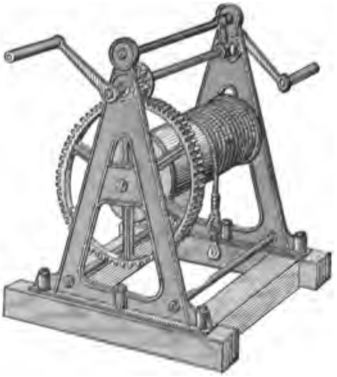


FIG. 187.

A compound machine like that shown in Fig. 187 is often employed to raise a heavy weight by hand power. For the purpose of estimating the mechanical advantage of this machine, we will assume that the radius of the crank is 24 in.; that the small wheel has 16 teeth, and the large wheel 96; and that the radius of the barrel round which the rope is wound is 6 in.

Since 96 ÷ 16 = 6, it is clear that the crank must make 6 revolutions while the large wheel, with the barrel, makes 1 revolution. During this motion, the distance described by the power is $6 \times 2\pi \times 24$ in.; and the distance described by the weight is $2\pi \times 6$ in.

Therefore, the mechanical advantage is 24.

If the power applied to each crank is 100 lb., and friction is neglected, the weight raised will be 200 × 24, or 4800 lb.

216. The Bicycle. Machines are generally employed for the purpose of increasing the effect of a force at the expense of distance. But sometimes we wish to produce the reverse; we have abundant force at command, and wish to apply this force so as to gain distance, or, what comes to the same thing, velocity. The *bicycle* is a good example.

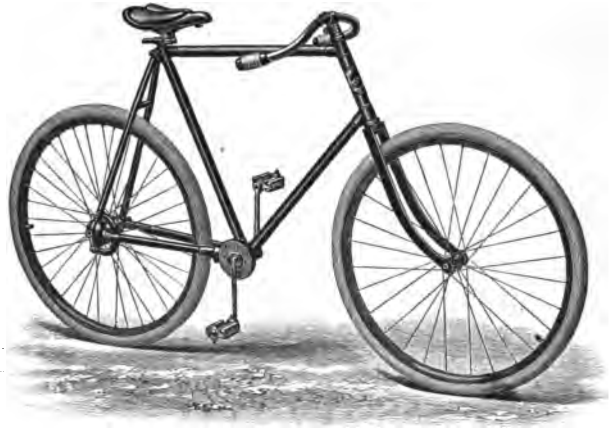


FIG. 188.

One of the latest styles of bicycle, called the *chainless* bicycle, is shown in Fig. 188. The rear wheel is driven by a bevel gearing and shaft, which is concealed from view by the framework. In the bicycle represented in the figure, one turn of the pedals gives $2\frac{4}{5}$ turns to the shaft, and also $2\frac{4}{5}$ turns to the rear wheel, provided the rear gear wheels have an equal number of teeth. If the radius of the pedal crank is 7 inches, and that of the rear wheel 14 inches, then during each revolution of the pedal the rider's foot moves through $2\pi \times 7$ inches, or 44 inches very nearly, and the rider himself is carried forward a distance of $2\frac{4}{5} \times 2\pi \times 14$ inches, or about 246 inches. By varying the relative size of the rear gear wheels, the distance described for one revolution of the pedals is varied. The rate of a bicycle is determined by its *gear;* if for one turn of the pedals the bicycle moves forward a distance equal to the circumference of a wheel 80 inches in diameter, the bicycle is said to be *geared to 80.*

CLASS-ROOM EXERCISES.

1. The handle of a claw hammer is 15 in. long, and the claw is 3 in. long. When a force of 50 lb. is applied to the handle, the nail is drawn out. What is the resistance of the nail?

2. A weight of 240 kg. is raised 20 cm. by means of a movable pulley. How much force is required? How far does the force move? How much work is done?

3. The radius of a wheel is 80 cm., and the radius of the axle is 12 cm. What weight can be supported by a force of 30 kg.? How much work is done if the weight is raised 60 cm.?

4. What must be the ratio of the radii of a wheel and axle in order that a force of 100 lb. may just support 1 ton?

5. A pair of pulley blocks contain each four pulleys. The rope is attached to the upper fixed block. What force is just sufficient to raise a weight of 1600 lb. if the friction amounts to a force of 50 lb.?

6. An inclined plane is 50 ft. long and 30 ft. high. What force is required to make a mass of 80 lb. ascend the plane, and how much work against gravity is done in the ascent, (1) if there is no friction, (2) if the coefficient of friction between the body and the plane is $\frac{1}{4}$?

7. A force of 75 lb. is exerted along a smooth inclined plane 5 yd. long and 3 ft. high. What weight will it raise?

8. A man can push with a force of 200 lb., and wishes to raise a body weighing 1000 lb. into a cart 3 ft. high. How long an inclined plane must he employ?

9. How great a weight can a force of 27 lb., applied horizontally on a smooth inclined plane 26 ft. long, raise to the top of a wall 10 ft. high?

10. The lever of a screw is 3 ft. 6 in. long, and the distance between two threads is $\frac{1}{5}$ of an inch. What pressure must be applied to the end of the lever to produce a pressure of 10 tons upon the press board?

11. The lever of a screw is 1 ft. 9 in. long, and the power applied at the end is 100 lb. What must be the thickness of the threads that a pressure of 5000 lb. may act upon the press board?

12. What pressure is exerted by a force of 1 lb. acting at a distance of 4 ft. 8 in. from the axis of a screw, if the distance between the threads is 1 in.?

13. A loaded car weighing 16 tons is to be drawn up an incline of 10 per cent grade by means of a rope coiled around an axle 2 ft. in diameter, the diameter of the wheel being 10 ft. Find the force that must be applied to the wheel.

The Two Types of Energy.

217. Kinetic Energy. *Energy* means power to do work. A body in motion has this power. A swift-moving bullet will overcome the cohesion of wood, and bore a hole through a board. Moving water will set a water wheel in motion, and this motion may be carried to a saw which saws wood, or to millstones which grind corn. Wind, which is air in motion, will drive windmills, and propel ships over the ocean.

Whenever a moving body encounters resistance, it will move against this resistance, and thus perform work. A moving body, therefore, has energy in virtue of its motion. Energy of this kind is called *kinetic* energy (energy of motion); and it is measured by the amount of work done by the body while being brought to rest.

Suppose that a mass of m pounds is moving vertically upwards at the rate of v feet per second. The mass will rise to a height h such that $h = \frac{v^2}{2g}$ (§ 173). The statical force with which gravity acts on the mass m is m pounds weight. Therefore, the work done against gravity is mh foot-pounds. This is equal to the kinetic energy of the mass, and if we substitute for h the value just given, we obtain

$$\textit{Kinetic energy of the mass} = \frac{mv^2}{2g}\ \textit{ft.-lb.}$$

In general, let any constant force equal to F poundals, and, acting along the line of motion, bring the mass to rest in t seconds, after it has moved through a distance of s feet. The impulse of the force is Ft, the momentum destroyed is mv, and the mean velocity during the time t is $\frac{1}{2}v$; therefore,

$$Ft = mv, \text{ and } s = \tfrac{1}{2}vt.$$

Multiplying these equations, and cancelling t, we obtain

$$Fs = \tfrac{1}{2}mv^2.$$

The product Fs is the work done against the force F, measured in foot-poundals. Therefore, the kinetic energy of the mass m is equal to $\frac{1}{2}mv^2$ foot-poundals. If we divide this value by g we obtain, as before, $\frac{mv^2}{2g}$ ft.-lb. Therefore, we have

Law 1. *The kinetic energy of a mass of m pounds moving with a velocity of v feet per second is* $\frac{mv^2}{2g}$ *foot-pounds.*

When other units than the pound and the foot are used, a corresponding change must be made in the unit of work. If, for example, the kilogram is the unit of mass, and the meter the unit of length, the kinetic energy of the moving mass will be equal to $\frac{mv^2}{2g}$ kilogram-meters.

In order that a body may possess kinetic energy, it must be set in motion by the action of some force; and before this force can give to the body a definite velocity it will cause the body to move through a certain distance. Hence, work must be done on a body before it can have kinetic energy. If F denotes the force (in poundals) which does this work, m the mass of the body, t the time, v the velocity acquired, s the space described; then, reasoning exactly as we have already done, $Ft = mv$, $s = \frac{1}{2}vt$, and, therefore, $Fs = \frac{1}{2}mv^2$. Now, Fs is the work expended in setting the body in motion, and $\frac{1}{2}mv^2$ is its kinetic energy (in dynamical measure); whence we have

Law 2. *The kinetic energy of a body is equal to the work expended in giving to it the velocity which it possesses.*

Thus, a freight car weighing 16 tons, standing on a track, has no kinetic energy. But now suppose that a steady pull, equal to 250 lb., act on the car for one minute, and that a force of 50 lb. is required to overcome friction. Then the moving force is equal to 200 × 32 poundals, and the

$$\text{acceleration} = \frac{\text{force}}{\text{mass}} = \frac{200 \times 32}{32{,}000} = \tfrac{1}{5} \text{ ft. per second each second.}$$

Velocity acquired = 12 ft. per second.

Kinetic energy acquired = $\frac{1}{2}mv^2 = 16{,}000 \times 144$ ft.-poundals.

218. Energy and Velocity. Law 1 in § 217 shows that the kinetic energy of a body varies directly as its mass and as the *square* of its velocity. Thus, doubling the mass doubles the kinetic energy, but doubling the velocity makes the kinetic energy *four* times as great. And if the velocity is made three times as great, the kinetic energy becomes *nine* times as great, and so on.

This explains why a rifle ball will pass through a two-inch plank of tough oak wood, while the recoil of the gun merely gives the shoulder a slight shock; also why a hammer striking a nail will drive the nail into the wood, while a body far heavier than the hammer, if placed upon the nail, will not move it; and other similar phenomena.

Let us consider the case of the rifle ball more closely. The momentum of the ball and that of the gun are equal (§ 185). Let the mass of the ball be 1 oz. ($\frac{1}{16}$ lb.), and that of the gun 200 oz. ($12\frac{1}{2}$ lb.); also let the velocity of the ball, when it leaves the gun, be 1600 ft. a second. The velocity of recoil of the gun will be only 8 ft. a second, since $1 \times 1600 = 200 \times 8$.

$$\text{The kinetic energy of the ball in ft.-lb.} = \frac{1600 \times 1600}{16 \times 2 \times 32} = 2500.$$

$$\text{The kinetic energy of the gun in ft.-lb.} = \frac{25 \times 8 \times 8}{2 \times 2 \times 32} = 12\tfrac{1}{2}.$$

Hence, the energy of the ball is 200 times as great as that of the gun.

Now let us suppose that each body loses its energy by doing work against a uniform resistance, acting through a distance of 6 in. Let P denote the value in pounds weight of this resistance in the case of the ball, P' its value in the case of the gun.

For the ball $P \times \frac{1}{2} = 2500$; whence $P = 5000$ lb.
For the gun $P' \times \frac{1}{2} = 12\frac{1}{2}$; whence $P' = 25$ lb.

Thus, the ball works its way 6 in. into a target against a resistance of 5000 lb., while the gun will merely force back a man's shoulder 6 in. against the small resistance of 25 lb.

A great variety of problems may be solved by the formula,

$$Ps = \frac{mv^2}{2g};$$

in which P = resistance overcome (in lb.), s = distance moved (in ft.), m = mass of body (in lb.), v = velocity of body (in ft. per second).

219. Energy in a Machine. Law 2 in § 217 shows that the kinetic energy of a moving body is equal to the work done in setting it in motion. Law 1 tells us that this energy is equal to the work which the body can do in being brought to rest. Therefore, the body is simply a *transmitter of energy.* It receives energy from some source, and delivers it unchanged in amount to some other body or bodies.

Now, every machine in action is a transmitter of energy. It receives energy by a force applied by means of a lever, or a rope, or in some other way, and transmits this energy unchanged in amount (except by friction) to some point where it is expended in doing work, after certain changes in the values of the two factors, force and distance, have been made. But the product of these factors remains unchanged.

When a machine is started, the energy supplied exceeds at first the work done. The excess is utilized in increasing the speed of the moving parts; it is *stored up* as kinetic energy in the machine.

The work done increases with the velocity, and when it becomes equal in amount to the energy supplied, the motion becomes uniform. All the energy supplied is now transmitted through the machine, and is doing useful or waste work. It is to this stage that the principle of work applies, which we may now state as follows:

Energy exerted = useful work + waste work.

When the supply of energy to a machine is cut off, the machine does not stop instantly, but keeps on doing work until the kinetic energy stored up in the machine has all been expended in doing work.

The fly wheel of a steam engine stores up the energy derived from the steam as kinetic energy when its velocity is increasing, and gives it back again when its velocity is decreasing.

220. Potential Energy. When a body is raised above the surface of the earth a certain amount of work is expended in overcoming the attraction between the body and the earth. The body in its elevated position has now the power to do work. It is only necessary to allow the body to fall; in falling the body acquires kinetic energy, and this energy may be expended in doing various kinds of work.

In the *pile driver*, a heavy mass of iron, called the *ram*, is allowed to fall from an elevated position upon a wooden pile. The ram does work by driving the pile deeper into the ground.

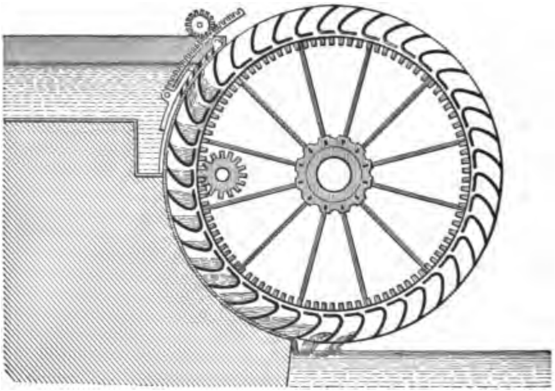


FIG. 189.

In the bucket *water-wheel* (Fig. 189), the weight of the water which fills the buckets on one side of the wheel causes the wheel to revolve. The water is constantly descending from the upper level to the bottom of the wheel, and doing work as it descends.

The *leaden weight* of a clock is a similar example. The energy of the clock is increased by winding up the weight, so that the clock is able to go for a day, or even a week, in spite of the friction of the wheels and the resistance of the air to the motion of the pendulum; moreover, the clock is able to give out energy in the form of vibrations to the air by which we hear the ticking of the clock.

Thus, a body in an elevated position, although at rest, possesses energy on account of its *position*. By raising a body above the surface of the earth, the energy of the material system, consisting of the body and the earth, is increased.

The energy of a body which depends on its position, and not on its velocity, is called *potential* energy.

Potential energy may be due to the action of other forces than gravity. Whenever the position of a body is changed by doing work upon it against a stress that tends to restore the body to its original position, potential energy is stored up in the body. This potential energy, when the body is free to obey the action of the stress, is either made to do work immediately, as in the case of a moving water wheel (Fig. 189), or is first converted into kinetic energy, and then made to do work, as in the case of a pile driver.

Electricity and magnetism are forces against the action of which work may be done and potential energy stored up.

A *magnet* and a piece of iron attract each other and stick together. Work is required to separate them; and when separated they still attract each other, and will move towards each other, and while so moving may be made to do work. Therefore, when separated, they have potential energy.

Electricity is like magnetism in this respect; by its means we are now able to store up potential energy, and then apply the energy to running street cars and in various other ways.

Potential energy is also stored up when a body is strained by the action of a stress of any kind.

A *watch spring* uncoiled has no energy. In the act of coiling it up we do work upon it, and alter its shape in opposition to elastic force. We thus store up in the spring a certain amount of potential energy, which is made to do work during the next 24 hours by driving the mechanism of the watch. A bent bow is a similar example.

When we *compress air* we store up potential energy against elastic force, and when the air is allowed to move the piston of an air engine, work is done.

221. Measure of Potential Energy. The potential energy of a body is measured by the amount of work required in order to move the body against the acting stress from some standard or zero position to the position which it occupies.

In the case of gravitation stress the zero position is usually assumed to be on the ground. Hence, the potential energy of a body, whose mass is m lb., and whose height above the ground is h ft., is equal to mh ft.-lb.

The ground is chosen as the zero position, because, as a rule, a body cannot do work by falling any farther. Strictly speaking, a stone lying on the ground has an amount of energy corresponding to its fall to the center of the earth; but it would be impossible to utilize this energy without first expending a great deal more energy in digging a hole.

For a spring coiled up the zero position is the spring uncoiled; for a bent bow, the zero position is the bow in its natural position.

222. Change of Form of Energy. A mass of m lb. at rest above the ground has potential energy, but no kinetic energy. If allowed to fall freely, it loses potential energy and gains kinetic energy. Let us suppose that it falls through a distance of h ft. The loss of potential energy is equal to mh ft.-lb. The gain in kinetic energy is $\frac{mv^2}{2g}$ ft.-lb. But $h=\frac{v^2}{2g}$ (§ 173).

Therefore,
$$mh=\frac{mv^2}{2g}.$$

Thus, the loss in potential energy is just equal to the gain in kinetic energy, or, to state the truth more completely,

The sum of the kinetic and the potential energies of a body, falling freely from rest under gravity, is constant; and is equal to the potential energy at the highest point, or to the kinetic energy acquired on reaching the ground.

This is a special case of a very general principle called the Conservation of Energy, which will come up later for further consideration.

CLASS-ROOM EXERCISES.

1. How much kinetic energy has a cannon ball weighing 64 lb. and moving with a velocity of 1600 ft. per second?

2. A mass of 4 lb. falls from rest to the ground in 3 seconds. How much kinetic energy does it have on reaching the ground? What becomes of this energy when the mass comes to rest?

3. A ball weighing 1 lb. is fired vertically upwards with a velocity of 160 ft. a second. Compute its kinetic and its potential energy at the start and at the end of each second during the time that it ascends.

4. A mass of 32 lb. falls from rest to the ground, a distance of 400 ft. What is its kinetic energy on reaching the ground?

5. How many ft.-lb. of energy does a ton of water possess at the height of 1 mile above the sea level? What kind of energy is it?

6. A stream of water is flowing at the rate of 4 miles per hour. Find the momentum and the kinetic energy per cubic foot of water.

7. Describe the transformations of energy that take place in the swinging of a pendulum.

8. A mass of 80 lb., moving with a velocity of 16 ft. a second, strikes an equal mass at rest. Both masses are inelastic, and move on together after the impact. Find the total kinetic energy of the masses, (1) before the impact, (2) after the impact. Explain the apparent loss of energy.

9. A mass of 100 grams, moving with a velocity of 24 cm. a second, overtakes and adheres to a mass of 60 grams, moving in the same direction with a velocity of 8 cm. per second. Find the common velocity and the total kinetic energy (in gram-centimeters) before and after the impact.

10. A mass of 60 lb., moving at the rate of 10 ft. a second, is acted upon by a force directly opposite to the motion until the mass has a velocity of 30 ft. a second in the direction of the force. How much kinetic energy has the force imparted to the mass?

11. A body whose mass is 3 lb. makes 42 revolutions per second in a circle whose radius is 14 ft. How much kinetic energy does it have?

12. A mass of 8 lb., starting from rest, is acted upon by a force of 2 lb. for 30 seconds. How much kinetic energy does it acquire?

13. A body whose mass is 10 lb. is moving at the rate of 24 ft. a second, when a constant resistance equal to 2 oz. begins to act directly against the motion. How far will the body move before it comes to rest?

14. A train whose mass is 120 tons is moving at the rate of 45 mi. an hour, when the steam is shut off, and a brake power equal to 4000 lb. is applied. How far will the train go before coming to rest?

Heat as a Form of Energy.

223. Conversion of Mechanical Energy into Heat. While a body is falling to the ground there is neither gain nor loss of energy; it is simply changed from the potential to the kinetic form. But after the body reaches the ground its energy is apparently put out of existence. It has no potential energy, for it can fall no farther; it has no kinetic energy, for it is at rest.

A similar destruction of energy appears to occur in other cases. What, for instance, becomes of the energy of a cannon ball after it has struck the target, or of the energy of a railway train after it has been brought to rest by the application of the brakes?

The answer to this question constitutes one of the great advances of science during the nineteenth century.

Whenever visible energy appears to be destroyed by impact or friction, *heat is produced.* In fact, the generation of heat by the expenditure of mechanical energy is a very common phenomenon.

Illustrations. 1. A piece of lead, if bent back and forth a few times, becomes warm, and, if hammered, it soon grows too hot to hold in the hand.

2. Saws and augurs become heated in cutting through wood; and the harder the wood, the sooner they are heated.

3. To prevent heating by friction, the axles of railway wheels, and the bearings of wheels in all kinds of machines, are kept well supplied with oil and other lubricants. Railway axles sometimes become so hot as to set the woodwork on fire.

4. On a dark night sparks are seen to issue from the wheels of a railway train when the brakes are put on to stop the train.

5. A cannon ball is greatly heated when its motion is suddenly arrested. Oftentimes a flash has been seen, even in broad daylight, when the ball strikes the target.

6. Savages obtain fire by the friction of two dry pieces of wood.

7. One of Professor Tyndall's experiments is illustrated in Fig. 190. A hollow brass tube is screwed on a whirling table, by means of which it can be set in rapid rotation. The tube is partly filled with cold water, closed by a cork, and set in rapid motion. A wooden squeezer *T* is made to clasp it tightly. The heat generated by the friction of the wood upon the brass is so great that the water in the tube is finally made to boil, and the cork is driven out, followed by a cloud of steam.

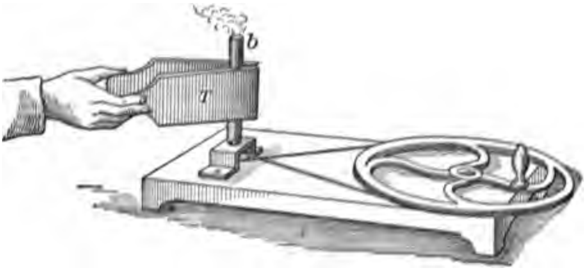


FIG. 190.

8. When air or any gas is suddenly compressed, its temperature is raised. This fact may be shown by means of the so-called *fire syringe* (Fig. 191). This consists of a strong glass tube in which a piston works up and down, air-tight. At the bottom of the tube a piece of tinder is placed. Then the piston is forcibly pushed down. The air is compressed, and heated to such a degree that the tinder is ignited. Tyndall used in this experiment a piece of cotton moistened with carbon disulphide (a very volatile and inflammable liquid). On repeating the experiment with the same piece of cotton, he obtained a flash of light many times in succession.

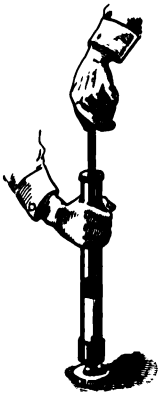


FIG. 191.

9. On a clear night we often see in the sky what we call a "shooting star." It is not a star, but a small mass of solid matter, which, drawn to the earth by gravitation, enters the earth's atmosphere with an enormous velocity. The resistance of the air suddenly stops its motion. The mass is thereby raised to a white heat and transformed into vapor, which condenses on cooling to minute particles of solid matter ("cosmic dust").

224. Conversion of Heat into Mechanical Energy. The converse change of heat into the energy of work is a very common phenomenon.

Illustrations. 1. When a body is heated, it expands. But in expanding it has to do work, if only to push away the atmosphere, which exerts upon the body a pressure of almost 15 lb. per square inch. If the body is a quantity of gas confined in a vessel, the expansive force increases with the temperature, until it becomes sufficient to burst the strongest vessel that can be made.

2. Fill a Florence flask about one third full with water, cork it with a rubber stopper, and then apply heat. After a time the stopper will be violently expelled from the flask, and if the water is sufficiently heated, a cloud of steam will issue from the mouth of the flask. Work has been done in propelling the cork from the flask, and the only source of this work is the heat which has been applied.

3. The steam engine is a machine which transforms heat into mechanical energy. If a sufficient supply of heat is furnished under the boiler, the engine will continue in action, and energy will be conveyed through the fly wheel to the machinery with which it is connected, where the energy is expended in doing work. If the supply of heat is cut off, the engine slows up, and soon comes to rest.

4. When air is pumped out of a vessel, we often see a sudden dimness fill the vessel. This is due to the formation of a visible cloud from the aqueous vapor which the air contains. Now, this cloud would not form unless the air were cooled below the dew-point (§ 127). Why should the air be cooled? The answer is that the air, in expanding against pressure, does work; and the only source for the energy required is the heat contained in the air itself. So a portion of the heat contained in the air does work, and therefore the temperature of the air falls below the dew-point. If a thermometer is placed in the vessel, the effect of exhausting the air in lowering the temperature is made manifest.

5. If moist air, compressed to 2 or 3 atmospheres, is allowed to issue through a small orifice and strike against a glass bulb, the bulb will soon be covered with hoar frost. Here work is done against the pressure of the atmosphere, and there is the disappearance of a large amount of heat.

Whenever a gas expands under pressure, and no heat is supplied from without, its temperature falls. The gas loses heat, and at the same time performs a certain amount of mechanical work.

225. True Nature of Heat. The old theory of heat maintained that heat was an invisible fluid without weight. This fluid was called *caloric*, and was supposed to enter the pores of a body as water enters the pores of a sponge. Count Rumford and Sir Humphrey Davy at the end of the eighteenth century proved by experiments that this theory was false; and at the same time they showed what the true nature of heat is.

Rumford (1798) was struck with the fact that a great amount of heat was developed in the process of boring a cannon, and made many experiments to measure the amount. In one of his experiments a steel borer was used to bore a brass cylinder surrounded by a known weight of water. After 2½ hours the temperature of the water was raised from the freezing to the boiling point. Rumford concluded from his experiments that the heat developed by friction was not squeezed out of the body (as the caloric theory maintained), but generated by the rubbing process. For a pound of brass at ordinary temperature could contain only a limited quantity of caloric, and therefore give out only a limited quantity when rubbed; whereas the quantity actually producible by friction appeared to be inexhaustible. He also observed that the quantity of heat generated was proportional to the quantity of work expended in generating it.

Davy showed that when two pieces of ice are rubbed together, some of the ice is melted. Now, water at 0° contains much more heat than ice at 0°, so that the experiment clearly proved that a fresh supply of heat was generated by the rubbing process.

The only rational conclusion to be drawn from these experiments of Rumford and Davy is that heat is not a substance, but a form of energy. There is only one way to account for this energy, and that is, to assume that the molecules of a body are in motion. *Heat is energy transformed into the invisible form of molecular motion.*

226. Mechanical Equivalent of Heat. In 1842, Dr. J. R. Mayer, of Heilbronn, Germany, asserted that an exact quantitative relation existed between mechanical work and heat, and attempted to calculate its numerical value.

Mayer's method was based on the fact that more heat is required to raise the temperature of a confined mass of air 1° when it expands under pressure and does work than when its volume is kept constant. Mayer assumed that this extra heat is all consumed in doing work as the gas expands. On this assumption (which Joule afterwards showed to be true), Mayer obtained a result nearly the same as that obtained by Joule, who used a wholly different method.

Between the years 1842 and 1849, Dr. J. P. Joule, of Manchester, England, performed a memorable series of experiments for the purpose of estimating the value of the heat unit in terms of work. He employed several methods; but the most important consisted in churning water with paddle wheels driven by a slowly descending weight. Joule found that 772 ft.-lb. of work are consumed in heating 1 lb. of water 1° F., or 1390 ft.-lb. ($\frac{9}{5}$ of 772) in heating 1 lb. of water 1° C. Later experiments have slightly increased these numbers. The final result, accurate enough for all practical purposes, may be expressed as follows:

The mechanical equivalent of one English heat unit is *778 foot-pounds* for 1° F., or *1400 foot-pounds* for 1° C. The mechanical equivalent of one calorie is *427 gram-meters.*

If, for example, a mass of 1 lb. should fall to the ground from a height of 1400 ft., and if all the kinetic energy thus acquired by the mass should be transformed into heat, the amount of heat thus generated would just suffice to raise the temperature of 1 pound of water 1° C.

If J denote the value of Joule's equivalent, H the number of heat units consumed or generated, W the corresponding number of units of work, the truth established by the labors of Mayer and Joule is concisely expressed by the formula

$$W = JH.$$

227. Mechanical Theory of Heat. We can conceive of only one way by which a body apparently in a state of rest can become a storehouse of kinetic energy. The body must consist of exceedingly small parts (molecules), and these parts must be capable of the motion of vibration or of rotation, or of both combined. Heat is the energy of this molecular motion. Thermal phenomena, therefore, are essentially mechanical phenomena. It is true that we do not know the exact nature of molecular motion; we cannot observe the motions of the molecules of a body as we can the motions of the wheels in a factory, or the motions of the planets around the sun. But even without this knowledge it is possible to apply the known laws of motion and energy to a great variety of thermal phenomena, especially to those manifested by matter in the gaseous state. This application has been made very thoroughly during the last half century, and the knowledge thus acquired forms a branch of science known as the Mechanical Theory of Heat, or Thermodynamics.

Only a portion of the heat which enters a body takes the form of kinetic energy. When a solid body is heated, three different effects are produced:

(1) The temperature of the body rises; its sensible heat is increased.
(2) The body expands; the forces of cohesion are overcome.
(3) The expansion takes place against external pressure.

Effect (1) is due to increase in the rate of the molecular motion. The heat thus expended takes the form of kinetic energy.

Effect (2) is internal work done against the force of cohesion.

Effect (3) is external work done against the external pressure. In (2) and (3) the heat applied is transformed into potential energy.

When a solid melts, the fact that its temperature does not change shows that the heat applied is producing effects (2) and (3) only. This heat is called the latent heat of fusion.

When a liquid changes into a gas, the same thing again happens; only effects (2) and (3) occur, and all the applied heat is changed into the potential energy of molecular separation. This heat is called the latent heat of vaporization.

When a hot body is brought in contact with a cooler one, energy is transferred from one to the other till their molecules have the same rate of vibration. The two bodies are then said to have the same temperature. We may still say (as in Chap. III) that heat flows from one to the other, that heat is lost by one and gained by the other; only we must remember that what is lost and gained is not matter, but energy.

When we touch a hot body, a transfer of energy from the body to the molecules of our skin takes place, and the sudden increase in their rate of vibration produces the sensation of heat. When we touch a cold body, energy passes from us to the cold body, and this change produces the sensation of cold.

228. Kinetic Theory of Gases. In solids, owing to cohesion, a molecule never moves to a sensible distance from its mean position. But in liquids, and still more in gases, the molecules are released from the restraining influence of cohesion, and can penetrate to any part of the space occupied by the body to which they belong. We must imagine the molecules of a gas as constantly colliding with one another and with the sides of the vessel that contains it, and then rebounding along new paths. This view is justified by the phenomena of diffusion as well as by those of heat. The pressure of the gas against the walls of the vessel that contains it is caused by the incessant impact of its molecules upon the walls, each square inch of surface receiving millions of blows every second.

It can be proved that a gas constituted as here described will obey the laws of Boyle and Charles, and behave in all respects just as the most perfect gases behave.

When a gas does work by expanding against external pressure, its temperature falls.

But Joule showed by experiment that if a gas expands into a vacuum, and therefore does no external work, its temperature does not change. This proves that a gas, in expanding, does no internal work.

The specific heat of air under constant pressure is 0.2375, while its specific heat under constant volume is only 0.1685. The difference is exactly equivalent to the amount of work done by the gas in expanding against the pressure of the atmosphere.

229. Heat of Chemical Combination. Chemical combination sets heat free. Combustion, or the union of oxygen with carbon or hydrogen, is the common mode of generating heat for useful purposes.

When 1 gram of carbon unites with oxygen to form carbon dioxide, about 8000 calories of heat are set free.

When 1 gram of hydrogen unites with oxygen to form the vapor of water, about 34,000 calories of heat are set free. These numbers are called the *calorific powers* of carbon and of hydrogen, respectively.

The doctrine of energy explains this enormous evolution of heat as due to the conversion of the potential energy possessed by unlike atoms attracted by chemical affinity into the kinetic energy of sensible heat.

In order that carbon dioxide or water may be decomposed into their elements, work must be expended to overcome the force of chemical affinity which binds the atoms into a molecule. Energy in the form of heat, or in some other form, must be supplied from an exterior source. In the case of carbon dioxide, this decomposition takes place on a grand scale in the vegetable world under the influence of the sun's rays (§ 160). The carbon is stored up by the plant, the oxygen is set free in the atmosphere. The heat of the sun is consumed in this process, but it reappears again when the wood or the coal is burned, and also in the slow oxidation of substances taken into the bodies of animals as food.

Men and animals have a higher temperature than the air that surrounds them. The average normal temperature of a man is about 37° C. (98.4° F.). Heat, therefore, is constantly leaving a man's body and passing to the cooler objects around him. A little passes away by conduction; but the most is lost by radiation, by evaporation from the skin, and by exhaling aqueous vapor in the breath. The sources of this heat are the chemical processes constantly going on in the body, whereby the food taken into the stomach is oxidized.

The Steam Engine.

230. Carnot's Principle. The energy of mechanical motion can be transformed entirely into heat (by friction, impact, etc.), whatever be the temperatures of the bodies concerned. But heat can be made to do mechanical work *only when it passes from a hot body to a cooler one, and then only partially.*

The power of heat to perform work may be compared to that of water to perform work. Just as you can get no work out of water unless it is falling from a higher level to a lower, so you can get no work out of heat except when it flows from a higher temperature level to a lower.

A machine contrived for the purpose of converting heat into work is called a *heat engine.* If H denote the amount of heat supplied to the engine, W the portion of this heat converted by the engine into work, both quantities being expressed in terms of the same thermal unit, then the ratio of W to H is called the *efficiency* of the engine.

The French physicist Carnot (1824) proved that *the maximum efficiency of a heat engine is independent of the material substance employed to carry heat into the engine, and depends only on the temperatures at which the heat enters and leaves the engine.*

This is called Carnot's Principle.

If t and t', respectively, denote the temperatures (Centigrade) at which the working substance in the gaseous state enters and leaves the engine, the maximum value of the efficiency of the engine is the following fraction:

$$\frac{t-t'}{273+t}.$$

The number 273 in the denominator of this fraction represents the *absolute temperature* of melting ice (§ 98).

231. The Steam Engine. By far the most important kind of heat engine is the *steam* engine. In Figs. 192 and 193 are represented sectional views of the cylinder and steam chest, where the heat of the steam is made to do work; the piston *P* moves to and fro in the cylinder, and the slide valve *V* to and fro in the steam chest.

Steam, generated in the boiler, enters the steam chest at *A*. In Fig. 192 the slide valve has such a position that the steam rushes down below the piston and moves it upward by its expansive force; at the same time the exhaust steam above the piston passes out of the cylinder through *B* either into the atmosphere, or into a closed vessel (the condenser), in which the steam is condensed.

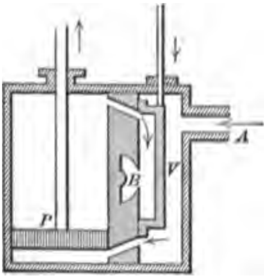


FIG. 192.

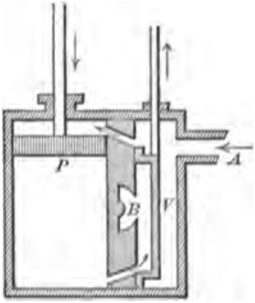


FIG. 193.

Before the piston reaches the end of the cylinder, the slide valve is moved by a rod connected with the shaft of the fly wheel into the position shown in Fig. 193. This change of position shuts off the steam from the lower side of the piston and admits it on the upper side. The piston is now pushed downwards, while the exhaust steam escapes through the opening *B*.

In this way the piston is made to move to and fro, and the piston rod carries this motion to a crank, where it is converted into the rotatory motion of the fly wheel. A belt passing round the fly wheel conveys the motion to the machinery, by which useful work is done.

The fly wheel, owing to its great mass and the distance of the mass from the axis, resists very effectually all tendencies to change of speed.

In the locomotive, the driving wheels answer the same purpose to some extent.

The governor. In order to secure as far as possible a uniform rate of rotation, stationary engines are often provided with a governor (Fig. 194), which regulates the supply of steam to the steam chest. A vertical shaft *A* carrying two arms with heavy balls *B*, *B* is made to rotate by the machinery. Centrifugal force tends to make the balls move away from the shaft; in so moving they raise, by means of connecting rods, a collar *C*; this collar moves a lever attached to a valve controlling the supply of steam. If the motion becomes too rapid, the collar rises and the valve closes.

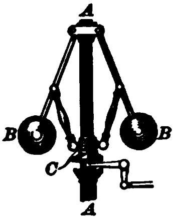


FIG. 194.

Condensing engines. In these the waste steam passes into a box or condenser, where it is condensed by contact with a jet of cold water, or by contact with pipes through which cold water is flowing. Therefore, there is always very nearly a vacuum on the exhaust side of the piston. Hence, a condensing engine will work at a lower pressure of steam than a non-condensing engine, or one in which the steam is blown out into the air; and is more economical, provided a plentiful supply of water is at hand. The engines of steamboats and steamships are condensing engines.

The locomotive. The locomotive is a high pressure non-condensing engine. The steam is generated at a pressure varying usually from 75 to 175 lb. per square inch, and, instead of being condensed, is blown in puffs into the air. In order to vaporize the water as quickly as possible, metallic tubes conveying the hot gases of the furnace pass through the boiler, thus exposing as great an area of the boiler as possible to the action of the heat. These tubes lead into the blast pipe, or pipe by which the waste steam escapes into the chimney. This arrangement creates a powerful draught for the fire; in fact, the fire is urged most when the engine is going rapidly, or at the very time when there is need of rapid combustion.

Factory and mill engines are, like the locomotive, non-condensing engines. Tubular boilers are now in general use.

Expansion Principle. To allow steam to escape at high pressure without doing work is a waste of energy. Considerably more work can be done with a given amount of steam by shutting off the communication with the boiler before the end of the stroke, and leaving the steam already in the cylinder to finish the stroke by its expansive action.

The expansion principle of working is applied in large marine engines, where the steam passes through two or more cylinders.

232. Source of the Energy. The energy of a steam engine is derived from the heat evolved by the combustion of the coal. The coal of the furnace and the oxygen of the air rush together, and their potential energy, originally obtained by the action of the sun on the coal-producing plants of former ages, is converted into the visible energy of mechanical motion.

When an engine is doing work it is found that the exhaust steam is cooler than the steam which enters the cylinder, and the researches of Hirn have shown that the amount of work done is exactly equivalent to the heat that has disappeared. When an engine moves without doing work, the steam remains hot, and the engine becomes heated.

233. Efficiency. The efficiency of an engine has been defined as the ratio of the heat consumed in doing work to the total amount of heat supplied. Carnot's Principle (§ 230) determines the maximum efficiency of an engine. The most perfect engine imaginable cannot convert a quantity of heat into the energy of work without reducing the temperature of another portion of the heat to a point where it is useless under the given conditions for the performance of work. The value of maximum efficiency is given at the bottom of page 250. In practice, owing to loss by friction, noise, conduction and radiation of heat, and the back pressure of the exhaust steam, the efficiency of the best-constructed engines is much less than the ideal maximum efficiency; it is never greater than $\frac{1}{10}$, and usually considerably less.

For a high-pressure non-condensing engine, working under a pressure of 5 atmospheres between the temperatures $t = 150°$ C., and $t' = 100°$ C., the maximum efficiency given by the formula on page 250 is $\frac{150 - 100}{273 + 150}$, or about $\frac{1}{8}$; that is, if all unnecessary waste of energy could be avoided, only $\frac{1}{8}$ of the heat supplied would be made to do mechanical work.

Energy Considered as Subject to Change.

234. Transformation of Energy. Let us consider a case in which energy undergoes certain transformations.

Let a bullet be fired from a gun vertically upwards. The energy received by the ball was stored up, before the discharge, in the gunpowder in the potential form of unsatisfied chemical affinity. This potential energy, when the powder is ignited, is instantly transformed into the energy of heat, which shows its power to do work by exerting great pressure on the ball and on the barrel of the gun.

Gunpowder is a mixture of niter (potassic nitrate), sulphur, and charcoal. When the powder explodes, chemical changes occur which set free at a high temperature a mixture of gases (nitrogen and carbon dioxide). The quantity of gas generated is such that at atmospheric pressure and temperature it would occupy some 1500 times the bulk of the powder.

The ball, yielding to the pressure, leaves the gun with great velocity. Here we see the energy of heat, evolved by the burning of the powder, transformed into the visible kinetic energy of a moving body.

As the ball ascends, this kinetic energy is transformed without loss or gain (§ 222) into the potential energy due to separation from the earth. During the descent of the ball the converse change takes place. The potential energy acquired by the ascent of the ball is reconverted into kinetic energy, and, if it were not for the resistance of the air, the ball would, on reaching the ground, have the same amount of kinetic energy as when it left the gun.

Owing, however, to the resistance of the air, a small portion of the energy has been converted into heat, and into the vibrations of air molecules which strike against our ears, and cause the sensation of sound (the whistling of the ball).

When the ball strikes the ground, all its visible energy disappears, but we have every reason to believe that here also there is no destruction of energy, but only a transformation. If the ball bore a hole into solid matter, then a part, at least, of the energy is clearly accounted for by the work done in overcoming the force of cohesion. And we know that whenever there is a sudden stoppage of motion the bodies concerned are heated; that is to say, the energy of visible motion is converted into the invisible form of heat.

Whenever two inelastic bodies, or two bodies imperfectly elastic, collide, the total kinetic energy is diminished by the impact. But the loss is explained by the fact that the bodies are heated by the collision, and moreover they suffer changes of form, or are broken in pieces, or in some way the force of cohesion is overcome.

Thus, let a mass of 8 lb., moving with a velocity of 40 ft. per second, strike a mass of 8 lb., at rest, both masses being devoid of elasticity. Then (§ 186) the common velocity of the masses after impact will be 20 ft. per second. The kinetic energy before impact is 200 ft.-lb., but after impact only 100 ft.-lb. In this case half the energy has apparently been destroyed, but really converted into heat or expended in doing work.

If two perfectly elastic bodies collide, they are not heated, neither do they suffer any change of shape, and, as we should expect, there is no loss of kinetic energy.

The steam engine affords an excellent example of the transformation of energy. The potential energy of the fuel is first converted into heat energy, which is communicated to the water in the boiler, and changes it into steam. The energy of the steam is transformed in the cylinder into the mechanical energy of the piston crank and fly wheel; and this energy is then, by means of suitable mechanism, made to do work; or, by means of a dynamo, it may be transformed into the energy of an electric current, which, at a distant place, is again transformed into mechanical energy, and made to do useful work.

235. Conservation of Energy. The very general principle that goes by this name may be stated as follows:

Energy can assume a great variety of forms, but cannot be created or destroyed.

More fully stated, the principle asserts that, if a system of bodies has a certain amount of energy, it must retain this energy in one form or another, unless it comes in contact with other bodies; in which case the energy may undergo changes of form and become differently distributed, but will remain the same in amount. If the system of bodies is so large that it comprises the whole universe, then the system cannot gain or lose energy by contact with other bodies. Hence, the total amount of energy in the universe is invariable, although changes of form are constantly occurring.

"The energy of nature," says Professor Tyndall, "is a constant quantity, and the utmost man can do in the pursuit of physical truth, or in the applications of physical knowledge, is to shift the constituents of the never varying whole, sacrificing one if he would produce another. The law of conservation rigidly excludes both creation and annihilation."

This principle does not admit of a direct proof like the similar one about the conservation of matter. We can confine the products of chemical combination so completely as to prove beyond a doubt that no matter passes out of existence. But we cannot directly show that no energy is destroyed; for during the process of combination some of the energy is sure to escape into the surrounding bodies. It is impossible to collect and measure all the energy which is thus apparently lost. We can, however, always be on the watch to notice whether any phenomenon occurs that is inconsistent with the truth of the principle in question. No such phenomenon has ever yet been observed.

Again, a very strong argument for the truth of the principle may be based on the uniform failure of all attempts to find what is known as "perpetual motion"; in other words, to devise a machine capable of doing work continuously without the aid of an external driving force. The principle of the conservation of energy leads to the conclusion that all such attempts are necessarily doomed to failure.

Still stronger evidence is furnished by the fact that predictions based on the principle respecting the behavior of bodies under certain conditions have always been found to be correct. The principle stands the same test as Newton's Laws of Motion, or the Law of Gravitation.

236. Sources of Energy. The chief natural sources of energy available to man may be summarized under the heads fuel, food, rain, wind, and the tides. In every case, except the tides (which is of little practical importance), the energy may be traced back to one main source, the *sun.*

By burning *fuel* we generate large quantities of heat at high temperatures. With this heat we warm ourselves, cook our food, and drive our heat engines. Now, the fuel, whether wood, coal, or other material, is produced by the action of the sun's rays. They decompose carbon dioxide on the leaves of plants, setting free the oxygen, while the carbon is stored up in the woody structure of the plant. In this process the energy of the solar rays is transformed into the potential energy of chemical separation. Coal is the remains of a luxurious vegetation that existed on the earth many thousand years ago, and was buried under vast masses of rock. When we burn coal we reproduce the light and heat emitted by the sun in those distant ages in which the plants grew that formed the coal.

Food, like fuel, possesses potential energy. After the food is eaten this energy is converted by gradual oxidation into animal heat, and into the muscular energy which men and animals put forth when they perform work. A man that works hard has to eat more than if he does no work at all.

The energy of vegetable food is obviously derived, like that of fuel, from the energy of the sun's rays. And in the case of animal food, the only difference is that the food has first passed into the body of an animal before coming to us. "The animal has eaten the grass, and we have eaten the animal."

Rain is the source of our ponds, lakes, and rivers. In these the water has potential energy by reason of its elevation above the sea level. As this water descends towards the ocean, it can be made to turn water wheels and to do useful work.

But if the sun did not vaporize water and raise it in the form of clouds into the air, there would be no rain. Therefore, the source of the energy of falling water is the sun.

So, too, the energy of the *wind*, whether displayed in the fury of a cyclone, or utilized in propelling a ship over the ocean, is directly traceable to the sun; for winds are currents of air set in motion by the unequal heating effect of the sun's rays on the atmosphere.

237. Energy of the Sun. The rate at which the sun radiates energy in the form of heat is simply astounding. "If," says Professor Young, "we could build up a solid column of ice from the earth to the sun, 2¼ miles in diameter, spanning the inconceivable abyss of 93,000,000 miles, and if then the sun should concentrate his power upon it, it would dissolve and melt, not in an hour or a minute, but in a single second; one swing of the pendulum, and it would be water; seven more, and it would be dissipated in vapor." The heat from *one square foot* of the sun's surface, if made to do work, would keep in full action an engine of 10,000 horse power.

Of this enormous outflow of heat, the earth receives not more than one part in two thousand million. And of the heat received by the earth, probably not more than a thousandth part is stored up by animals and vegetables, or in water above the sea-level; and this constitutes practically the whole revenue of energy available to man.

238. Source of the Sun's Energy. The enormous expenditure of energy by the sun has been going on for ages without sensible change. "There is not the slightest reason," says Professor Ball, "for the belief that the sun has been during geological times either appreciably hotter or appreciably colder than at the present moment." How, then, is this tremendous outflow of energy maintained? The idea that the sun is a white-hot body cooling down may be dismissed at once; were this the case, its temperature would fall very perceptibly during two or three centuries.

Nor will the supposition that the sun is a huge fire answer the purpose. "Were the sun a block of burning coal, and were it supplied with oxygen sufficient for the observed emission, it would be utterly consumed in 5000 years" (Tyndall).

It has been suggested that the solar heat is maintained by the impact of meteoric matter falling into the sun (meteoric theory, Mayer, 1848); and it has been computed that a quantity of matter equal to that of the moon, falling each year into the sun, would maintain the solar radiation.

It has also been suggested that solar energy is due to a very slow contraction of the sun, caused by gravitation, and accompanied by a gradual liquefaction and solidification of the gaseous mass (contraction theory, Helmholtz, 1853); and Helmholtz has shown that the entire emission of heat may be accounted for by a rate of contraction so slow that about 10,000 years must elapse before any sensible diminution of the sun's bulk could be detected.

Both these theories are probably true to a certain extent, but astronomers at the present time are generally of the opinion that the main source of the sun's heat is due to the contraction of its bulk.

We may now go on and ask, How did the meteorites get their energy of motion, or the material of the sun its potential energy? A discussion of this question would take us outside the field of Elementary Physics.

239. Dissipation of Energy. When we rub together two pieces of wood, they become warmer. We have converted a certain amount of muscular energy, derived from the potential energy of food, first into visible kinetic energy, and then into heat. This heat is no doubt proportional to the work performed, but it soon diffuses by conduction and radiation through surrounding bodies, and then it is no longer available for the performance of work. It is like water which has fallen from an elevated position to the sea level, and so is rendered powerless to do work of any kind.

This example illustrates two truths of deep meaning:

1. We can easily transform the energy of mechanical motion into heat, but we cannot, by any means in our power, transform all the heat back again into the energy of mechanical motion (§ 230).

2. There is a general tendency in nature for potential energy of every kind to assume first the form of visible kinetic energy, and finally the invisible form of uniformly diffused heat. In this last form the energy is unavailable for the performance of work.

We are, therefore, compelled to believe that the mechanical energy of the universe is slowly but surely passing into the form of heat. By this change the energy is unaltered in *quantity*, but it has degenerated in *quality*, as estimated by its ability to do work.

If this is true, then the available energy of the universe must at last be reduced to that point where the existence of living beings will no longer be possible. But this question is one which need give no anxiety for many thousands of years to come.

REVIEW EXERCISES ON CHAPTER VI.

1. Define *work*, and the common unit of work; also give a numerical example to illustrate the formula, work = force × distance.

2. How is the work done by a force which acts obliquely on a moving body estimated?

3. Name and define the common unit employed to measure rate of working.

4. Define the *foot-poundal* and the *erg*.

5. What is the use of a machine? Illustrate your answer by some special case.

6. State the *Principle of Work.* Show by an example the meaning of the statement that what is gained in power is lost in speed.

7. Define the terms *efficiency* and *mechanical advantage* and illustrate by an example.

8. Apply the principle of work to the *wheel and axle.*

9. Apply the principle of work to the *screw.*

10. What is the simplest way of finding the mechanical advantage of a compound machine?

11. Prove the formula for kinetic energy (given in § 217).

12. Explain why a rifle ball will bore a hole through a two-inch plank while the recoil of the gun only gives the shoulder a slight shock.

13. Give examples of bodies possessing potential energy.

14. How is potential energy in the case of gravity measured?

15. Give examples of the generation of heat by work.

16. Give examples of the conversion of heat into mechanical energy.

17. What is the true nature of heat?

18. Explain the statement that the mechanical equivalent of heat is 1400 ft.-lb.

19. Explain the statement that the calorific power of carbon is 8000 calories.

20. State Carnot's Principle.

21. Explain with diagrams how the heat of steam is converted into mechanical energy by a steam engine.

22. Give an example of energy undergoing transformations.

23. State the Law of Conservation of Energy. What are some of the grounds for believing it to be true?

24. Name some of the sources of energy, and show that the sun is the main source.

25. What is the general tendency of potential energy of every kind? What does it compel us to believe?

CHAPTER VII.

MAGNETISM AND ELECTRICITY.

Magnetism.

240. Magnetic Attraction. Magnets are bodies that have the power to attract iron (and in much weaker degree, nickel and cobalt). This power or property is called *magnetism.* The name is derived from Magnesia, a city in Asia Minor, where the property was first observed.

The mineral magnetite (magnetic oxide of iron, lodestone) is a natural magnet. The magnets in common use are bars of iron or steel, either straight or bent in the form of a horseshoe, to which magnetism has been imparted artificially.

Magnetic attraction is rendered evident by very simple experiments.

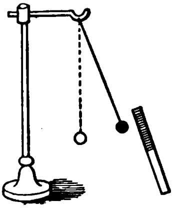


FIG. 195.

(1) When a magnet is brought near a small bit of iron suspended by a thread, the iron moves towards the magnet (Fig. 195). Paper or glass held between the two bodies does not seem to interfere with the attraction, but a piece of sheet iron weakens it.

(2) When a magnet is placed in iron filings, on raising the magnet the filings cling to it.

241. Poles of a Magnet. When a magnet is placed in iron filings, the filings adhere to the magnet much more abundantly near the ends than at the middle. The points of strongest action are, therefore, near the ends. These points are called the *poles,* and in a bar magnet the straight line which joins the poles is called the *magnetic axis.*

242. Magnetic Action of the Earth. A small steel magnet mounted by means of a brass cap on a pointed rod is called a *magnetic needle* (Fig. 196). If a magnetic needle is removed from the action of all other bodies, it will point nearly north and south; and if moved from this position it will return to it again. We shall call the pole of the magnet towards the north the *north* pole, and the other pole the *south* pole.

The vertical plane passing through a magnetic needle at rest under the earth's action is called a *magnetic meridian*, and the angle which it forms with the geographical meridian is called the *declination* or *variation* of the magnetic needle.

If the needle is suspended so that it can move in a vertical plane as well as a horizontal one (Fig. 197), the north pole will be depressed in the northern hemisphere.

The angle which the needle makes with a horizontal plane is called the *dip* or *inclination* of the needle.

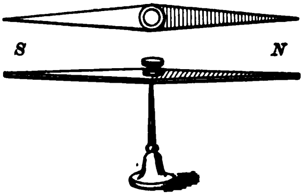


FIG. 196.

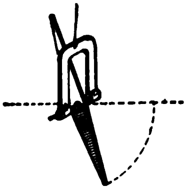


FIG. 197.

The variation of the needle differs in different places, and at the same place it undergoes slow changes, both irregular and periodic. Its present value on the coast of Maine is about 20° W.

Isogonic lines are lines drawn on a map through places of *equal* variation. An *agonic line* is a line through places of *zero* variation.

A *mariner's compass* is a magnetic needle which moves over a circular disc on which are engraved the points of the compass. A mariner can ascertain his true north and south line by observing the direction of the needle and making proper allowance for the variation.

The dip also varies with the locality. In latitude 40° on this continent it is about 70°. At a certain place in northern British America it is 90°.

243. Mutual Action of Two Poles. Two poles of the same name (both north or both south) are called *like* or *similar* poles, and two poles, one north and the other south, *unlike* or *dissimilar* poles. Then the mutual action of two poles may be thus stated:

Like poles repel each other; unlike poles attract each other.

This law is easily verified by simple experiments with two magnetic needles, or with two magnetized sewing needles floating on water.

It is often said that there are two kinds of magnetic force, but the difference in kind means nothing more than is expressed by this law.

244. Strength of a Pole. Magnetic poles differ in strength. A pole A is twice as strong as a pole B, if A exerts twice as great a force upon a bit of iron near it as B exerts upon the same bit of iron at the same distance. The force exerted by two poles upon each other depends not only on their strength, but also on their distance apart. Coulomb proved that the law of attraction or repulsion of two poles is identical in form with that of gravitation, namely:

The force exerted between two magnetic poles varies directly as the product of their strengths, and inversely as the square of their distance apart.

If m and m' denote the strengths of two poles, d their distance apart, and f the force acting between them, then

$$f = \frac{mm'}{d^2}.$$

Let $d = 1, f = 1$, and $m = m'$; then $m = 1$; hence

A pole of unit strength is a pole which repels with a force equal to one unit a like pole of the same strength placed at a distance of one unit.

In the C. G. S., or centimeter-gram-second system of units, the unit of length is 1 cm., and the unit of force 1 dyne; in this system, the unit pole is one which *repels a similar pole placed at a distance of 1 cm. with a force of 1 dyne.*

245. Magnetic Induction. An iron nail, held near one end of a strong magnet (Fig. 198), becomes itself a magnet; as is proved by its power to sustain iron filings applied to its lower end.

If the nail is allowed to touch the pole of the magnet, it will be held fast; hence we infer that the upper pole of the nail is *unlike* the pole of the magnet.

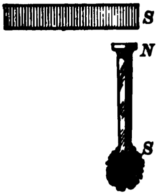


FIG. 198.

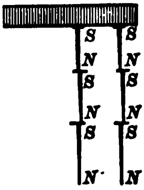


FIG. 199.

This phenomenon is called *magnetic induction;* we say that the magnet has induced magnetism in the nail. The attraction between a magnet and soft iron is always due to induction; it is the attraction of unlike magnetic forces.

Magnetic induction explains why a chain of nails can be supported from the pole of a strong magnet (Fig. 199); also why, if two such chains are formed, we cannot by swinging them bring their ends together.

246. Retentive Power. The nail in Fig. 198 retains its magnetism only while it is near the inducing magnet; if it is moved away, the filings drop off and the poles disappear.

If *hard steel* is used instead of soft iron, the steel will also become magnetic, but in a weaker degree, and only after some time. The steel, however, will retain its magnetism when taken away from the inducing magnet; it has become a *permanent* magnet.

Steel offers a strong resistance both to gaining and to losing magnetism; this resistance is sometimes called its coercive power, but a better term is *retentive power.*

The retentive power of soft iron is very small.

The inductive action of a magnet upon steel is increased by striking the steel with a hammer. If a long steel rod is held parallel to the direction of a dipping needle (Fig. 197) and struck with a hammer, it will become a magnet by the inductive action of the earth.

247. Magnetization by Contact. A bar of steel may be converted into a permanent magnet as follows :

Stroke the bar several times from the center to one end with one pole of a magnet, and then an equal number of times from the center to the other end with the other pole of the magnet. Take care to turn the bar over during the process, so that both sides may be magnetized. The poles of the bar will be *unlike* those of the magnet which they touch.

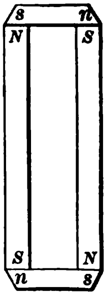


FIG. 200.

Magnets gradually lose their strength unless they are kept in constant action upon pieces of soft iron called *keepers*. The proper arrangement of a pair of bar magnets with their keepers is shown in Fig. 200.

248. Theory of Molecular Magnets. If a magnet be separated into parts, each part will be found to be a complete magnet with two poles ; and the poles of each part will face the same way as the like poles of the entire magnet (Fig. 201). This is true, however small the parts may be. Thus we are led to the conclusion that *every molecule of a magnet is itself a complete magnet.*

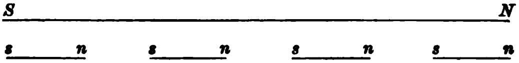


FIG. 201.

If we treat a glass tube full of iron filings as described in § 247, the filings will arrange themselves lengthwise, and the tube will behave like a magnet. On shaking the tube all signs of magnetism will disappear. The inference is that the molecules of ordinary iron are really magnets, but so disorderly arranged that they neutralize one another. Magnetizing the iron turns the molecules around so that their like poles all face the same way.

This theory explains retentive force as the resistance which molecules oppose to rotation, and enables us to understand why heat, vibrations, and blows tend to destroy magnetism.

249. Magnetic Field. The space around a magnet through which its action extends is called a *magnetic field.* A magnetic field is explored by experiment as follows :

(1) Place a bar magnet horizontal, and move a small compass needle slowly round it, stopping occasionally to observe the direction of the needle when at rest. At points equally distant from the poles of the magnet the direction of the needle is parallel to the axis of the magnet.

The nearer the needle is to either pole of the magnet, the more strongly is the *unlike* pole of the needle attracted towards the pole which it approaches, while the *like* pole is repelled. The needle, when at rest, always points in the direction of the resultant of all the magnetic forces that act upon it. At a comparatively small distance, the magnet ceases to exert any perceptible influence upon the needle.

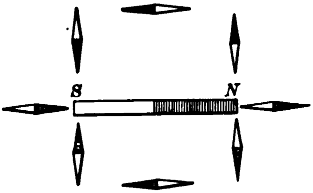


FIG. 202.

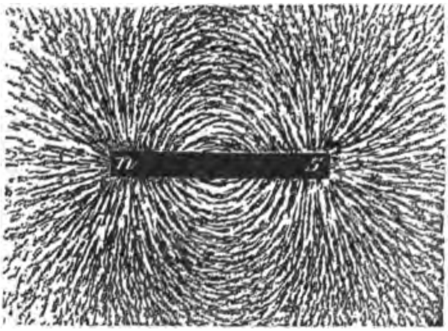


FIG. 203.

(2) A map of a magnetic field, showing at a glance its general nature and properties, may be made by placing a sheet of paper over a magnet, and sprinkling iron filings upon the paper. When the paper is gently tapped, the filings arrange themselves in curved lines, as shown in Fig. 203. The bits of iron are converted by induction into magnets, which cling to one another, end to end, and thus indicate the direction of the resultant magnetic force at every point.

250. Lines of Force. The curved lines in Fig. 204 formed by the iron filings are called *lines of magnetic force.*

A line of magnetic force is a line so drawn in a magnetic field that its direction at every point is the same as the direction of the resultant magnetic force at that point.

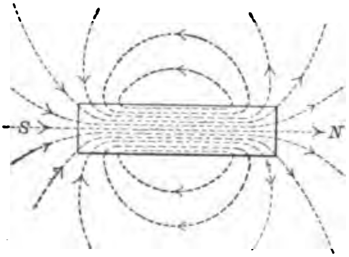


FIG. 204.

The positive direction of a line of force is assumed to be that in which a north pole would be urged, if placed on the line.

The lines of force are, therefore, to be regarded as proceeding from the north end of a magnet through the air to the south end. But they do not stop here; they continue through the iron, so as to form complete *closed circuits,* as shown in Fig. 204.

The conception of lines of force enables us to describe magnetic induction very briefly. When a piece of iron is placed in a magnetic field, the lines of force make the end which they *enter* a south pole, and the end from which they *emerge* a north pole.

Very few words are now required to explain why the earth tends to make a magnet turn on its axis, but has no tendency to move the magnet either north or south. The magnetic field produced by the earth is so enormous in extent that the lines of force at any place are sensibly parallel lines. The earth, therefore, exerts upon the poles of the magnet two equal parallel forces in opposite directions. Such a pair of forces is called in Mechanics a *couple,* and it tends to cause rotation but not translation (§ 165); for instance, we apply a couple when we make an auger enter wood, or a corkscrew enter a cork.

Lines of force were first employed in the study of magnetic and electric phenomena by Faraday (about 1830). Magnetism and electricity, as we shall see, are very closely related. Probably nothing has contributed so much to our present knowledge of this relationship, and the uses we make of it, as the conception of lines of force.

251. Permeability. Lines of force have many curious properties. One is that they pass through iron much easier than through air. In other words, iron has much greater magnetic *permeability* than air.

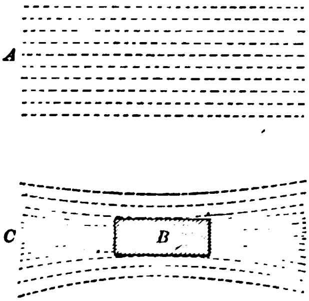


FIG. 205.

Suppose that we have a magnetic field of uniform intensity, that is, a field in which the lines of force are parallel and equidistant (Fig. 205, *A*), and that we introduce into this field a bar of soft iron *B*. Then the field will take the aspect shown in the lower part of Fig. 205; most of the lines of force will crowd together and pass into the iron, creating there an intense field, while the field on each side of the iron will be weakened.

Watches are sometimes protected from magnetic action by enclosing the works in a case of soft iron; then the lines of force pass into the case, and around it, but not across it.

LABORATORY EXPERIMENTS.

1. Perform such experiments as suggest themselves to you with a magnet and some iron nails for the purpose of illustrating magnetic induction (§ 245).

2. Magnetize by contact a steel knitting needle (§ 247); then separate it into parts, and examine the magnetic properties of each part.

3. Perform the experiment with a glass tube full of iron filings which is described in § 248.

4. With the aid of iron filings make a map of the lines of force around a bar magnet (§ 249).

The filings may be fixed in their positions by applying a thin coating of shellac varnish to the paper beforehand, and then directing a gentle current of steam upon the paper after the filings have arranged themselves into lines of force. When the varnish hardens, the filings stick to the paper.

5. Make a map of the lines of force around and between two bar magnets placed parallel to each other.

CLASS-ROOM EXERCISES.

1. Why does a magnetic needle, floating on water, point towards magnetic north and south, but not move bodily in either direction?

2. If you have a bar magnet and a knitting needle, one end of which is notched with a file, describe how you would proceed in order to convert the needle into a magnet with its north pole at the notched end.

3. Two sewing needles are magnetized so that their eyes are north poles. They are then stuck through bits of cork and thrown into water, so that they float with their eyes upward. How will they behave towards each other, and how if the south pole of a magnet is held over them?

4. How can you tell whether a steel rod is magnetized or not, (1) by means of a compass needle, (2) without the aid of a magnet or of iron?

5. Two iron rods hang from the same pole of a magnet. Will they hang parallel? Give a sketch and reasons for your answer.

6. You have two similar rods, one of steel, the other of soft iron. You have also a bar magnet and some iron nails. Describe how you would tell which was the steel and which the iron rod.

7. A bar magnet is laid on a table with its north pole projecting over the edge. An iron nail clings to the projecting end. State and explain what happens when the south pole of another magnet is brought above and near the north pole of the first.

8. You have three equal bar magnets without keepers. How would you arrange them so that when not in use they will best retain their magnetism? Give a sketch.

9. If a long bar of very soft iron is held upright, why does its lower end repel the north pole of a compass needle?

10. The beam of a balance is made of soft iron. When it is placed at right angles to the magnetic meridian, two equal weights placed in the pans just balance. Will the weights still appear to be equal when the beam is placed in the magnetic meridian? Explain.

11. If a compass needle were carried around the equator, would it point in the same direction at all places? If not, state as nearly as you can what changes would be observed in its behavior during its journey.

12. A tall iron mast stands just in front of the compass on a wooden ship. What effect will be produced in the declination of the compass when the ship is sailing due east (1) in the northern hemisphere, (2) in the southern hemisphere?

Note. Many of the exercises on Magnetism and Electricity have been selected from papers set by the Science and Art Department, South Kensington, England.

Electrostatics.

252. Electric Attraction. If a dry glass rod is rubbed with warm dry silk, the glass acquires the property of attracting light bodies, such as bits of paper or cork. If a vulcanite rod (or a hard rubber ruler) is rubbed with fur or flannel, the vulcanite acquires the same properties. The glass and the vulcanite are said to be *electrified*, or in a state of *electrification.*

In general, two unlike substances, when rubbed together, are both more or less strongly electrified. A part of the energy expended in rubbing them together is changed into the form of electrification.

253. Electrification by Contact. A thin disc of metal provided with a glass handle is called a *proof plane.*

If we apply a proof plane to the surface of an electrified rubber ruler, and then bring the proof plane near bits of paper or cork, they will be attracted (Fig. 206).

This experiment proves that electrification can be communicated from one body to another by direct contact.

We naturally think that something has passed from the ruler to the proof plane. This something is indeed invisible, but it manifests its existence by the fact that the proof plane acquires the power to attract bits of paper. This invisible something we call *electricity;* and we say that the proof plane has received a charge of electricity from the ruler.

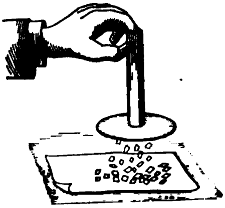


FIG. 206.

In the study of electric phenomena it is sometimes found useful to regard electricity as a fluid capable of flowing from one body to another; but we must remember that this idea is nothing more than a mere hypothesis.

254. Conductors and Insulators. If we rub one end of a glass rod with silk, the end rubbed will attract light bodies, but the rest of the rod will not. If we treat a metal rod in the same way, no part of the rod will show signs of electrification. But if we hold the metal rod by means of a glass handle, and then rub one end, the entire rod will acquire the power to attract light bodies.

To explain these facts, we speak as though electricity were a fluid, and say that the electricity developed by friction at any part of a metal rod quickly flows or spreads to all the other parts. On the contrary, electricity cannot flow through glass. More briefly, the metals are *good conductors* of electricity, but glass is a *bad conductor*. This language is only a concise way of expressing facts, and implies no assertion as to the real nature of electricity.

Since bad conductors prevent the escape of electricity from a body, they are also called *insulators*.

Bodies may be classified roughly as conductors and insulators, but no hard and fast line between them can be drawn. No body is a perfect conductor, and no body is a perfect insulator. Between the best conductor (silver) and the best insulator (dry air), every grade of conducting power is found.

Some of the best conductors are: the metals, charcoal, acidulated water, and the human body.

Some of the best insulators are: dry air, glass, paraffine, vulcanite, shellac, silk, wool, and dry wood.

When we hold in our hands a metal rod, we cannot electrify it by friction because the electricity escapes as fast as generated through our body to the floor or the ground; as the electricians say, "the rod is connected to earth."

An electric charge will not remain on a conductor unless we interpose between it and the earth an insulator such as glass or vulcanite. The air around the conductor must also be dry; otherwise the charge will soon escape into the air.

255. Electric Repulsion. Electrify a glass rod by rubbing it with silk, and then bring it near an electric pendulum, or small ball of cork suspended by a dry silk thread (Fig. 207). The cork will be attracted to the glass; but after touching it (and being charged by it), the cork will be *repelled.* If now we bring near the cork a vulcanite rod electrified by rubbing it with fur, the rod and the cork will *attract* each other.

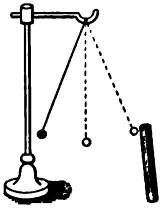


FIG. 207.

If we first electrify the cork by contact with the vulcanite, we again observe repulsion; and if we then bring near the cork a charged glass rod, we again find attraction.

256. Two Kinds of Electrification. The facts just mentioned lead us to say that the electrification of glass is different from that of vulcanite. By experimenting with other bodies, we find that some behave like glass rubbed with silk, while others behave like vulcanite rubbed with fur.

To distinguish these two kinds of electrification, we say that the glass is *positively* electrified, and the vulcanite *negatively* electrified. Two bodies, both of which are charged positively or both negatively, are said to be *similarly* charged. Two bodies, one of which is charged positively and the other negatively, are said to be *dissimilarly* charged.

We may now condense the facts thus far learned respecting electric attraction and repulsion into the following statements:

1. *Similarly charged bodies repel each other.*
2. *Dissimilarly charged bodies attract each other.*
3. *Bodies, however charged, attract unelectrified bodies.*

We must not fall into the error of thinking that there are two essentially different kinds of electric force. All that is meant by distinguishing electric charges as positive and negative is the difference in their mutual action.

257. Electrostatic Induction. The space through which an electrified body exerts its action is called an *electric field.* If we bring into the field of a positively charged body *A* (Fig. 208) an insulated unelectrified brass cylinder *BC*, a phenomenon similar to magnetic induction, and called *electrostatic induction*, occurs. On testing the cylinder, we find that the end *B* has a *negative* charge, and the end *C* a *positive* charge, while the middle part remains unelectrified. If *A* is removed, both of the induced charges disappear.

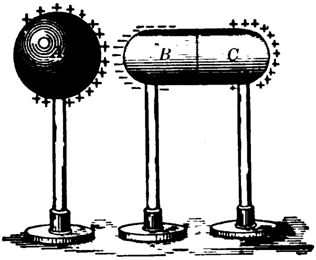


FIG. 208.

If, before removing *A*, we connect the cylinder for an instant to the earth (by a chain, or by touching it with the finger), the positive induced charge at *C* will escape to the earth, because it is repelled by the similar charge on *A*; but the negative induced charge at *B* will remain, because it is attracted or "bound" by the charge on *A*. If we now remove *A*, the cylinder will be left with a negative charge. This is called *charging by induction.*

It may seem that we have stored up electric energy in this way without doing any work. But work has to be done to overcome the attraction between *A* and the cylinder *BC*.

Induction explains why an electrified body always attracts an unelectrified body. The former induces on the nearer side of the latter a charge unlike its own, and on the farther side a charge like its own. The unlike charge, being nearer, makes the resultant effect one of attraction. By contact, the original charge is distributed over both bodies.

If *BC* is divided into two parts by a plane through its middle, and the parts are separated, while under the influence of *A*, *B* will carry away a negative, and *C* a positive, charge.

258. The Electrophorus. This instrument illustrates very well the process of charging a body by induction. It consists of a vulcanite plate *A* (Fig. 209), and a metal disc *B* provided with a glass handle. When we place the disc upon the plate, they touch only at a few points, and elsewhere there is a thin film of air between them.

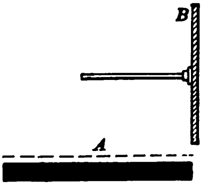


FIG. 209.

First, we charge the plate negatively by rubbing it with fur.

Secondly, we place the disc upon the plate; induction then occurs, as shown in Fig. 210, *A*.

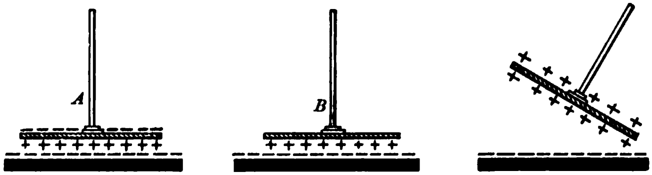


FIG. 210.

Thirdly, we remove the induced negative charge from the disc by touching it with the finger (Fig. 210, *B*); in other words, we connect the disc with the earth.

Lastly, we raise the disc by means of its handle from the plate. The disc will now be found to have a positive charge. If we discharge the disc in any way, we can repeat the whole operation; and we can thus charge and discharge the disc many times without sensibly diminishing the charge originally given to the plate.

The source of the energy possessed by the charged disc is not the charge given to the plate, but the *work done* in removing the disc from the plate which attracts it. The case is similar to that in which we impart potential energy to a body by raising it into the air.

259. Inductive Capacity. Faraday discovered that inductive action depends on the nature of the intervening medium, or *dielectric*, as he called it. The inductive power of glass, for example, is greater than that of air. The power of a dielectric to transmit induction, referred to the power of air as the unit, is called its *specific inductive capacity.*

Faraday also showed that the insulating or dielectric substances that surround charged bodies are in a state of molecular strain. Hence he concluded that induction is not action at a distance, as gravitation seems to be, but the transmission of force from molecule to molecule through the dielectric. We know that mechanical force can be transmitted from one end of a rope to the other, but not without straining the rope. Similarly, electric force can be transmitted across a dielectric, but not without straining the dielectric.

Faraday explained, in fact, all the phenomena of electric attraction and repulsion by the aid of lines of force, traversing the fields of force round electrified bodies, and straining the dielectric media through which they pass.

260. The Electric Spark. If we bring a body connected with the earth, say our hand, near enough to a charged body, a spark passes between them, and a sound is heard. On examination, we find that the charged body is no longer electrified; it has lost its charge, or has been *discharged.*

As the hand approaches the charged body, the charge induced on the hand becomes stronger, and the intervening air is more and more strained. At last the air is no longer able to withstand the strain; the air gives way, and allows the opposite charges to neutralize each other. In the act of union most of the energy stored up in the charged body is transformed into heat.

A similar phenomenon occurs when two insulated conductors, having unlike charges, are brought very near each other.

261. Gold-Leaf Electroscope. This instrument is used to detect an electric charge, and also to determine whether the charge is positive or negative.

It consists of two gold leaves, enclosed in a glass flask and attached to the lower end of a brass rod. The rod passes through the cork of the flask, and ends in a metal disc.

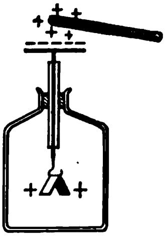


FIG. 211.

Suppose that a positively charged body is brought near the disc; the gold leaves will diverge, because each receives by induction a positive charge. If we now touch the disc with the charged body, the leaves will be charged by contact, and will retain a positive charge when the body is removed. If we do not touch the disc with the charged body, but touch it with the finger and then remove the body, the leaves will be charged *negatively* by induction. In both cases the leaves will remain separated.

Now, suppose that we wish to learn what kind of a charge a body has. We first charge the electroscope, say positively, so that the leaves slightly but distinctly diverge. We then bring the body near the disc. If the leaves diverge *more,* the body has a positive charge. If they diverge *less,* the body probably has a negative charge; if they collapse utterly, but diverge again as soon as the body touches the disc, then we may be sure that the charge is negative.

The reasons for these conclusions should be apparent to the learner after what has been said respecting electrification by induction and by contact.

The gold leaf electroscope is an imperfect instrument for testing the kind of electrification. *Increased divergence* of the leaves is the only sure sign. If the body tested is unelectrified, the leaves will partially collapse, owing to the inductive action of the electroscope upon the body.

Instruments called *electrometers* have been devised, by means of which the kind of electrification, and also its intensity, can be ascertained with the greatest ease and certainty.

262. Plate Electric Machine. This machine converts mechanical energy into electric energy by means of friction.

A circular glass plate *A* (Fig. 212) is made to revolve on a horizontal axis, and rub against a pair of leather cushions or rubbers *B* which are coated with a metal amalgam. Thereby the glass is positively charged, and the rubbers negatively charged.

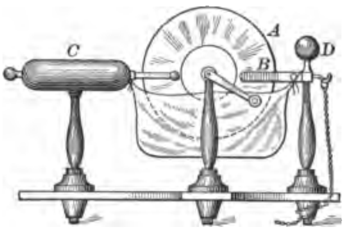


FIG. 212.

The charge in the glass is carried through half a revolution, and then acts inductively on the teeth of two metallic combs, which are attached to a brass cylinder *C* called the *prime conductor.* The negative charge induced on the teeth of the combs unites with the positive charge on the glass as fast as it is produced. The induced positive charge is repelled, and spreads over the prime conductor. Very soon the prime conductor becomes charged so strongly that a large spark can be taken from it by bringing the hand or any conductor near it.

The negative charge received by the rubbers passes to another insulated conductor *D.* Thus the machine may be employed to furnish a supply of either kind of electricity.

Usually positive electricity is desired; in this case the rubbers should be connected to earth, as shown in Fig. 212. Unless this is done, the negative charge on the rubbers soon becomes so strong that it will neutralize the positive electricity on the glass as fast as it is produced by friction. But if the rubbers are joined to earth, they are kept in their initial state by a steady flow of negative electricity to earth. For this reason, much more powerful sparks can be obtained from the prime conductor when the rubbers are joined to earth than when they are insulated.

263. Quantity of Electricity. The existence of electricity on a body is shown only by the *force* which it exerts upon neighboring bodies. According as this force is large or small, we conceive that the quantity of electricity is large or small. And we measure the quantity of electricity by reference to the force which it exerts under specified conditions. We say that two small bodies, *A* and *B*, have *equal quantities* of electricity, if when placed at the same distance from a third body *C*, they exert upon *C* the same force. If *A* exerts upon *C* twice as great a force as *B*, then we say that *A* has twice as much electricity as *B*. Thus the notion of electric quantity is attached to purely mechanical effects, and made independent of the nature of electricity.

The C. G. S. electrostatic unit of electric quantity is defined as *that quantity which must be communicated to each of two small bodies placed 1 centimeter apart in order that they may repel each other with a force of 1 dyne.*

264. Distribution of a Charge. If an electric charge is something made up of parts that repel one another, we should naturally expect all the electricity of a charged conductor collected on its outer surface. There are many ways of proving this fact. One of them is illustrated in Fig. 213.

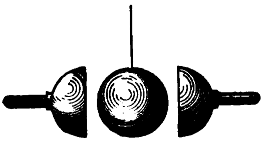


FIG. 213.

An insulated metal sphere is electrified, and then two hemispherical metal shells, held by glass handles, are placed around it and in contact with it. When the shells are removed from the sphere, they are both found to be electrified, while the sphere is no longer electrified.

The electricity on the sphere and the unlike charges induced on the inner surfaces of the shells are neutralized by contact; the outer surfaces of the shells are left charged.

Faraday, in one of his experiments, employed a conical bag of muslin, attached to a wire ring around its base, and having silk threads tied to its apex (Fig. 214).

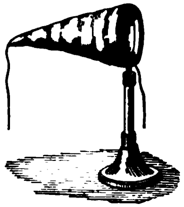


FIG. 214.

The ring was supported on an insulating stand. When the bag is electrified, it can be shown by a proof plane and an electroscope that the charge is entirely on the outside. If the bag is turned inside out, the charge passes through the muslin so as to be again on the outer surface.

To test the matter in another way, Faraday had a large wooden cube (edge 12 ft.) made and completely covered with copper wire and bands of tinfoil. After the metallic covering had been strongly charged, Faraday entered the cube, taking with him his most delicate electroscopes. But, to quote his own words, "I could not find the least influence upon them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of the outer surface."

265. Electric Density. Experiment shows that the distribution of a charge on the surface of a conductor depends on the shape of the conductor, and is not uniform except in the case of a sphere. The term electric density is used to denote the quantity of electricity per unit area of a conductor.

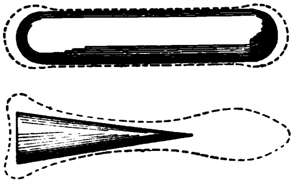


FIG. 215.

The electric density on a cylinder with rounded ends is greater at the ends than at the middle. On a cone the density increases as we approach the apex; if the cone is very acute, almost the whole charge is found near the apex. These facts are represented to the eye by the dotted lines in Fig. 215.

The force with which a charge tends to escape from a conductor increases with the density of the charge. For this reason a pointed conductor soon loses its charge. An insulated sphere retains a charge better than a body of any other shape.

266. Condenser. An electric condenser consists essentially of two conducting surfaces separated by a thin layer of air or other dielectric. It stores up electric energy by means of induction. Its action is illustrated in Fig. 216.

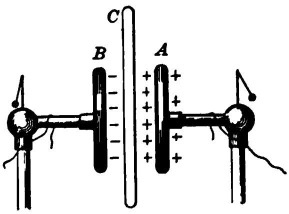


FIG. 216.

A and *B* are brass discs mounted very near each other on glass stems. *A* is joined by a wire to an electric machine, and *B* is joined by another wire to the earth. When the machine is worked a positive charge spreads over *A*, and induces a nearly equal negative charge on the nearer face of *B*. This negative charge will attract a large part of the positive charge on *A* to the face of *A* that is opposite *B*, and will hold it "bound" there, so that there is left very little "free" electricity on *A*. Under these conditions a fresh supply of positive electricity will pass from the machine to the disc *A*. This new supply will induce more negative electricity on *B*, which in its turn will react on *A*, and bind more positive electricity on the face opposite to *B*. Thus the reciprocal action goes on for some time, and a far greater quantity of electricity is collected on *A* than *A* is capable of holding when standing by itself; in other words, the capacity of *A* to hold electricity has been greatly increased.

The condenser is discharged by placing one end of a *discharging rod* (a bent metal rod with an insulating handle) in contact with *A*, and then bringing the other end near *B*; before it touches *B*, the union of the opposite charges is shown by a large, brilliant spark, accompanied with a sharp report.

The capacity of a condenser depends on the area of the plates, on their distance apart, and on the specific inductive capacity of the dielectric. The unit of capacity is called a *farad*.

267. Leyden Jar. The best-known condenser is the *Leyden jar.* It is a wide-mouthed bottle, coated with tinfoil both inside and outside about three quarters of the way to the top. The inner coating is connected by a chain to a metal rod which passes through the cover of the jar, and terminates in a metal knob.

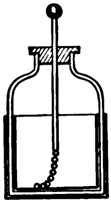


FIG. 217.

To *charge* the jar, we connect the outer coating with the earth, and the inner coating through the metallic knob with a source of electricity, such as an electric machine. The condensing action already described then takes place (§ 266). The jar can be *discharged* by a discharging rod in the manner already mentioned.

Thin jars sometimes break when heavily charged. This fact proves that the glass or dielectric is in a state of strain. Another indication of strain in the glass is the fact that soon after a jar is discharged, a much weaker second charge is found to exist upon it; this second charge is called the *residual* charge. It appears to be due to the *gradual* return of the strained molecules of the glass to their normal condition.

Franklin proved that the charges of a Leyden jar reside on the surface of the glass, not on the tinfoil coatings. To repeat his proof, a jar with movable coatings is required (Fig. 218). This jar is charged and then placed on an insulator. Then the coatings are removed (taking care to preserve their insulation). The coatings are now found to be almost free from charge, but the glass will attract pith balls and show other signs of electrification. On putting the jar together again it is found to be charged almost as strongly as at first. The inference is that the charge remained on the glass all the time. In the figure, *B* is the glass, *C* the outer coating, and *D* the inner coating and rod.

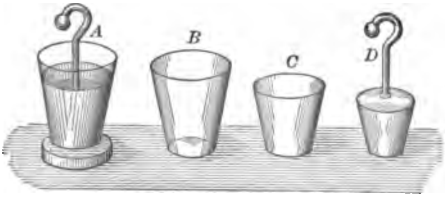


FIG. 218.

268. Atmospheric Electricity. Franklin (1750) was impressed by the analogy between a flash of lightning and the electric spark. One day, when a thunder cloud was approaching, he sent up a kite, to which a pointed wire was attached. After the kite string was wet, he obtained a series of sparks with which he charged a Leyden jar. With this jar he was able to perform the same experiments as with a jar charged in the ordinary way. Hence he concluded that lightning is simply an enormous electric spark.

During a thunder shower there is inductive action between the earth and the cloud. We are placed between the two coatings of a huge Leyden jar; the earth and the cloud are the coatings, and the intervening air is the dielectric. The danger of a disruptive discharge is increased by the moisture of the air and the downfall of rain; its occurrence is made known to us by a flash of lightning and a crash of thunder.

The noise of thunder is caused by the sudden expansion and contraction of the heated air along the line of the discharge. The noise is a sudden crash or a prolonged roar, according to the distance of the observer. The interval of time between seeing a flash of lightning and hearing the sound of the thunder increases with the distance of the flash, because sound travels much more slowly than light.

In a thunder shower the electric density is greatest on objects that project from the earth (§ 265); for this reason lightning very often strikes such objects. Also, as a rule, lightning will choose the easiest path. A pointed metal conductor, projecting above a house, and carried downwards till it meets water or moist earth, offers in most cases the easiest path. A conductor so placed, or *lightning rod,* if properly made and set up, suffices usually to protect a building from strokes of lightning. But it is found that powerful flashes have a tendency to choose some other path than the easiest or that which is afforded by the rod.

LABORATORY EXERCISES.

Devise, perform, and describe an experiment to prove that:

1. Two bodies if rubbed together become electrified with opposite charges.

2. Brass is a conductor and glass a non-conductor of electricity.

3. A moist thread conducts electricity better than dry thread.

4. There is no electricity on the inside of a charged conductor.

5. Electricity escapes at points faster than from a flat surface.

6. Construct a gold-leaf electroscope, and describe how you would use it to determine whether a body had a positive or a negative charge.

CLASS-ROOM EXERCISES.

1. How would you proceed in order to electrify a rod given to you? How would you prove that it is electrified? How would you ascertain whether the charge is positive or negative?

2. Describe a gold-leaf electroscope. How would you charge it positively, and how negatively?

3. You have a rod of unknown material. How would you determine whether it is a good or a bad conductor of electricity?

4. An electroscope is charged negatively, and an insulated brass ball is brought near the disc. What conclusion as to the electrical state of the ball do you come to (1) when the leaves slightly collapse, (2) when they slightly diverge?

5. How can a series of sparks be obtained from an electrophorus?

6. Describe a Leyden jar, the method of charging it, and the method of discharging it.

7. Explain the action of a condenser of electricity.

8. One person holds a charged Leyden jar in his hand by its outer coating, and another person holds similarly an uncharged jar. What will happen when the knobs of the two jars are brought together?

9. A brass rod held in the hand and beaten with catskin shows no electricity when it is made to touch an electroscope. How would you prove that it was really electrified when so beaten?

10. If a cent is fastened to the end of a stick of sealing wax, how could you give it a negative charge by help of a positively charged rod?

11. A copper vessel is insulated and electrified. If touched at different parts with a proof plane, what part will show the strongest charge, and what part the weakest charge?

12. Why do you get no shock on touching the knob of a charged Leyden jar which stands on a vulcanite plate?

13. Why is it a surer test that a body is electrified, if it repels an electrified pith ball suspended by a silk thread than if it attracts the ball?

14. A piece of dry brown paper, laid on a warm metal tray, is rubbed with catskin. The tray is then placed on a dry glass tumbler, and the brown paper is removed. Why can you now get a spark on bringing your knuckle near the tray?

15. Why is it that a plate electrical machine will not work well in damp weather?

16. If you have a positively charged brass rod, and a piece of gilt paper fastened to the end of a dry glass rod, how would you charge the gilt paper with negative electricity?

17. Three insulated metal balls, *A*, *B*, and *C*, are placed in a line, *A* and *B* in contact, *C* a little way off. *C* is positively electrified, and then *A* and *B* are separated. What are now the electrical states of *A* and *B*?

18. Two pith balls hang side by side by two damp cotton threads. State and explain what happens when an electrified glass rod is brought gradually near the two pith balls.

19. If an electrified piece of metal is made to touch a gold-leaf electroscope, the leaves separate; and on taking the metal away, the leaves remain separated. But if the electrified metal is only brought near to the electroscope, and then taken away, the leaves separate when the electrified metal is near, but fall together when it is taken away. Why is there a lasting effect on the gold leaves in one case and only a temporary effect in the other case?

20. Two Leyden jars charged in the ordinary way are held one in each hand by the outer coatings. What takes place when the knob of one is made to touch the outer coating of the other, and what is the subsequent condition of each jar?

21. An insulated conductor *A* is brought near to the cap of a gold-leaf electroscope charged positively. Explain what will happen (1) if *A* is unelectrified, (2) if *A* is charged positively, (3) if *A* is charged negatively.

22. If a rod of sealing wax is rubbed with a piece of flannel, the leaves of an electroscope diverge when either the wax or the flannel is separately brought near to it, but not if the wax and the flannel are presented to it together before being separated. What conclusions do you draw from these facts?

23. A gold-leaf electroscope is placed upon an insulated brass plate. When the plate is electrified, will the leaves of the electroscope show any divergence? Give reasons for your answer.

24. Why is it impossible to charge a Leyden jar strongly unless its outer coating is connected with the earth?

The Electric Current.

269. First Notions. An electrified body possesses potential energy, or power to do work, in virtue of its electrification.

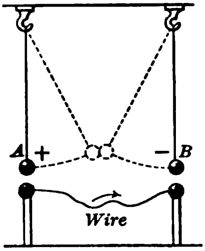


FIG. 219.

Let us suppose that two very small metal spheres are electrified with equal unlike charges, and suspended near each other by silk threads. They will move up to each other, and when they touch, their charges will disappear. The potential energy of electrification has been converted partly into kinetic energy, and partly into mechanical work, since the spheres are raised a little.

But now suppose that we mount the same two spheres, electrified as before, on glass legs, and connect them by a wire. The charges again disappear. This is because the wire affords an easy path by which they can unite and neutralize each other. It is customary to confine our attention to the positive electricity only. So we say that there has been a *flow* of positive electricity from one sphere to the other. This flow is called an *electric current.*

Although the charges disappear, neither sphere moves, and no visible mechanical work is done. What has become of the energy of electrification? The answer is that it has been expended in heating the wire that connects the two spheres.

The heat thus generated is precisely equivalent to the kinetic energy produced and mechanical work done in the case in which the two spheres are suspended by threads.

We shall find that an electric current, besides heating a wire, can do work of various kinds. All that our simple example is meant to illustrate is the truth that *the energy of an electrified body is not destroyed by allowing the electricity to flow along a wire, but reappears in other forms.*

270. Electric Potential. Suppose we join by a wire two electrified conductors, *A* and *B*, and find that positive electricity flows from *A* to *B*. Consider the condition of a positive unit of electricity on each of the conductors. The unit on *A* is able to do work by passing along the wire, while the unit on *B* is not. This difference of condition is expressed by saying that there is a *difference of potentials* between *A* and *B*, and that the potential of *A* is *higher* than that of *B*. The difference of potentials is measured by the amount of work done by the transfer of a positive unit of electricity from *A* to *B*.

The quantity of electricity transferred from *A* to *B* will be such that the potentials of *A* and *B* are made the same. The flow of electricity then ceases; electric equilibrium is established. A great quantity of electricity will be transferred, if the difference of potentials is great and the conductors are large; a small quantity, if the opposite conditions exist.

In order to compare potentials, the earth is chosen as a convenient standard of reference, and its potential is assumed to be zero. A positively charged body has a positive potential, and a negatively charged body has a negative potential; in the former case positive electricity tends to flow from the body to the earth, and in the latter case positive electricity tends to flow from the earth to the body.

Difference of potentials is analogous to difference of water levels or pressures. Water will not flow through a pipe from one vessel to another, if the water level is the same in both vessels. But if the level is higher in vessel *A* than in vessel *B*, then water will flow from *A* to *B*, whatever be the quantity contained in either vessel.

Again, potential and temperature have similar meanings, while quantity of electricity corresponds to quantity of heat. Just as heat will flow from a small body at a high temperature to a large body which contains more heat than the small body but at a lower temperature, so electricity will flow from a small body at a high potential to a large body which contains more electricity but at a lower potential.

271. Voltaic Cell. If we join two conductors, *A* and *B*, by a wire, and by any device keep the potential of *A* higher than that of *B*, a *continuous* current of electricity will flow from *A* to *B*. Methods of producing continuous currents were first devised by Galvani (1786) and Volta (1792).

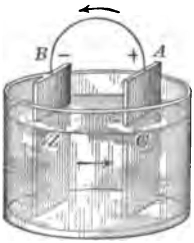


FIG. 220.

The *voltaic cell* (Fig. 220) illustrates Volta's method. Plates of copper and *pure* zinc are partly immersed in dilute sulphuric acid contained in a glass vessel. If the upper ends, *A* and *B*, of the plates are not connected, nothing to attract attention happens; but delicate tests show that *A* has a positive potential and *B* a negative potential.

If we join *A* to *B* by a wire, changes begin to occur. The zinc in the cell slowly wastes away, and hydrogen gas collects on the surface of the copper. A chemical change is taking place, the nature of which has been described (see § 153).

At the same time the wire becomes capable of producing (in a weak degree) the known effects of an electric current. If thin, it becomes warmer; if held near a magnetic needle, it makes the needle move; if we cut it anywhere and apply the ends to a file, minute sparks are seen.

Evidently a difference of potentials between *A* and *B* is in some way maintained. The only conceivable way is by means of the chemical action going on in the cell. This action constantly drives positive electricity to *A*, and negative electricity to *B*, or (what is equivalent) takes positive electricity from *B* as fast as it arrives there. Choosing the latter view as simpler, we say that a current of electricity flows from the copper to the zinc outside the cell, and from the zinc to the copper inside the cell, thus forming a *closed circuit*.

If *A* and *B* are not connected, the circuit is said to be *open*; *A* is called the *positive pole*, and *B* the *negative pole*.

272. Electromotive Force. The cause of an electric current is called *electromotive force* (E. M. F.). The E. M. F. of a closed voltaic circuit is measured by the work done in driving a unit of electricity round the entire circuit. It is also proportional to the potential difference of the poles of the cell, when the circuit is open. E. M. F. and difference of potentials, however, are not identical terms, but stand to each other in the relation of cause and effect.

E. M. F. is sometimes called *electric pressure* from its analogy to water pressure. In a certain sense the analogy exists. A voltaic cell may be compared to a pump which forces water through a pipe; the E. M. F. of the cell corresponds to the pressure produced by the action of the pump.

But E. M. F. is not, properly speaking, a force. It does not tend to move matter, but to move electricity. It is a form of energy, and is always measured as such. An electric current expends energy. The source of the energy is found in the consumption of zinc in the cell. For every gram of zinc consumed a definite amount of chemical energy is transformed into the energy of an electric current. The E. M. F. of the cell is the portion of this energy needed to drive a unit of electricity around the circuit.

The E. M. F. of a voltaic cell depends on the nature of the materials used, and is wholly independent of the size and the shape of the plates.

273. Strength of Current. The conductor, which joins the poles of a cell, constantly tends to bring the two poles to the *same potential*, and the chemical action of the cell constantly tends to produce between the poles a *difference of potentials* equal to the electromotive force. As long as these conditions are maintained the conductor possesses certain properties which are briefly expressed by saying that it is traversed by an electric current.

Experiment shows that these properties are possessed in equal degree by every point in the circuit formed by the cell and the conductor; whence it follows that there is no accumulation of electricity at any place. Therefore, the same quantity of electricity must pass every cross-section of the circuit in the same time. In this respect, an electric current exactly resembles the steady flow of water through a pipe.

The *strength* of the current is proportional to the *rate* of the electric flow, and is measured by *the quantity of electricity that passes any cross-section of the circuit in one second.*

In speaking of a quantity of electricity, we need not conceive electricity to be a fluid or a substance of any kind. We speak of quantities of heat to avoid tedious circumlocution, and, similarly, we speak of quantities of electricity without for a moment imagining that any real electric fluid exists.

274. Local Action. If pure zinc is used in a voltaic cell,

(1) When the circuit is open, the acid does not attack the zinc, except just enough to coat its surface with a thin film of hydrogen.

(2) When the circuit is closed, the acid attacks the zinc, and the hydrogen is set free on the surface of the copper.

If ordinary impure zinc is used,

(1) When the circuit is open, the acid actively attacks the zinc, and hydrogen is copiously evolved upon its surface.

(2) When the circuit is closed, the same action as before continues, but hydrogen also collects on the surface of the copper.

Therefore, if impure zinc is used, a portion is consumed without contributing to the production of a current. This waste of zinc is said to be caused by *local action.*

Local action is prevented by *amalgamating* the zinc; that is, rubbing its surface with mercury. After a time the amalgam on the surface wears away, and must be renewed.

275. Polarization. If a voltaic cell is set up as described in § 271, the current rapidly diminishes in strength, and soon almost ceases. The cause is found in the *film of hydrogen* which adheres to the surface of the copper; for if this film is brushed away, the current instantly becomes stronger. This film weakens the current in two ways:

(1) It increases the resistance of the circuit, because hydrogen (unless rarified) is a non-conductor of electricity.

(2) It creates a *counter* E. M. F., which tends to neutralize the E. M. F. due to the oxidation of the zinc.

A cell in the state just described is said to be *polarized.*

Polarization is prevented by putting into the cell a second liquid that will combine with the hydrogen.

The existence of the counter E. M. F., due to the hydrogen film, is proved by allowing the current to flow for some time, and then replacing the zinc plate by one of clean copper. A weak current is then found to flow through the wire in the *reverse* direction of the original current.

276. Cells in Common Use. *Daniell Cell.* This cell is shown in section in Fig. 221. *Z* represents a zinc bar, *P* a porous jar, *C* a cylindrical sheet of copper. The porous jar contains dilute sulphuric acid. The copper is surrounded with a saturated solution of copper sulphate ($CuSO_4$).

The action is as follows: the zinc displaces the hydrogen in the acid, and zinc sulphate ($ZnSO_4$) is formed. The hydrogen set free travels with the current through the porous jar; then it displaces copper in the copper sulphate, and sulphuric acid (H_2SO_4) is formed. The free copper is deposited on the sheet *C*. Thus the zinc decreases in weight, the copper increases, and polarization is prevented. The E. M. F. of this cell remains nearly constant for a long time.

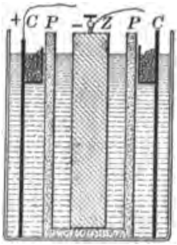


FIG. 221.

Grove Cell. In this cell platinum is used instead of copper, and strong nitric acid (HNO_3) instead of copper sulphate. The nitric acid acts as the depolarizing agent; it gives up a part of its oxygen, and converts the hydrogen into water. The E. M. F. of a Grove cell is almost double that of a Daniell, but the evolution of poisonous fumes by the decomposed nitric acid is a serious objection to its use.

Bunsen Cell. This is simply a Grove cell in which a plate of gas carbon is substituted for the expensive platinum plate.

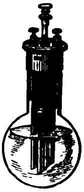


FIG. 222.

Bichromate Cell. One form of this cell is seen in Fig. 222. Two plates of gas carbon are supported parallel to each other in a glass bottle with a wide mouth. Midway between them is a plate of zinc. The liquid is a solution of sodium bichromate acidified with sulphuric acid. As this liquid attacks the zinc even when the circuit is open, the zinc plate is attached to a sliding rod, so that it can be raised out of the liquid when the circuit is open. The E. M. F. of the cell is at first fully double that of a Daniell, but it falls away by degrees, the depolarization being imperfect. The cell is very compact and convenient, and gives off no fumes.

Leclanché Cell. One of the latest forms of this cell is shown in Fig. 223. The only liquid is a saturated solution of ammonium chloride. In this liquid stands a cylinder composed of gas graphite and manganese dioxide, with a core of gas carbon. Separated from the cylinder by rubber bands is placed a cylindrical sheet of zinc. The E. M. F. of the cell is 1.4 times that of a Daniell. Polarization soon reduces its value if the circuit is closed. But the cell depolarizes rapidly in open circuit, and is excellently adapted for purposes that require a current for a short time only, such as ringing an electric bell. When used for such purposes, the cell, once set up properly, will require no further attention for a year or two except the occasional addition of water to replace that lost by evaporation.

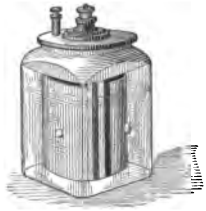


FIG. 223.

In all these cells zinc is the fuel consumed. But zinc is an expensive fuel. A pound of zinc costs more than 15 times as much as a pound of coal, and yields by oxidation not more than one-fifth as much energy. Hence the voltaic battery, which is so useful in telegraphy, for ringing bells, and for laboratory experiments, is unable to supply the large quantities of electricity required for electric lighting and for the performance of mechanical work, except at a cost which prevents its use.

277. Voltaic Battery. When a stronger current than that furnished by one cell is required, two or more cells are combined; the combination is called a *voltaic battery.*

A battery of five Daniell cells, joined "in series," is shown in Fig. 224. The copper of the first cell is joined to the zinc of the next cell, and so on. Finally the circuit is closed by joining the copper of the last cell to the zinc of the first cell. The E. M. F. of the current is thus made *five* times as great as that produced by a single cell.

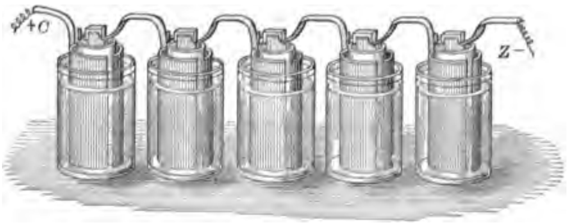


FIG. 224.

LABORATORY EXERCISE.

1. Set up a Daniell cell. Allow the current to flow for one hour through a galvanometer, and observe the variations in the strength of the current every 10 minutes. Determine the change in weight of the plates by weighing them at the beginning and end of the hour.

CLASS-ROOM EXERCISES.

1. Describe with a sketch a Daniell cell. What is the source of the energy of the current?

2. Describe a Leclanché cell. Which cell would you use for ringing an electric bell, the Leclanché or the Daniell? Why?

3. What is meant by *polarization?* How is it prevented?

4. Why and how are zinc plates amalgamated?

5. How would you connect a poker by wires to the plates of a Daniell cell so as to make the current pass through the poker from the handle to the point? Give a sketch.

6. If a charged battery is to be kept ready for use, why should the ends of the battery be kept disconnected outside the battery?

7. What is meant by the terms *open circuit* and *closed circuit?*

Magnetic Effects of a Current.

278. Oersted's Discovery. Oersted (1820) discovered that an electric current is able to deflect a magnetic needle. This discovery is memorable as the first of a series that established the common nature of magnetism and electricity.

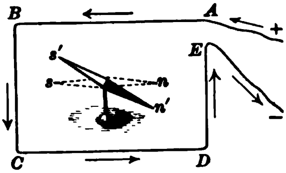


FIG. 225.

Oersted found that when he held a wire carrying a current parallel to and just above a magnetic needle, the needle instantly swung out of position, and was permanently deflected. If the wire is held below the needle, or if the current is reversed in direction, the direction of the deflection is reversed. In every case, the direction of the motion may be found by the following *rule of thumb:*

Imagine your right hand placed on the wire with the fingers pointing in the direction of the current, and the palm facing the needle; then the north pole of the magnet always turns in the direction of the extended thumb.

If the wire is carried completely round the needle, as shown in Fig. 225, the effects of the upper and lower portions of the wire combine to increase the deflection of the needle.

The behavior of the needle proves that the current is exerting force upon each of its poles along lines of action which are perpendicular to the plane of the wire and the needle. These two forces, a, a (Fig. 226), are equal, but opposite; they form a *couple*, and their effect is to turn the needle towards an east and west line, until their turning effect or *moment* is balanced by the moment of another couple, b, b, due to the directive action of the earth.

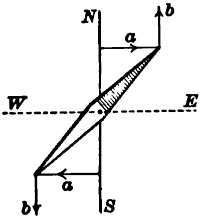


FIG. 226.

While the needle is moving, there is a direct conversion of the energy of an electric current into visible kinetic energy.

279. Galvanoscope. If we carry an insulated wire many times round a magnetic needle, the effect of the current on the needle is greatly increased. In this way it is easy to construct instruments which will detect the existence of a current, determine its direction, and measure roughly its magnitude. Such instruments are called *multipliers* or *galvanoscopes.*

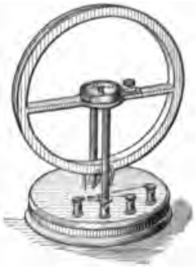


FIG. 227.

For school laboratory uses a simple multiplier, made by carrying insulated copper wire (No. 30) fifteen times round a circular frame, will answer. A compass needle is placed at the center of the circle (Fig. 227). Binding screws should be so connected with the coil that the current may be sent through five, ten, or fifteen turns at pleasure.

280. Galvanometers. An instrument designed to *measure* the strength of a current is called a *galvanometer.* The way in which a galvanometer fulfills its purpose may be understood now, although the units of current strength have not yet been defined.

In all galvanometers the two essential parts are a magnet or magnetic needle, and a coil of wire; and the moving force is the mutual action between the magnet and the coil.

In one class of instruments the coil is fixed and the needle moves (as in the multiplier); in this case the earth's magnetism is the controlling force, or force tending to restore the needle to its original position. In instruments of this class the angle of deflection (except when small) is not proportional to the strength of the current. To this class belong the *tangent* galvanometer, the *astatic* galvanometer, and the *mirror* galvanometer.

The *tangent* galvanometer resembles in appearance the multiplier (Fig. 227). The needle is very short, and is provided with a long pointer which moves over a graduated scale.

The radius of the coil is made so great that the field of magnetic force around the needle is sensibly uniform. Under these conditions the strength of the current is proportional to the tangent of the angle of deflection, the term tangent having the meaning given in works on Trigonometry. The strength of the current is found by multiplying the tangent of the angle of deflection by a constant whose value depends on the earth's magnetic force, the dimensions of the coil, and the number of turns of wire in the coil.

In the *astatic* galvanometer great sensitiveness is gained by suspending two equal magnetic needles by a fiber of unspun silk *with their poles reversed.* This arrangement almost, yet not entirely, neutralizes the directive action of the earth. The coil of wire is arranged as seen in Fig. 228, and both needles tend to turn in the same direction.

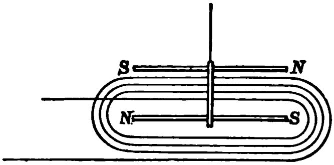


FIG. 228.

In Thomson's *mirror* galvanometer, very great accuracy of measurement is secured by attaching to the needles a light mirror, which reflects a beam of light to a distant scale (see Fig. 249, page 311).

In another class of galvanometers the magnet is fixed and the coil of wire moves. D'Arsonval's galvanometer is a type of this class (Fig. 229). The fixed magnet is so powerful that the earth's action may be left out of account. The controlling force is the elasticity of the wire by which the coil is suspended. These galvanometers are not affected by magnets near them, and are *aperiodic* or *dead-beat;* that is, free from oscillations.

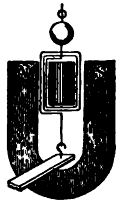


FIG. 229.

For industrial uses portable direct-reading instruments, called *amperemeters* or *ammeters* are now much employed. The essential parts consist of a needle, a coil of wire, and a controlling magnet. A pointer attached to the needle moves over a graduated scale, and points directly to the number of amperes (§ 289) of current flowing through the coil.

281. Magnetic Field around a Wire. The French physicists, Arago and Ampere, on learning of Oersted's discovery, very quickly extended our knowledge of the connection between magnetism and electricity. Arago found that a copper wire, when traversed by a strong current, will attract iron filings so that they adhere to it in clusters. Ampere magnetized knitting needles by placing them in glass tubes around which copper wire was wound, and then sending a strong current through the wire.

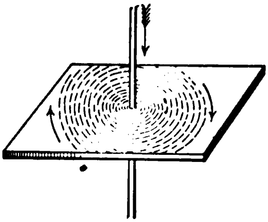


FIG. 230.

If iron filings are sprinkled upon a sheet of cardboard through which a vertical wire passes (Fig. 230), and a strong current is sent through the wire, the filings will group themselves in rings around the wire; they become for the time *small magnets.*

In short, *an electric current produces in its neighborhood a magnetic field of force. The lines of force are concentric circles perpendicular to the direction of the current.*

The direction of the lines of force may be found by the rule of thumb (§ 278), or by the following *clock rule*:

If you look along the wire in the direction in which the current flows, the force at every point in the field tends to carry the north pole of a magnet round the wire in the direction of the motion of the hands of a clock.

When a wire carrying a strong current is bent into the form of a circular ring (Fig. 231), lines of force will issue from one face of the ring, as if it were the north pole of a magnet, and enter the other face, as if it were the south pole. On bringing a bar magnet near, we find that the ring *actually is* a weak magnet.

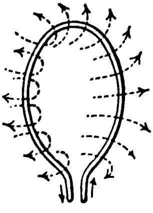


FIG. 231.

282. The Solenoid. Let us now suppose that a strong current is sent through a wire bent round and round in the form of a spiral (Fig. 232). A wire of this form, and carrying a current, is called a *solenoid*. The lines of force are modified by their mutual action, as shown in the figure, and experiment shows that *the solenoid behaves in all respects like a cylindrical bar magnet.*

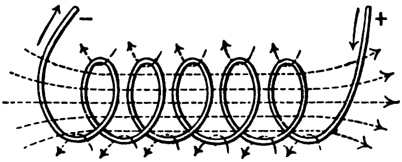


FIG. 232.

We can examine the air within the solenoid (the "magnet made of air"), and prove that it is a magnetic field with lines of force flowing through it from end to end. This is one reason for believing that lines of force continue through the iron of a bar magnet, although we cannot examine them.

The end of the solenoid from which the lines of force issue is the north pole; the other end is the south pole.

The end of a solenoid around which the current circulates *clock-wise* is always the *south* pole.

Fig. 233 illustrates one method by which the magnetic properties of a solenoid may be examined. The solenoid is suspended so that it can turn freely on a vertical axis by means of small conical metal pieces which dip into small cups containing mercury. The current passes through the mercury to the solenoid.

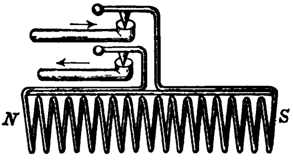


FIG. 233.

When the wire is traversed by a strong current, the solenoid will set itself with its axis pointing north and south, just like a suspended magnet. If a small bar magnet, suspended by a paper stirrup, is held with its south pole near the end *N* of the solenoid, it will be pulled into the solenoid. If the other pole of the magnet is presented, strong repulsion is observed. Thus in every way the solenoid behaves just as a magnet would behave if put in its place.

283. Mutual Action of Currents. Two solenoids act upon each other exactly like two magnets. Suppose that we mount a solenoid *AB* (Fig. 234) in a fixed position, and a smaller solenoid *ab*, so that it is coaxial with *AB*, and can move freely to and from *AB*. On sending currents through the solenoids, *ab* will be strongly attracted, and drawn inside *AB*, if the adjacent poles *B* and *a* are *unlike*; but *ab* will be strongly repelled if *B* and *a* are *like* poles.

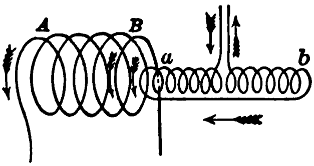


FIG. 234.

Compare now the direction of the parallel currents at *B* and *a* in the two cases. When there is attraction, they have the *same* direction; when there is repulsion, they have *opposite* directions. From this point of view the experiment illustrates the truth of a general law, discovered by Ampere, respecting the mutual action of currents:

1. *Parallel currents in the same direction attract one another.*

A second law established by Ampere is as follows:

2. *Two currents that form an angle with each other tend to become parallel and to flow in the same direction.*

The *vibrating spiral* (Fig. 235) furnishes an interesting illustration of the attraction of parallel currents.

A solenoid is suspended vertically so that the lower end of the wire just dips into mercury. The wires from the battery are connected with the mercury and with the upper end of the solenoid. When the current flows, the successive turns of the spiral attract one another, thus causing the spiral to shorten, and drawing the wire out of the mercury. This breaks the circuit and stops the current. The wire now drops back into the mercury, and the current again flows. A vibratory motion of the spiral is thus kept up as long as the connection with the battery is maintained.

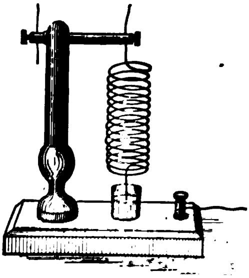


FIG. 235.

284. The Electromagnet. We now come to a fact of the highest practical importance. *The strength of the magnetic field within a solenoid is enormously increased by substituting in place of air a cylinder of soft iron.*

The iron not only prevents lines of force from leaking out of the sides of the solenoid, but has the power to call into existence a great number of new lines. A rough idea of its effect is obtained by comparing Fig. 236 with Fig. 232.

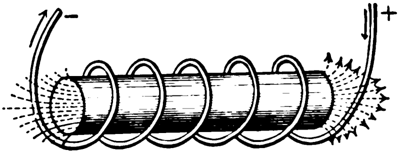


FIG. 236.

We say that the *permeability* of iron to lines of force is much greater than that of air. Why it should be so we do not know; any more than we know why iron should be heavier than air, or harder than wax But the fact exists, and it can be turned to very practical uses.

A soft iron cylinder, round which insulated wire is coiled in the form of a spiral, is called an *electromagnet.* Its poles are determined by the rule given on page 298.

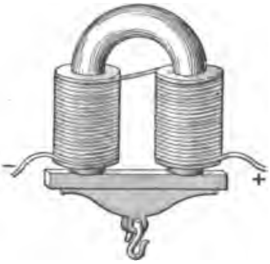


FIG. 237.

Electromagnets may have any form, but the form usually chosen is that of a horseshoe (Fig. 237). This form greatly shortens the air path which the lines of force must traverse in going from pole to pole, thereby increasing their number and the strength of the magnet.

If a bar of soft iron, called an *armature,* is placed across the poles, the lines of force lie wholly within iron, and their number becomes the greatest which a given strength of current can produce. If the strength of the current is increased, the power of the electromagnet is increased until finally the iron core becomes *saturated* with magnetism.

Electromagnets far exceed permanent steel magnets in strength. Joule constructed one capable of exerting an attraction upon its armature of 200 lb. per square inch. There is an electromagnet at the Stevens Institute at Hoboken which weighs about 1600 lb., and has a lifting power of nearly forty tons.

But the great practical value of the electromagnet consists, not in its lifting power, but in the fact that *its magnetism is controllable at will.* An electromagnet is a magnet only when we allow an electric current to flow through its coil. When the current is stopped, the iron core returns to its natural condition.

The loss of magnetism, however, is usually not quite complete. Usually a little remains for a longer or shorter time. This is known as *residual* magnetism.

285. The Electric Bell. The application of the electromagnet to ringing a bell is illustrated in Fig. 238. When the push button is pressed in, a current flows through the coils of the electromagnet, then to the armature, which is pressed gently by a spring against the screw *C*, thence back to the battery. The electromagnet draws the armature forward, thus causing the hammer to strike the bell. But the circuit is thereby broken at the screw *C*. Therefore the armature moves back again till contact with *C* is again made. Then the cycle of changes begins again.

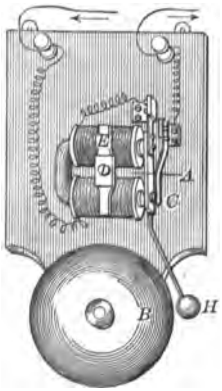


FIG. 238.

The push button turns on the current by pressing the metal end of a spring connected with one pole of the battery against a metal plate which is connected with the other pole.

286. The Electric Telegraph. A diagram of the essential parts of a telegraph line between two stations is shown in Fig. 239. The essential parts are a *line wire*, a *transmitter* or *key*, a *relay* or *receiver* or *sounder*, and the *batteries*.

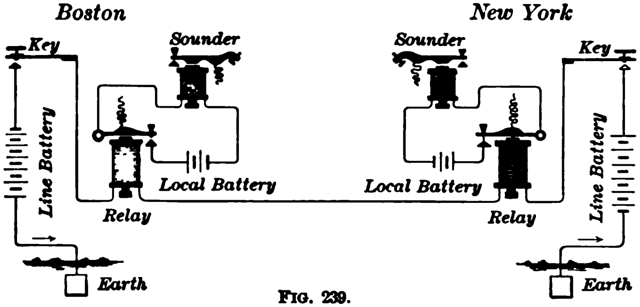


FIG. 239.

The circuit is completed by soldering the ends of the line wire to metallic plates buried in the earth.

The *key* (Fig. 240) is a brass lever, *A*, which completes the circuit when it is pressed down. A *switch*, *H*, can be turned so as to keep the circuit closed when the key is not in use.

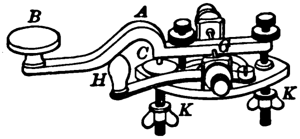


FIG. 240.

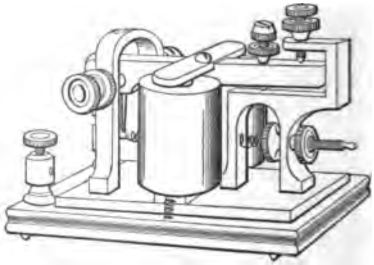


FIG. 241.

The *relay* is an electromagnet, the coil of which forms part of the line wire. Its armature, as it moves to and fro, opens and closes a local circuit which includes the local battery and the sounder. The local battery operates the sounder.

The current in a long line wire is too weak to operate the sounder; hence the necessity for a relay.

The *sounder* (Fig. 241) is an electromagnet whose armature, as it moves to and fro, makes a clicking sound. The intervals between the sounds are called *dots* or *dashes* according as they are short or long. The dots and dashes are combined so as to express letters and words.

The motion of the armature away from the poles of the electromagnet, both in the relay and in the sounder, is produced by the aid of a steel spring.

The *batteries* are usually of the Daniell type, and the cells of the battery are joined in series.

When an operator in Boston wishes to send a message to New York, he turns the switch of his key, so as to interrupt the steady current, and then makes the proper series of dots and dashes with the lever of the key. These dots and dashes are faithfully reproduced by the sounder in New York.

LABORATORY EXERCISE.

1. Make a solenoid by winding insulated wire round a cylindrical lamp chimney, and study its magnetic properties by the method suggested in Fig. 233, (1) when the solenoid contains no iron core, (2) when it contains a soft iron core.

CLASS-ROOM EXERCISES.

1. A wire lies east and west directly over a compass needle. How is the needle affected when a strong current flows through the wire (1) from west to east, (2) from east to west?

2. If you have a voltaic cell, insulated wire, and a bar of soft iron with a notched end, how would you proceed to magnetize the iron so that the notched end should be a N. pole? Give a diagram.

3. Describe a *relay* in telegraphy and its use.

4. Describe and sketch an electric bell which will ring as long as the current is closed.

5. Give a diagram of an *astatic* needle, and explain why the use of such a needle makes a galvanometer very sensitive.

Ohm's Law.

287. Statement of the Law. Suppose we have a current generated by a battery and flowing round a circuit. By means of a galvanometer two general facts are easily established.

(1) If we keep the external part of the circuit unchanged, but change the nature or the number of the cells, the strength of the current will change. We find that the strength of the current increases when the E. M. F. is increased and diminishes when the E. M. F. is diminished.

(2) If we make no change in the battery, but change the external conductor by altering its length, or cross-section, or material, or even its temperature, the strength of the current is in general changed. For example, the current is weakened if we substitute for the conducting wire a longer or thinner wire of the same material, or if we substitute for a copper wire a poorer conductor such as iron.

Every conductor obstructs the flow of electricity to an extent depending on its length, cross-section, material, and temperature. This property of a conductor to obstruct the flow of electricity is called its *electric resistance.*

The strength of a steady current depends on two things only: the E. M. F. which causes it, and the resistance which it has to overcome. The exact law of dependence for *steady currents* was discovered by Dr. G. S. Ohm, of Berlin (1827):

Strength of current varies directly as the electromotive force, and inversely as the resistance overcome.

Let C denote strength of current, E electromotive force, and R the total resistance; then, if suitable units are chosen,

$$C = \frac{E}{R}.$$

For a conductor forming part of a circuit, let D denote the difference of potentials between its ends, R its resistance; then

$$C = \frac{D}{R}.$$

288. Application of the Law. In applying Ohm's Law to a closed circuit or to a conductor in which there is more than one source of E.M.F., we must count each E.M.F. as positive if it acts in the direction of the current, and as negative if it acts in opposition to the current. With this understanding, the law may be stated in a general form as follows:

$$C = \frac{\textit{sum of electromotive forces}}{\textit{total resistance}}.$$

Suppose that the circuit consists of two cells and an external conductor. Let E and E' denote the electromotive forces of the cells, R the total resistance of the circuit. If the zinc of one cell is joined to the copper of the other (Fig. 242), E and E' will act in the same direction. But if the two zincs are joined by one wire, and the two coppers by another wire (Fig. 243), E and E' will act in opposite directions.

According as the first or the second arrangement is chosen,

$$C = \frac{E + E'}{R}, \text{ or } C = \frac{E - E'}{R}.$$

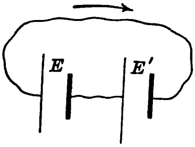


FIG. 242.

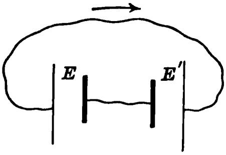


FIG. 243.

In the case of a single cell of electromotive force E and resistance r, with an external conductor of resistance R

$$C = \frac{E}{R + r}, \text{ whence } E = CR + Cr.$$

If D denotes the potential difference between its two *poles*,

$$C = \frac{D}{R}, \text{ whence } D = CR, \text{ and } E - D = Cr.$$

The difference between E and D, or Cr, represents the E.M.F. expended in forcing the current through the cell.

289. Electromagnetic Units. The electric units in common use are called *electromagnetic* units, because they are based on the mutual action between currents and magnets. The effect of this action is purely mechanical; and like any other mechanical effect it may be measured and expressed in terms that involve only the fundamental units of **length, mass,** and **time.** The unit of length, chosen by general agreement, is the *centimeter*, the unit of mass is the *gram*, and the unit of time is the *second.* Any other unit derived directly or indirectly from these three is called a *C. G. S. unit.* Thus, the dyne and the poundal are C. G. S. units of force.

The C. G. S. electromagnetic units of current strength, quantity of electricity, electromotive force, and resistance are defined as follows:

Unit of Current. *A current has unit strength when 1 cm. of its length, bent into an arc of a circle of 1 cm. radius, exerts a force of 1 dyne upon a magnetic pole of unit strength placed at the center of the circle.*

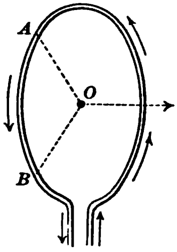


FIG. 244.

The meaning of this definition is illustrated in Fig. 244, which represents a ring of thin wire traversed by a current, and a pole of unit strength placed at the center O. The current exerts upon the pole a force tending to move the pole perpendicular to the circle in the direction of the arrow.

If the radius $AO = 1$ cm., and the arc $AB = 1$ cm., then the current has unit strength if each portion of the ring equal in length to AB exerts upon the pole a force of 1 dyne.

Unit of Quantity. If C denotes the strength of a current, and Q the quantity of electricity that flows past any cross-section in the time t, then

$$Q = Ct.$$

If $C = 1$, and $t = 1$, then $Q = 1$; therefore,

Unit quantity of electricity is that quantity which is conveyed by a current of unit strength in one second.

Unit of E.M.F. The electromotive force of a closed circuit is measured by the work required to carry 1 unit of electricity round the circuit (§ 272). If E denotes its value, and if W denotes the amount of work required to carry Q units of electricity round the circuit, then

$$W = EQ.$$

If $W = 1$, and $Q = 1$, then $E = 1$; therefore,

Unit electromotive force is an E.M.F. such that it performs 1 erg (dyne-centimeter) of work in carrying unit quantity of electricity round a closed circuit.

Also, unit difference of potentials between the ends of a conductor is a difference such that 1 unit of electricity performs 1 erg of work in passing from one end to the other.

Unit of Resistance. If in the formula of Ohm

$$C = \frac{E}{R}$$

we make $C = 1$, and $E = 1$, then $R = 1$; therefore,

Unit resistance is the resistance of a conductor in which unit current is produced by unit E. M. F. between its ends.

The units of E. M. F. and resistance, as above defined, have values far smaller than occur in practice. To avoid useless arithmetical work, another set of units, derived from the C. G. S. units and called the **practical units**, are in common use. Their names and relations to the corresponding C. G. S. units are as follows:

Kind of Unit.	*Name.*	*Value.*
Strength of current	AMPERE	$\frac{1}{10}$ of the C. G. S. unit.
Quantity	COULOMB	$\frac{1}{10}$ of the C. G. S. unit.
Electromotive force	VOLT	100 million (10^8) C. G. S. units.
Resistance	OHM	1000 million (10^9) C. G. S. units.

The volt is about 6% less than the E. M. F. of a Daniell cell. The ohm is equal to the resistance at 0°C. of a column of mercury 1 sq. mm. in cross-section and 106.3 cm. long.

290. Resistance. The resistance of that part of an electric circuit which is outside the battery is called the *external* resistance, and the resistance of the battery itself is called the *internal* resistance. The external resistance is usually that of a metal wire, and the internal resistance is mainly that offered by water holding certain substances in solution.

Experiment shows that the resistance of a metal wire depends on the kind of metal; and *varies directly as the length of the wire, and inversely as the area of its cross-section.*

Let l denote the length of a wire *in meters*, s the area of the cross-section *in square millimeters*, R the resistance of the wire *in ohms;* then

$$R = k \times \frac{l}{s}$$

when k denotes the resistance in ohms of a wire of the same material 1 meter long and 1 square millimeter in cross-section.

The value of k for any material at a temperature of 0°C. is called the *specific resistance* of the material.

The value of k for metals *increases* with the temperature; the increase between 0° and 100° C. amounts to about 40%.

The value of k for liquids *decreases* with the temperature.

The resistance of a liquid conductor is modified by its length and cross-section in the same general way as that of a metal conductor. Thus the resistance of a voltaic cell is diminished either (1) by bringing the plates nearer together or (2) by increasing their size. But the changes do not obey the exact laws expressed in the above formula.

Values of k in fractions of an ohm: silver, 0.015; copper, 0.016; brass, 0.068; iron, 0.100; platinum, 0.110; German silver, 0.236; mercury, 0.943.

Example. What is the resistance at 0° C., and also at 100°, of a copper wire, size No. 24 (diameter = 0.511 mm.), and 560 meters long?

$$\text{Cross-section} = \tfrac{1}{4} \times \tfrac{22}{7} \times (0.511)^2 = 0.2052 \text{ sq. mm.}$$

$$\text{Resistance at } 0^\circ = \frac{0.016 \times 560}{0.2052} = 43.18 \text{ ohms.}$$

$$\text{Resistance at } 100^\circ = 43.18 + (0.4 \times 43.18) = 60.45 \text{ ohms.}$$

291. Divided Circuits. When a wire traversed by a current is divided into two branches, the current divides, a part flowing by one branch and the rest by the other. Either branch is called a *shunt* of the other branch.

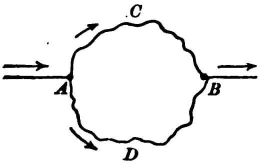


FIG. 245.

Since there is no accumulation of electricity anywhere in a circuit, the sum of the two partial currents is equal to the undivided current. In Fig. 245 the two branches are *ACB* and *ADB*. Let

C denote the strength of the undivided current.
C_1 " " " " " current in the branch *ACB*.
C_2 " " " " " current in the branch *ADB*.
R " the total resistance between *A* and *B*.
R_1 " the resistance of the branch *ACB*.
R_2 " the resistance of the branch *ADB*.
D " the difference of potentials between *A* and *B*.

Then, $$C = C_1 + C_2. \quad (1)$$

By Ohm's Law, $$C = \frac{D}{R}, \quad C_1 = \frac{D}{R_1}, \quad C_2 = \frac{D}{R_2}. \quad (2)$$

Therefore, $$D = CR = C_1R_1 = C_2R_2. \quad (3)$$

Hence, $$C_1 : C_2 = R_2 : R_1, \quad (4)$$

or, *the partial currents vary inversely as the resistances through which they pass.*

By substitution in (1) of values in (2), and cancellation,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2};$$

whence $$R = \frac{R_1R_2}{R_1 + R_2}. \quad (5)$$

The value of R is less than that of either R_1 or R_2.

From equations (3) and (5) it follows that

$$C_1 = \frac{CR_2}{R_1 + R_2}; \quad C_2 = \frac{CR_1}{R_1 + R_2}.$$

292. Fall of Potential. Ohm's Law is applied to a conductor, forming part of a circuit, as follows: Let D denote the potential difference between the ends of the conductor AB (Fig. 246), R the resistance of the conductor, C the strength of the current; then, if there is no source of E. M. F. between the ends of the conductor,

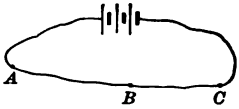


FIG. 246.

$$C = \frac{D}{R};\ \text{whence}\ D = CR.$$

Let BC be another portion of the same circuit, D' the potential difference between its ends, R' its resistance. Since the strength C of the current is constant throughout the circuit,

$$D' = CR'.$$

Therefore, $D : D' = CR : CR' = R : R'$; that is,

The fall of potential along a conductor is directly proportional to the resistance passed over.

293. Wheatstone's Bridge. The principle just stated is applied in *Wheatstone's Bridge*, an arrangement in common use for the measurement of electric resistance.

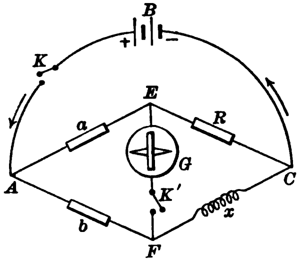


FIG. 247.

In Fig. 247 B represents the battery (the short thick line being the — pole, the longer thin line the + pole); a and b fixed resistances whose values in ohms are known; R a resistance which can be varied at the will of the operator, G a galvanometer, K and K' keys for making or breaking the circuits, x the resistance to be measured.

When the key K is pressed down the circuit is closed. A current flows to A where it divides into two branches, part taking the course AEC, and the rest the course AFC.

In general the potentials of the points E and F will be different; hence if the key K' is pressed down, a current will also flow through the galvanometer G, and the needle will be deflected. But suppose we vary the resistance R until the needle is *not* deflected. Then no current flows through G, and therefore we know that the potentials of E and F are the same. Therefore the potential difference between A and E must be equal to that between A and F; let it be denoted by D. Likewise the potential difference between E and C must be equal to that between F and C; let it be denoted by D'. Then (§ 292)

$$D : D' = a : R.$$

$$D : D' = b : x.$$

Therefore, $$a : R = b : x,$$

And $$x = \frac{b}{a} \cdot R.$$

Since a, b, and R are known, the value of x is easily found.

The variable resistances R are usually coils of wire enclosed in a *resistance box*. The ends of each coil are soldered to massive brass pieces which are separated by circular openings. When a brass plug is put into the opening, it provides a path of practically no resistance at all. When the plug is taken out, the current is obliged to pass through the coil below (Fig. 248).

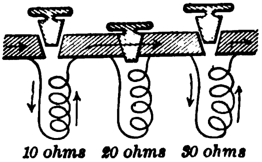


FIG. 248.

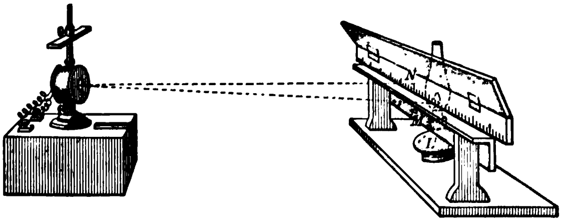


FIG. 249.

The most sensitive galvanometer for use with Wheatstone's Bridge is Thomson's Mirror Galvanometer (Fig. 249).

294. Method of Substitution. Fig. 250 illustrates this method of measuring resistance. The circuit includes a battery, a galvanometer *G*, a resistance box *R*, and the wire whose resistance x is to be determined. We note the deflection of the needle, then throw the wire out of the circuit and unplug resistance in the box till the deflection of the needle is the same as before. The resistance x of the wire is equal to the resistance which has been unplugged in the box.

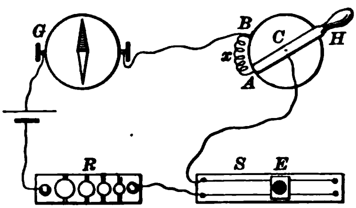


FIG. 250.

It is convenient to employ a *switch* *C* provided with a handle *H*. The ends of the wire to be measured are attached at *A* and *B*. By moving the handle from *A* to *B* we throw the wire out of the circuit.

It is also convenient to make the final adjustment in adding resistance by means of a *rheostat* *S* connected with the resistance box. Two platinum wires are stretched parallel to each other, and pass through a piece of hollow ebonite *E* containing mercury, and capable of sliding along the frame. The platinum wires are joined in circuit as shown, so that the contrivance enables us to vary the resistance continuously by an amount equal to twice that of either wire.

295. Battery Resistance. The resistance of a cell may be found in various ways, two of which will be mentioned:

(1) *Method of opposition.* Join two precisely similar cells in opposition to each other so that they will cause no current of their own. Measure their united resistance just as that of a wire is measured, and halve the value obtained.

(2) *Half-current method.* Place the cell in circuit with a known resistance r and a galvanometer of known resistance g. Note the deflection of the needle. Then add a resistance r' till the tangent of the angle of deflection is halved. The resistance x of the cell is equal to $r' - (r + g)$.

296. Measurement of Electromotive Force. Several methods may be used. Three of them are the following:

(1) *Equal-deflection method.* Allow the current from the cell, whose electromotive force E is to be measured, to flow through a *large* resistance R and a *sensitive* galvanometer of resistance g, taking care to make $R+g$ so great that the internal resistance of the cell may be neglected. Note the deflection of the needle. Then substitute for the cell another cell whose electromotive force E' is known, and change R till the deflection is the same as before; let R' denote the new value of R. Then if C denotes the strength of the current in each case,

$$E = C\,(R+g)\,;\; E' = C\,(R'+g).$$

Therefore,
$$\frac{E}{E'} = \frac{R+g}{R'+g}.$$

From this equation E may be found.

(2) *Voltmeter method.* By using many turns of thin wire, the resistance of a galvanometer may be made so great that the total resistance of the circuit remains sensibly constant, when different cells are connected with the galvanometer. In this case the current will be very weak, but whatever current there is will vary directly as the E.M.F. that causes it. The galvanometer, therefore, will measure the E.M.F. directly, and indicate its value in volts, if provided with a suitable scale. Sensitive, dead-beat galvanometers constructed to indicate directly the E. M. F. of a current in volts are called *voltmeters.*

(3) *Electrometer method.* The preceding methods are based on the application of Ohm's law to a *closed* circuit. When a circuit is *open,* the E.M.F. of the battery may be directly measured as a difference of potentials by means of an instrument called the *quadrant electrometer,* invented by Lord Kelvin (Sir William Thomson).

297. Arrangement of Cells. In a circuit formed with a single cell, let R denote the external resistance, and r the internal resistance; then by Ohm's law

$$C\ (\textit{in amperes}) = \frac{E\ (\textit{in volts})}{R + r\ (\textit{in ohms})}.$$

Suppose now that we have several perfectly similar cells, and wish to join them so as to form a battery. There are two different ways of doing this:

(1) We may join the negative (zinc) pole of the first cell to the positive pole of the second cell, and so on; the positive pole of the first cell and the negative pole of the last cell forming the two poles of the battery (Fig. 251, *A*). This mode of arrangement is called *joining in series.* Its effect is *to multiply both E and r by the number of cells.*

(2) Or we may join together all the positive poles, and also all the negative poles, thus forming in effect a single large cell with plates equal to the sum of the separate plates (Fig. 251, *B*). This is called *joining in surface* or *joining in parallel circuit.* The effect is *to divide r by the number of cells, E* remaining unaltered.

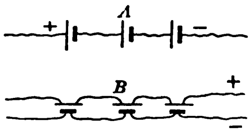


Fig. 251.

If *economy of working* is the main object, the surface arrangement should be chosen, for the reason that it reduces the internal resistance to a minimum, and leaves a larger proportion of the energy of the current to be expended outside the battery. When there is great external resistance to be overcome, the current produced by many cells thus arranged is weak, but very little of it is wasted in the battery.

If, however, we wish to obtain a *current of maximum strength* from a given number of cells, the relative values of R and r must be considered. The influence which they have may be illustrated by taking some numerical values.

Let $E = 1$ volt, $R = 1000$ ohms, and $r = 2$ ohms. Here R is so great in comparison with r that r may be neglected.

If we neglect r, then for one cell, $C = \frac{1}{1000}$ ampere.

Let ten cells be arranged (1) in series and (2) in surface.

In case (1) $C = \frac{10}{1000}$, or *ten* times as strong as before.

In case (2) $C = \frac{1}{1000}$, or no stronger than before.

When the external resistance is large compared with the internal resistance, the cells should be joined in series.

Now suppose that $E = 1$ volt, $R = 2$ ohms, and $r = 1000$ ohms. In this case R may be neglected without sensible error. Let the cells again be joined (1) in series and (2) in surface.

In case (1) $C = \frac{10}{10000} = \frac{1}{1000}$, or no stronger than at first.

In case (2) $C = \frac{1}{100}$, or *ten* times as strong as at first.

When the external resistance is small compared with the internal resistance, arrangement in surface is the best.

Sometimes the two methods combined will give the strongest current. The maximum effect is obtained *when the total internal resistance is equal to the external resistance.*

Suppose that for each cell in Fig. 252 $E = 1$ volt, $r = 2$ ohms; and let $R = 4$ ohms. For one cell $C = \frac{1}{6}$ ampere. The different possible arrangements are as follows:

(1) The 8 cells in series.

Then $C = \dfrac{8}{4 + 16} = \frac{2}{5}$.

(2) 4 cells in series, 2 cells in surface.

Then $C = \dfrac{4}{4 + 4} = \frac{1}{2}$.

(3) 2 cells in series, 4 cells in surface.

Then $C = \dfrac{2}{4 + 1} = \frac{2}{5}$.

(4) The 8 cells in surface

Then $C = \dfrac{1}{4 + 0.25} = \frac{4}{17}$.

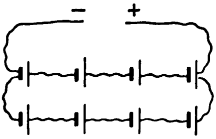


FIG. 252.

Arrangement (2) gives the strongest current, and the method of joining the cells is shown in the figure.

LABORATORY EXERCISES.

1. Measure the resistance of a coil of wire by the method of Wheatstone's Bridge.

2. Prove by experiment that the arrangement of cells in series gives the strongest current when the external resistance is large.

CLASS-ROOM EXERCISES.

1. Find the resistance of 8000 meters of iron wire, the diameter of which is 0.2 cm., specific resistance of iron 0.1 ohm.

2. Compare the resistances of two copper wires, one of which is 100 times as long as the other and has a cross-section one fourth as large as that of the other.

3. An iron wire is 200 meters long and 3 mm. in diameter. A copper wire is 50 meters long and 1 mm. in diameter. Compare their resistances. Specific resistance of iron = 0.1, of copper = 0.016.

4. What is the resistance of a column of mercury 1 meter long and 1 sq. mm. in cross-section? Specific resistance of mercury = 0.943.

5. Two exactly equal pieces of copper are drawn into wire; one wire is 10 ft. long, the other 20 ft. long. If the resistance of the shorter wire is 0.5 ohm, what is the resistance of the longer wire?

6. A piece of copper wire 100 ft. long weighs 1 lb. Another piece of copper wire 500 ft. long weighs 4 oz. What are the relative resistances of the two wires?

7. The E. M. F. of a Daniell cell is 1.02 volts; the internal resistance is 2 ohms; the external resistance 1 ohm. What is the strength of the current?

8. If 100 cells like that in Ex. 7 are joined in series, and the external resistance is 200 ohms, what is the strength of the current?

9. A battery of 12 Grove cells is arranged in 3 rows of 4 cells each. The 4 cells of each row are joined in series, and the cells of each row are joined to those of the other rows in surface. The E. M. F. of each cell is 1.9 volts and its internal resistance is 3 ohms. The external resistance is 6 ohms. Find the strength of the current.

10. Fifty cells of a battery in which the E. M. F. of each cell is 1 volt, and the internal resistance 4 ohms, are arranged in 5 rows of 10 each. Find the current when the external resistance is (1) 12 ohms, and (2) 32 ohms.

11. How would you arrange 10 equal cells, each having a resistance of 2 ohms, in order to obtain the strongest current through an external resistance of 2 ohms?

12. A wire whose resistance is 4 ohms is bent into the form of a square *ABCD*, the two ends being soldered together. Find the resistance of the system when the current enters at *B* and leaves at *D*. Will the resistance be altered if the corners *A* and *C* are connected by another wire?

13. Two wires equal in length and thickness, one of iron and the other of platinum, are soldered together and a current is sent through them. The potential difference between the ends of the entire wire is 6 volts. What is the potential difference between the free iron end and the soldered junction?

14. The external part of a circuit consists of two wires joined to the battery in multiple arc (see Fig. 245). Their resistances are 10 and 16 ohms respectively. The strength of the current in the battery is 2 amperes. Find the strength of the current in each wire.

15. The total resistance of a circuit is 18 ohms. What change in the strength of the current will be produced if two points of the circuit between which the resistance is 12 ohms are joined by a wire of 4 ohms resistance?

16. A galvanometer of 90 ohms resistance is shunted by a shunt of 10 ohms. If the potential difference between its terminals is 45 volts, find the resistance of the shunted galvanometer and the strength of the current which flows through it.

17. Twelve similar cells are arranged in series, and the poles of the battery are connected by a wire whose resistance is 240 ohms. E. M. F. of each cell = 1 volt; internal resistance = 3 ohms. By accident three of the cells are placed with their poles inverted. What is the strength of the current (see § 288)?

18. Six similar cells are arranged in series, and the circuit completed through a coil of wire and a galvanometer. The resistances of the battery, the coil, and the galvanometer are 10, 50, and 20 ohms respectively. If the potential difference between the terminals of the galvanometer is 2 volts, find the E. M. F. of each cell.

19. An insulated wire is wound round a glass tube *AB* from end to end and a current is sent through it so that to an observer looking at the end *A* the current flows round the wire clock-wise. A rod of soft iron is held (1) inside the tube, (2) outside but parallel to the tube. What in each case will be the magnetic pole of the end nearest the observer?

20. An insulated wire is wound round a wooden cylinder *AB* from *A* to *B*. How would you wind it back from *B* to *A* (1) so as to increase, (2) so as to diminish the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram.

Thermal Effects of a Current.

298. Production of Heat by a Current. Electric currents heat the conductors through which they pass. This effect is due to the resistance of the conductor. When matter in motion is stopped by friction, the energy of motion is converted into heat. When electricity in motion is stopped by resistance, the energy of the flow is also converted into heat. Resistance might be defined as that property of a conductor in virtue of which it transforms electric energy into heat.

For the same strength of current, the heating effect increases with the resistance of the conductor. The specific resistance of platinum is about 7 times that of copper. If a chain, composed of alternate links of platinum and copper, is traversed by a current of suitable strength, the platinum links will become red-hot, while the copper links will remain dark.

The heating effect also depends on the strength of the current. If a thin platinum wire is inserted in a circuit, and we go on increasing the strength of the current, the wire will become red-hot, then white-hot, and finally it will fuse.

Suppose that we arrange two equal wires, one of platinum and the other of copper, so that the current divides and passes partly through each wire (Fig. 253). The resistance of the platinum wire is 7 times that of the copper wire; hence the strength of the current in the copper wire will be 7 times as great as in the platinum wire. When a current of suitable strength is turned on, the copper wire will become red-hot, while the platinum wire will remain dark, or just the reverse of what happens when the whole current traverses each wire. This experiment shows that the influence of strength of current in heating a wire is much greater than that of resistance.

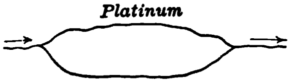


FIG. 253.

299. Joule's Law. Dr. J. P. Joule, who measured the mechanical equivalent of heat, performed a series of experiments to determine the law by which electric energy is transformed into heat. In these experiments the wire conveying the current was surrounded with alcohol (an insulator) into which a thermometer dipped. In this way the number of heat units produced in wires of known resistance, traversed by currents of known strength, was measured (Fig. 254). The result of the experiments is expressed by the following law:

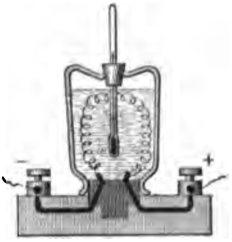


FIG. 254.

The quantity of heat developed in a conductor by the passage of an electric current is proportional

(1) *To the resistance of the conductor,*
(2) *To the square of the strength of the current,*
(3) *To the time of flow.*

If the energy of the current is wholly transformed into heat, the quantity of heat is given by the equation

$$H = C^2Rt \times 0.24$$

when H denotes the number of *calories of heat* generated, C the current strength in *amperes*, R the resistance of the conductor in *ohms*, t the time of flow in *seconds*, and 0.24 a factor obtained as will be explained in the next section.

Example. A coil of wire, whose resistance is 1000 ohms is found to develop heat enough in 2 minutes to raise the temperature of 12 kilograms of water 10° C. What is the strength of the current and the potential difference between the ends of the coil?

Number of calories of heat developed $= 12{,}000 \times 10 = 120{,}000$.
By substitution in the above formula we obtain
$120{,}000 = C^2 \times 1000 \times 120 \times 0.24$, whence $C = 2.04$ amperes.
Difference of potential $= C \times R = 2040$ volts.

300. Units of Electric Energy. If the electric pressure E between the ends of a conductor drive Q units of electricity through the conductor, then the amount W of electric energy expended is given by the equation

$$W = EQ.$$

The value of W when $E = 1$ volt and $Q = 1$ coulomb is the practical unit of electric work, and is called the *joule.*

Since 1 volt $= 10^8$ C. G. S. units and 1 coulomb $= 10^{-1}$ C. G. S. units, therefore 1 joule $= 10^8 \times 10^{-1}$ ergs $= 10{,}000{,}000$ ergs, the erg being defined as explained in § 205.

If C denote the strength of the current, and *if the electric energy is wholly expended in overcoming the resistance R of the conductor,* then by Ohm's law, $E = CR$; if t is the time of flow, then $Q = Ct$ (§ 273); whence by substitution we have

$$W = C^2Rt.$$

If P denote the work done by an electric current in 1 second, or the *power* of the current, then

$$P = EC = C^2R.$$

The practical unit of power is called a *watt;* it is the power of a current of 1 ampere flowing under a pressure of 1 volt.

1 watt = 1 joule per second = 10,000,000 ergs per second.

The joule and watt are reduced to gravitation measure by dividing 10,000,000 by 980 (§ 180); and to heat equivalents by dividing their gravitation values by the mechanical equivalent of heat expressed in gram-centimeters, or 42,700 (§ 226).

Hence,

1 joule = 10,204 gram-centimeters = 0.24 calories.
1 watt = 10,204 gram-cm. per sec. = 0.24 calories per sec.

In English units,

1 joule = 0.727 foot-pounds.
1 watt = $\frac{1}{746}$ of a horse power.

When a larger unit than the watt is more convenient, the *kilowatt* (1000 watts) is used. 1 kilowatt = 1.34 H. P.

301. Distribution of Power in a Circuit. Let E denote the E.M.F. of the current generator (battery or dynamo), C the strength of the current, R the resistance of the outside circuit, r the resistance of the generator, P the total power generated (in watts), P_1 the portion of P expended in overcoming r and therefore wasted in heating the generator, P_2 the portion of P expended in the outside circuit either in heating it or in performing useful work of any kind. Then

$$P = EC; \qquad P_1 = C^2r.$$

And, $$P_2 = P - P_1 = EC - C^2r = C(E - Cr).$$

This is the value of P_2 in all cases. If P_2 is wholly consumed in heating the outside circuit, then $P_2 = C^2R$. But if the current is used to do work of any kind, then the quantity C^2R represents only a portion of P_2.

In the case of a simple outside conductor of resistance R,

$$E = C(R + r),$$

$$P = EC = C^2(R + r).$$

Therefore, $$C^2 = \frac{P}{R + r},$$

whence by substitution we obtain

$$P_1 = C^2r = \frac{r}{R + r}P; \qquad P_2 = C^2R = \frac{R}{R + r}P;$$

or P_1 and P_2 are to each other as r and R.

Example. A battery of 12 Daniell cells arranged 6 in series and 2 in parallel circuit sends a current through a conductor whose resistance is 5 ohms. The E. M. F. of each cell is 1.1 volts, and its resistance is 1 ohm. Find (1) total power generated, (2) power wasted in heating battery, (3) power expended in heating the conductor.

$$C = \frac{6.6}{5 + 3} = 0.825 \text{ amperes (see § 288).}$$

Therefore, (1) $P = 6.6 \times 0.825 = 5.445$ watts.

(2) $P_1 = \frac{3}{8}P = 2.042$ watts.

(3) $P_2 = \frac{5}{8}P = 3.403$ watts.

302. Glow Lamp. By means of the *incandescent* or *glow lamp* (Fig. 255) an electric current is made to illuminate a room. The current brings to vivid incandescence a very thin, tough thread of carbonized bamboo or cellulose, enclosed in a glass bulb. The air is removed from the bulb so that the carbon may not be consumed. The ends of the thread are attached to platinum wires which are fused into pieces of glass at the base of the bulb, and connect with the wires conveying the current.

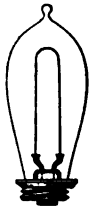


FIG. 255.

The current is generated, not by batteries, but by dynamos. The expense of batteries prevents their use.

The potential difference or *voltage* of an ordinary 16-candle Edison lamp is about 100 volts, and the strength of the current 0.6 of an ampere; hence the rate of consumption of energy is 3.75 watts per candle power.

Wattmeters have been devised, which register in watts the consumption of energy by pointers moving round dials, just as the consumption of gas is recorded by a gas meter.

Carbon is used for the incandescent thread because of its great infusibility and high resistance, and because, unlike the metals, the resistance of carbon *decreases* as the temperature increases.

When the lamp is lighted the carbon neither fuses nor burns, but slowly throws off minute particles which gradually blacken the glass and destroy the illuminating power of the lamp. After a certain time (about 1000 hours for a well-made 16-candle lamp) the thread breaks, and the lamp becomes worthless.

Increasing the voltage of a glow lamp rapidly shortens its life; a moderate voltage gives the best results when both lighting power and life are considered.

It is usual to group glow lamps in parallel circuit between mains kept at constant voltage (Fig. 256). Under this arrangement, when a lamp is lighted it opens a new path for the current, and causes a stronger current to flow from the source of supply, the current in the other lamps not being affected.

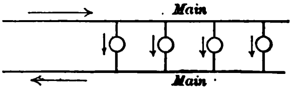


FIG. 256.

303. Arc Lamp. When two pointed carbon rods are put in circuit with a powerful battery, and their ends brought in contact, the ends become red from the heat produced.

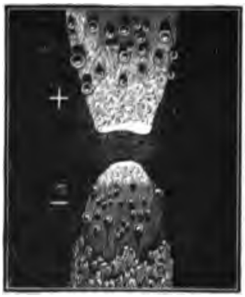


FIG. 257.

If the ends are then slowly separated by a space of 3 or 4 mm., the current continues to pass in the form of a luminous arc, known as the *voltaic arc* (Fig. 257). A brilliant white light is emitted, coming mostly from the white-hot ends of the rods. If the action is allowed to continue, the carbons are slowly consumed by oxidation; at the same time a cavity called the *crater* is formed at the end of the positive rod, while the negative rod becomes cone shaped; this is due to a transfer of minute particles of carbon from the positive to the negative rod.

The temperature of the arc is so high (about 4000° C.), that a diamond, placed in the crater, is melted, and gold or platinum is quickly vaporized.

In the common *arc lamp* the voltaic arc is applied for the purpose of lighting streets, railway stations, etc. For economical reasons the required current is generated by a dynamo instead of a battery. If a continuous current is used, the E.M.F. required for a good light is from 40 to 50 volts. The strength of current for a lamp of 1000 candle power varies from 7 to 10 amperes.

The carbons are made from a very dense coke-carbon. As they burn away, the air-space between them widens; hence some kind of automatic mechanism is required to keep the distance between their ends constant.

Various kinds of regulators are in use; in most of them the action of an electromagnet excited by the current, or by a part of the current, performs the work of regulation.

Arc lamps are usually connected in series.

304. Electric Firing of Explosives. The heating effect of the electric current finds an important application in the firing of blasts, mines, and torpedoes. The current is transmitted on good conductors to the point where the explosive is placed. The good conductors are connected by a thin platinum wire passing through the explosive. The passage of the current develops a white heat in the platinum wire, and the explosion instantly follows.

Submarine mines and torpedoes are exploded by means of currents which can be controlled from the shore. In one kind of torpedo a *circuit-closer* is placed in the torpedo or in a buoy moored to the torpedo, and automatically closes the circuit when the torpedo or buoy comes in contact with a vessel.

305. Thermo-Electricity. If the ends of two bars A and B (Fig. 258), made of different metals, are soldered together so as to form a complete circuit, and one junction is heated, a weak current will flow through the circuit. Currents thus produced are called *thermo-electric* currents.

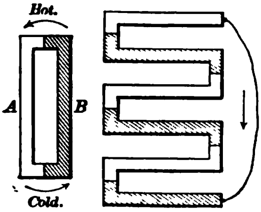


FIG. 258.

For the same difference of temperature, the combination of bismuth and antimony gives a stronger current than almost any other combination. The current flows across the heated junction from the bismuth to the antimony, and across the cooler junction from the antimony to the bismuth.

The strength of the current is increased by joining several pairs of metals, forming what is called a *thermopile.*

The E. M. F. of a thermopile composed of 90 bismuth-antimony pairs for a temperature difference of 100° C. is 1.08 volts, or the same as that of a Daniell cell.

A thermopile in connection with a galvanometer is used for the purpose of detecting very small changes of temperature.

CLASS-ROOM EXERCISES.

1. Why is it that if the poles of a battery are joined by a long, thin wire the battery does not get so hot as when a short, thick wire is used ?

2. How could you boil water by means of a current from a battery ? Give a sketch of the arrangement you would use.

3. The wire which connects the plates of a Daniell cell gets hotter when the plates are brought nearer each other. Explain why.

4. The same current flows through a thick copper wire and a thin copper wire, joined end to end and of such lengths that they have the same resistance. Which wire will become hotter ? What is the reason ?

5. The resistance of two wires *A* and *B* made of the same metal are as 2 : 3. What are the relative quantities of heat developed in the wires (1) when fastened end to end so that the same current passes through both wires, (2) when arranged in multiple arc ?

6. The poles of a cell are joined by two wires similar in all respects except that one is longer than the other. In which wire is the greater amount of heat produced, and why ?

7. The E. M. F. of a battery is 18 volts, and its internal resistance is 3 ohms. The difference of potential between its poles when they are connected by a wire *A* is 15 volts, and falls to 12 volts when *A* is replaced by another wire *B*. Compare the quantities of heat developed in *A* and *B* in equal times.

8. A coil of wire whose resistance is 35 ohms is traversed by a current whose strength is 0.4 amperes. What amount of heat is generated in the wire each second ?

9. How much heat is developed in 1 hour by a current of 9 amperes in a wire the potential difference of whose ends is 300 volts ?

10. What horse power is required to maintain through a resistance of 373 ohms (1) a current of 5 amperes, (2) a current of 10 amperes ?

11. What is the equivalent in horse powers of a current of 1000 amperes flowing under a pressure of 1 volt ?

12. The strength of the current flowing through a coil of wire is 30 amperes, and the potential difference between the ends of the coil is 60 volts. How much energy is consumed per hour in joules, and also in foot-pounds ? How much heat is developed ?

13. How many watts must be expended to send a current of 10 amperes through a resistance of 15 ohms ?

14. A current of 2 amperes is allowed to flow for 20 minutes through a coil of wire of 8 ohms resistance, immersed in 300 grams of water. What is the rise in the temperature of the water?

15. In order to determine the strength of a current, it is made to pass through a coil of wire of 40 ohms resistance, placed in a calorimeter containing 200 grams of water. After 10 minutes the temperature of the water had risen 15°. Find the strength of the current.

16. The resistance of a conductor is halved and the current is doubled. How is the heat generated in the conductor affected?

17. If the current in a conductor is quadrupled, how must the resistance of the conductor be altered in order that the amount of heat generated may remain unchanged?

18. An arc lamp takes a current of 10 amperes, and its voltage is 50 volts. What power does it absorb?

19. A glow lamp of 16-candle power takes a current of 0.75 ampere, and its voltage is 100 volts. Find the number of watts per candle power absorbed by the lamp and the amount of heat generated in 1 hour.

20. If a current of 0.75 ampere flows through an Edison lamp when 108 volts are maintained at its terminals, how many foot-pounds of work per minute are expended on the lamp? How many such lamps can be made to glow with the expenditure of 5 horse power?

21. A 16-candle-power glow lamp takes a current of 1 ampere with an E. M. F. of 52 volts. Find the number of watts absorbed per candle, and the H. P. required to supply 200 of these lamps in parallel circuit.

22. Calculate the power required to light 80 glow lamps, the voltage of each being 65 volts, and the strength of current required being 0.8 ampere.

23. A conductor carrying a current divides into two branches whose resistances are in the ratio of 4 : 5. Compare the amounts of heat generated in the branches (see § 291).

24. A battery of 10 cells, the resistance of each of which is 4 ohms, is arranged in series. What must be the resistance of a wire in order that the current from the battery when sent through the wire may expend nine tenths of its energy in heating the wire?

25. The internal resistance of a battery of 12 cells joined in series is 36 ohms. The E. M. F. of each cell is 2 volts. The strength of the current when the poles of the battery are joined by a certain wire is 0.4 ampere. Find the resistance of the wire, and the number of calories of heat set free in the wire in one hour.

Chemical Effects of a Current.

306. Electrolysis. A metal wire, when traversed by a current, suffers no apparent change except a rise of temperature. But when a current passes through a chemical compound dissolved in a liquid, or through certain compounds in a state of fusion, the compound is *decomposed.*

Decomposition effected in this way is called *electrolysis.*

The body decomposed is called an *electrolyte,* and the liquid holding the electrolyte in solution is called the *bath.*

The battery wires connect with the bath by means of plates called *electrodes;* the plate by which the current enters the bath is called the *anode,* and the plate by which the current leaves the bath is called the *kathode.*

As a simple example of electrolysis, suppose we pass a current through a solution of hydrochloric acid (HCl) in water, using as electrodes platinum plates. The electrodes soon become covered with bubbles of gas. When tested, the gas on the kathode is found to be hydrogen, and the gas on the anode is found to be chlorine. Thus the acid has been separated into its two elements, hydrogen and chlorine.

These are the facts. Just what takes place in the liquid is a matter of speculation, but the general result is the same as if a stream of hydrogen atoms traveled through the liquid with the current to the kathode, and a stream of chlorine atoms traveled in the opposite direction to the anode.

It is generally believed that in some way the hydrogen conveys the electricity across the liquid, and that the passage of electricity through an electrolyte is not true conduction, but a phenomenon analogous to the convection of heat.

The two most striking facts about electrolysis are:

(1) The products of the decomposition *appear at the electrodes only.*

(2) *Hydrogen and the metals* are deposited on the *kathode;* while *oxygen, chlorine,* and the *other products* of the decomposition are deposited on the *anode.*

307. Secondary Action. In electrolysis it often happens that, besides the decomposition directly effected by the current, other changes occur. These are called *secondary actions.*

In the case of copper sulphate $CuSO_4$, if *platinum* electrodes are used, the kathode becomes coated with a bright red deposit of copper, and oxygen is set free at the anode. The current separates $CuSO_4$ into Cu and SO_4, depositing the Cu on the kathode, the SO_4 on the anode. A secondary action now occurs at the anode; the SO_4 decomposes a molecule of water H_2O, uniting with the H_2 to form a molecule of H_2SO_4, and setting the atoms of oxygen free. Thus the kathode becomes covered with copper, and the copper sulphate is gradually converted into sulphuric acid.

In Fig. 259 the upper row of symbols represents molecules of copper sulphate before the current is turned on. The middle row shows the effect of the current; the Cu of each molecule of $CuSO_4$ is transferred to the next molecule in the direction of the current, and the last atom of Cu is deposited on the kathode. The lower row shows the effect of the secondary action.

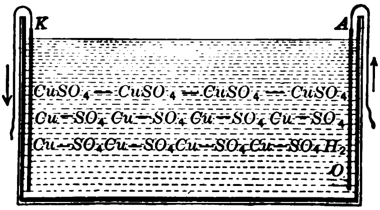

K
A
CuSO4 — CuSO4 — CuSO4 — CuSO4
Cu — SO4 Cu — SO4 Cu — SO4 Cu — SO4
Cu — SO4 Cu — SO4 Cu — SO4 Cu — SO4 H2
O

FIG. 259.

If, however, a *copper* anode is used, the secondary action is different. No oxygen gas is set free. The group of atoms SO_4 attacks the copper anode, unites with an atom of copper, and is converted into a molecule of copper sulphate.

The effect is to transfer copper from the anode to the kathode, the strength of the copper sulphate solution remaining unchanged. If the kathode is made of iron or carbon or any conducting material, it becomes *copperplated.*

If the bath contains in solution a salt of silver instead of a salt of copper, and the anode is a silver plate, then the kathode will be coated with a thin layer of silver.

308. Electrolysis of Water. The electrolysis of water has already been described (see p. 163). The apparatus employed (Fig. 144) is called a *water voltameter.* Hydrogen is set free at the kathode, and oxygen at the anode. The current will not decompose pure water. The presence of sulphuric acid is essential. The current acts directly, not on the water, but on the acid, decomposing each molecule H_2SO_4 into H_2 and SO_4. The hydrogen is set free on the kathode. The remaining group of atoms SO_4 is set free on the anode, but instantly decomposes a molecule of water, uniting with hydrogen to form sulphuric acid, and setting the oxygen free, as explained in § 307.

309. Counter Electromotive Force. When a current decomposes an electrolyte, work is done against the force of chemical affinity. The potential energy stored up is manifested by the tendency of the separated elements to reunite, and also by the existence of an E.M.F. contrary to that which causes the current. This is known as the *counter E.M.F.* or *E.M.F. of polarization.*

In order that electrolysis may occur, the E.M.F. of the current must exceed that due to the decomposition of the electrolyte. For every electrolyte there is a minimum E.M.F. of current necessary to produce continuous decomposition, no matter what the strength of the current may be.

In the case of water this minimum value is 1.495 volts. Therefore, one Daniell cell will not decompose water, however large it is; but two Daniell cells joined in series will decompose water, however small they may be.

If, however, the electrolysis takes place in such a way that no change in the chemical composition of the electrolyte is produced, then only a current of very weak E.M.F. is required. Whenever the two electrodes are composed of the same metal, and are immersed in a salt of the same metal, we have a case of this kind.

310. Secondary Currents. Suppose we join in circuit a constant battery, a galvanometer, and a voltameter containing acidulated water. At first the galvanometer shows that the current grows weaker, but soon it assumes a steady strength. If now we remove the battery from the circuit, the swing of the needle in the opposite direction proves that a current is passing in the opposite direction. This current is caused by the counter E. M. F., due to the formation of oxygen on the anode and hydrogen on the kathode; and it is termed a *secondary* current.

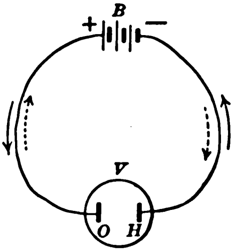


FIG. 260.

The tendency to establish a secondary current exists from the beginning, and causes the weakening of the primary current when it first begins to flow. If we indicate by a full arrow the direction of the primary current (Fig. 260), the broken arrow shows the direction of the current produced by the voltameter. But with platinum electrodes the secondary current is weak and of short duration.

If lead plates are substituted for the platinum electrodes, the secondary current is much stronger, and lasts much longer.

Secondary or *storage* cells consist of lead plates immersed in dilute sulphuric acid, and charged or polarized by a current from a battery or a dynamo.

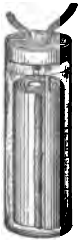


FIG. 261.

In the Planté cell two pieces of sheet lead, separated by strips of gutta-percha, are rolled up as seen in Fig. 261. The charging process oxidizes the anode plate and reduces the kathode plate to a spongy metallic state. When the plates are joined by a wire, a current flows opposite to the direction of the charging current. The flow continues till both plates are reduced to the same state of oxidation. The E. M. F. of each cell is about 2 volts.

A storage battery does not store up electricity, but energy in the potential form of chemical separation.

311. Faraday's Laws. The quantitative laws of electrolysis were discovered by Faraday (1833), and are as follows:

1. *The weight of an element deposited by a current is proportional to the quantity of electricity that passes through the voltameter or bath.*

2. *When the same current passes through several electrolytes, the weights of the different elements deposited are proportional to their chemical equivalents.*

The *chemical equivalent* of an element is equal to its atomic weight (§ 155), if it unites with or displaces *one* atom of hydrogen in chemical reactions, but equal to *half* its atomic weight, if it unites with or displaces *two* atoms of hydrogen.

The weight of an element deposited by a current of 1 ampere in 1 second, is called the *electro-chemical* equivalent of the element. Let z denote its value in grams, and W the weight of the element deposited by a constant current of C amperes in t seconds; then

$$W = Ctz.$$

If the value of z for any element is known, and we find the values of W and t by experiment, we can compute the value of C in amperes. This method of measuring the strength of a current is often employed.

Values of z in grams: hydrogen, 0.00001038; oxygen, 0.0000829; silver, 0.001118; copper, 0.000327; zinc, 0.000337; nickel, 0.000304.

The electro-chemical equivalent of an element may be found by multiplying the chemical equivalent by 0.00001038.

Fig. 262 illustrates one of Faraday's experiments. A circuit divides into two branches of equal resistance, so that the current is compelled to pass through two precisely similar water voltameters A and B. One half of the current then flows through each branch. The quantity of gas set free in each voltameter in the same time is found to be half of that set free by a third voltameter C, traversed by the entire circuit, and therefore by *twice* as much electricity as flows through either A or B.

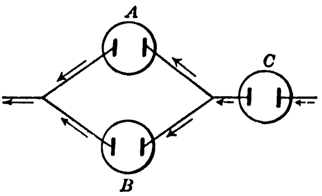


FIG. 262.

312. Uses of Electrolysis. Electrolysis is employed,

(1) To cover with gold, silver, nickel, or copper, objects made of cheaper metals. This process is called *electroplating.*

(2) To make copies of printing types, woodcuts, casts, medals, etc. This process is called *electrotyping.*

(3) To extract metals from their ores, or from combination with other metals. Copper and aluminum are now mostly obtained in a pure state by the aid of electrolysis.

A sectional view of a bath for *silvering* is shown in Fig. 263. The vessel is a wooden box, lined inside with gutta-percha. Copper rods extend in pairs across the bath, one pair only being seen in the figure. The anodes are plates of pure silver suspended from one rod; and the kathodes are the objects to be plated suspended from the other rod. The action of the current causes pure silver to be taken from the anodes and deposited on the kathodes (§ 307). The strength of the current should be about 0.5 amperes per square decimeter. The operation lasts three or four hours with a dynamo, and eight to twelve hours with a battery. A deposit of 1.5 grams of silver per square decimeter gives a good coating. The materials of the bath are potassium cyanide 500 grams, silver cyanide 250 grams, distilled water 10 liters. Potassium cyanide is a *deadly poison.*

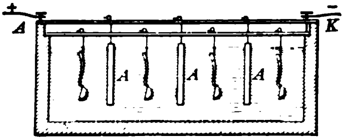


FIG. 263.

In theory electroplating is simple enough, but in practice success cannot be gained without careful attention to various details.

Electrotyping differs from electroplating in the fact that the coating of metal must be removed from the surface on which it is deposited. To reproduce a wood engraving, for example, first a wax or plaster *mould* is made, in which the elevations of the object appear as depressions. The mould is then coated with plumbago to render it a good conductor, and placed as kathode in a saturated solution of copper sulphate. A copper plate serves as anode. The layer of copper deposited on the mould gives a faithful reproduction of the original object. When it is of the proper thickness, it is separated from the mould, and "backed" by attaching it to a plate of type metal. Printed impressions can then be made from it just as well as from the original object.

LABORATORY EXERCISES.

1. Cover a silver coin with a coating of copper. After the coin is well coated with copper, reverse the current. Describe and explain what then takes place.

2. Make a copperplate copy of a medal or similar object.

CLASS-ROOM EXERCISES.

1. Sketch and describe an arrangement by means of which a piece of platinum can be coated with copper.

2. Explain with a diagram how you would proceed in order to plate an article with silver.

3. A battery is hidden from view, but the ends of wires connected with the poles of the battery are in sight. How can you ascertain which wire is connected with zinc and which with the copper pole of the battery by observing what takes place when the ends of the wires are dipped into a solution of copper sulphate?

4. How much copper will be deposited by a current of 3 amperes in 1 hour? Electro-chemical equivalent of copper, 0.000327 grams.

5. How long will it take a current of 1 ampere to deposit 10 grams of silver? Electro-chemical equivalent of silver, 0.001118 grams.

6. What is the strength of a current that will deposit 1 gram of copper in 2 hours?

7. A battery of 8 cells joined in series is used to decompose water. How much zinc is consumed by the battery while 1 gram of hydrogen is set free? Chemical equivalents: hydrogen 1, zinc 32.5.

8. A copper voltameter and a silver voltameter are included in the same circuit. After a certain time it is found that 1 gram of copper is deposited. How much silver will be deposited during the same time?

9. A current divides into two branches, each of which is carried through a solution of copper sulphate. If all the circumstances are the same except that the electrodes are in one case copper and in the other case platinum, will the two currents through the solutions be equally strong? Give reasons for your answer.

10. A current is passed through a coil of wire and through a voltameter arranged in series with it. If the strength of the current is altered so that the heat produced per minute in the coil of wire is doubled, what change will be produced in the rate at which chemical action takes place in the voltameter?

Induced Currents.

313. Electromagnetic Induction. Faraday (1831) discovered that a current is produced in a closed coil of wire, when the coil and a magnet are made to approach each other or made to recede from each other.

If a magnet SN (Fig. 264) is thrust quickly into a coil of wire connected with a galvanometer, the needle swings. Therefore a current is flowing through the coil.

While the magnet is at rest, no current flows.

When the magnet is withdrawn, a current flows through the coil in a direction opposite to that of the first current.

These transient currents, which flow only while the magnet is in motion, are called *induced currents.*

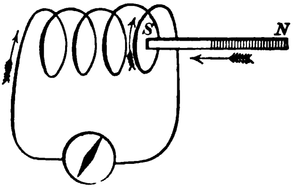


FIG. 264.

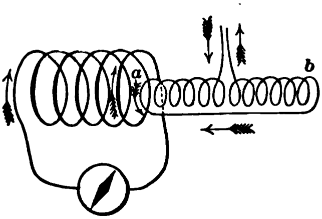


FIG. 265.

The direction of the induced currents is such that the coil of wire, acting as a solenoid, *opposes* the motion of the moving magnet. Therefore, when the magnet is thrust into the coil, the direction of the induced current is *contrary* to clock motion; and when the magnet is withdrawn, the direction of the induced current is *the same* as clock motion; the observer looking in the direction of the lines of force.

If in place of a magnet we use a solenoid ab traversed by a current, the same effects are observed; and the same rule determines the direction of the induced currents (Fig. 265).

314. Induced Currents and Lines of Force. Induced currents and lines of force are related as follows:

1. *When from any cause the number of lines of force that pass through a closed circuit undergoes a change, a current is induced in the circuit; if the number of lines increases, the induced current is counter clock-wise; if the number of lines decreases, the induced current is clock-wise; the observer being supposed to look in the direction of the lines of force.*

2. *The E.M.F. of the induced current is proportional to the rate per second of the increase or decrease in the number of lines of force which pass through the circuit.*

Consider, for example, a wire rectangle *ABCD* (Fig. 266) as it revolves about the axis *EF* in a uniform field of force. During the first quarter of a revolution the number of lines of force passing through the rectangle is decreasing; during the next quarter the number is increasing, but the opposite face of the rectangle is now presented to the observer. Hence, during the *first half* of the revolution, a current circulates in the direction of the arrows. During the second half of the revolution the current circulates in the opposite direction.

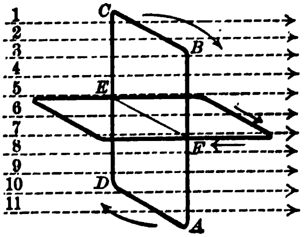


FIG. 266.

315. Lenz's Law. When we move the magnet in Fig. 264 we meet with a resistance much greater than that due to the mere inertia of the magnet. The cause of this resistance is the induced E.M.F., which acts in opposition to the motion. The law which governs this peculiar kind of action and reaction was stated by Lenz (1834) as follows:

Induced currents have such a direction that their reaction tends to stop the motion which produces them.

316. Eddy Currents. Arago found that a copper disc, when held below an oscillating magnetic needle, checked the oscillations; he also found that if the disc was made to rotate in its own plane, it dragged the needle round with it, although the two bodies were not in contact. Foucault rotated a copper disc rapidly between the poles of an electromagnet, and found that much more force was required to produce the motion than was needed merely to overcome friction; moreover, he observed that the disc soon became very hot.

These phenomena were explained by Faraday. He showed that whenever a continuous mass of metal is moved in a magnetic field of force currents are induced in the metal in directions such that they tend to stop the motion (by Lenz's law). They circulate in the metal 'till their energy is frittered down into heat. They are called *eddy currents.*

In constructing the iron core of a dynamo armature, eddy currents are avoided as far as possible by building up the core out of thin metal strips separated by some insulating material. Care is taken to have the planes of division parallel to the lines of force so as to render impossible any electric flow in a direction perpendicular to the lines of force.

317. The Dynamo. The *dynamo* is a machine which converts mechanical energy into the energy of an electric current. The chief parts of a dynamo are a *field-magnet* and an *armature.*

The field-magnet is a massive electromagnet, made of wrought iron or cast steel so prepared as to have great magnetic permeability. The armature consists of coils of wire wound round a soft iron core and revolving between the poles of the magnet.

The simplest conceivable armature is a single loop of wire, like that seen in Fig. 266. The effect of rotating such a loop in a field of force has already been described (§ 314). The current thus induced reverses in direction every half revolution, or is an *alternating* current. It is transformed into a direct or continuous current by means of a *commutator.*

318. Commutator. An arrangement or *commutator* suitable for converting the alternate currents induced in a revolving loop of wire into a continuous current is shown in Fig. 267.

Two semicircular segments of metal *a* and *b* are mounted on a cylinder of ebonite which revolves with the loop of wire. The ends of the loop are joined, one to each segment. Two flat strips of metal, called *brushes*, press upon the segments, and form the terminals of the exterior circuit. The segments are so placed that they change contact with the brushes when the current reverses in the loop (*i.e.*, when the plane of the loop is vertical). Therefore, the current in the exterior circuit always has the same direction.

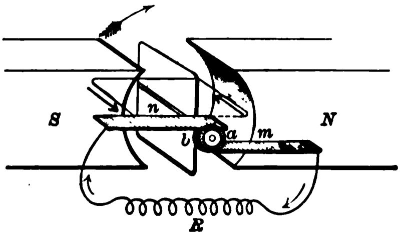


FIG. 267.

319. Drum Armature. If we substitute for the wire loop in Fig. 267 many coils of wire, each coil being wound longitudinally round a massive iron drum (Fig. 268) built so as to avoid eddy currents, and if we provide a suitable commutator,

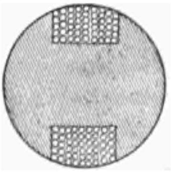


FIG. 268.

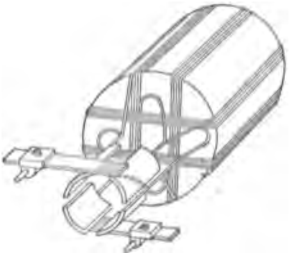


FIG. 269.

we have the drum armature now in use. A four-coil armature with the commutator is shown in Fig. 269.

320. Ring Armature. A *ring* armature (Gramme ring) with eight coils is represented in Fig. 270. The coils of wire are wound upon the ring at equal intervals and connected within the ring to metal bars separated by mica, thus forming a closed circuit. The effect of rotating the ring is to induce currents, as shown by the arrows. The currents induced all around the ring flow to the brush *m*, whence they pass through the exterior circuit, returning to the brush *n*. This arrangement gives a current continuous in direction and also constant in strength, provided the velocity of rotation is kept constant.

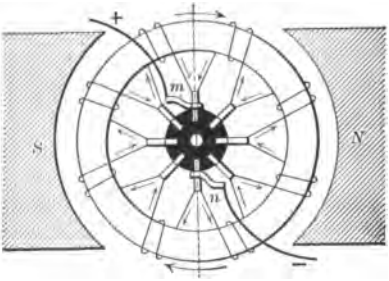


FIG. 270.

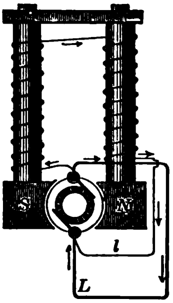


FIG. 271.

321. Series and Shunt Dynamos. The field-magnet is excited by the current generated in the machine itself. The minute traces of magnetism which exist in all iron are utilized. The coil of wire around the field-magnet is joined in circuit with the coils of the armature. When the armature is set in rotation, the very feeble current first induced acts on the core of the field-magnet and strengthens the magnetic field. This strengthens the induced current, and so the mutual action goes on till the core is strongly magnetized.

In a *series* dynamo the whole current is allowed to flow around the field-magnet; in a *shunt* dynamo only a part of it.

322. Alternators. A single coil of wire, revolving in a magnetic field, generates *alternating* currents. In each revolution the E.M.F. rises to a maximum, dies away, reverses to a negative minimum, and returns to zero again. By means of a commutator we can make the current continuous in direction, but it will *pulsate* or undergo fluctuations in strength. If several coils are used, the fluctuations will be lessened; and if the number of coils is great enough, the current will be sensibly constant.

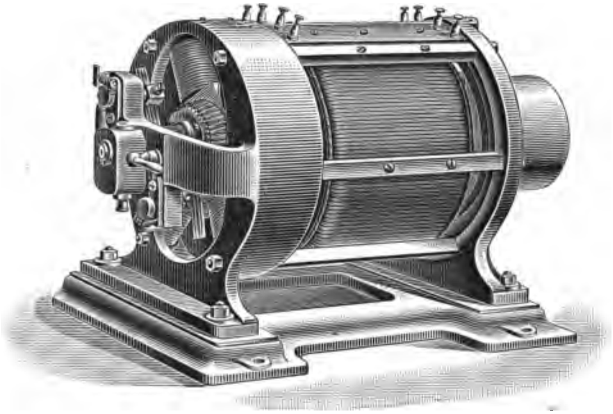


FIG. 272.

For incandescent lighting it is advantageous to employ alternating currents. Dynamos constructed without a commutator, so that they furnish alternating currents, are called *alternators.* In these machines a high voltage is often desirable, and for the sake of better insulation, it is usual to keep the armature fixed and allow the field-magnet to rotate. The magnet is excited by a continuous current from a separate small direct-current machine called the *exciter.*

Fig. 272 represents an alternator. The field-magnets revolve within the ring armature, which is stationary.

323. Efficiency of a Dynamo. The *electric efficiency* of a dynamo is the ratio of the *useful electric power* developed in the exterior circuit to the *total electric power.*

The *commercial efficiency* is the ratio of the *useful electric power* to the *mechanical power* expended in turning the armature of the machine.

Let C denote the strength of the current, E the total electromotive force of the dynamo, D the potential difference of its terminals. D is always less than E; if the machine is a series dynamo, and r denote the total interior resistance, then, as in the case of a voltaic cell (§ 288), $D = E - Cr$. In this case

$$\textit{Total electric power (in watts)} = EC.$$
$$\textit{Useful electric power (in watts)} = DC.$$
$$\textit{Electric efficiency} = \frac{D}{E}.$$

The commercial efficiency is less than the electric efficiency on account of friction, eddy currents, heating effects upon the iron cores, and the periodic changes of magnetism in the armature core. But the commercial efficiency of the best machines is not less than 90%.

Example. Let the data for estimating the efficiency of a series dynamo, working under normal conditions, be as follows:

Resistance of the armature, $r = 0.13$ ohm.
Resistance of the field-magnet, $r' = 0.37$ ohm.
Potential difference of terminals, $D = 210$ volts.
Strength of the current, $C = 50$ amperes.
Power expended in turning armature, $P = 16$ H. P.
∴ Useful electric power, $DC = 10{,}500$ watts.
Total internal resistance, $r + r' = 0.5$ ohm.
Loss of potential in machine, $C\,(r + r') = 25$ volts.
Total electromotive force, $E = 235$ volts.
Mechanical power expended, $746\,P = 11{,}936$ watts.

$$\text{Electric efficiency} = \frac{210}{235} = 93\tfrac{1}{8}\,\%.$$

$$\text{Commercial efficiency} = \frac{10{,}500}{11{,}936} = 88\,\%.$$

$16x^2 = .9128$

$x^2 = .5705$

x

27 : 1626 = 1 : x

27x = 1625

x = 37.9

mile = 80 sec

mile : 1 m. 20 sec.

3042 0

6084

(16) .9128

8 0

1 1 2

8 0 0 0

.5705

√.05 7 0 5 | 2 4

4

44) 1 7 0 9

the
een
in
s a
ses
the
of
sis
ter
nto
y of

e
t
e
o
e
in
lies

nos
tor.

325. Efficiency of a Motor. Let E denote the voltage of the generator, E' the counter voltage of the motor, C the strength of the current, R the resistance of the circuit; then, if we neglect minor losses, due to friction, eddy currents, etc.,

$$\textit{Power applied to motor} = EC.$$

$$\textit{Power yielded by motor} = E'C.$$

$$\textit{Electric efficiency of motor} = \frac{E'}{E}.$$

Hence, if E is constant, the efficiency of the motor is directly proportional to E', and therefore to the speed of the armature of the motor.

As regards the *power* $E'C$ yielded by the motor the case is altered. For as E' increases C decreases; they are, in fact, so related that when E' becomes equal to *half* of E, the product $E'C$ attains its *maximum value.* If E' is greater than half of E, then, although the efficiency of the motor is increased, the power yielded by it is diminished.

The product $E'C$ may be otherwise expressed; for, since

$$C = \frac{E - E'}{R}, \text{ therefore } E' = E - CR.$$

$$\text{Power yielded by motor} = E'C = EC - C^2R.$$

$$\text{If } E' = \frac{E}{2}, \text{ then } C = \frac{E}{2R}, \text{ and}$$

$$\text{Maximum power of motor} = \frac{E^2}{4R}$$

for given fixed values of E and R.

The efficiency is now only $\frac{1}{2}$; that is, 50 % of the power supplied by the generator is wasted in heating the circuit.

Also the strength of the current $\left(\frac{E}{2R}\right)$ is just *half* of the value it would have $\left(\frac{E}{R}\right)$ if we allowed the current to flow through the armature of the motor but prevented the armature from moving.

326. Electric Transmission of Power. A generator and a motor, joined in circuit, form a very effective arrangement for the transmission of mechanical power from place to place. At any convenient spot a generator is driven by steam or water power; the current generated is carried by a copper conductor to the place where the power is wanted; there it is restored to the form of mechanical power by a motor.

The chief difficulty encountered is the waste of power in heating the conductor. This waste amounts to C^2R watts if C denote the strength of the current, and R the resistance of the conductor.

To keep this waste within economic limits on a long line we must diminish the factor C; for R can be reduced only by the use of a larger wire, and the price of copper puts this out of the question. But by diminishing C, we diminish the product EC, which measures the power supplied to the line by the generator. The only way to meet this evil is to raise the voltage E, so that the magnitude of E may compensate for the feebleness of C. Hence, the economic transmission of power by electricity to considerable distances requires a *high voltage.*

The remarkable installation of electric power at Niagara Falls, not yet completed, has furnished new data respecting the possible efficiency of this mode of transmitting power. Enormous alternating generators, weighing 80 tons each, and revolving 250 times per minute, are driven by turbine water wheels, the fall of water being 140 ft. Each generator yields a current of 5000 H. P., and its commercial efficiency is estimated to be about 97%. It is claimed that this power can be carried by cable to Buffalo (15 miles distant), and there distributed to customers at a good profit, and without a loss of more than 10% of the power.

Aside from the question of economy, the electric transmission of power has enormous advantages over the transmission by belts, shafts, water pressure, or compressed air. "An electric conductor is a perfectly flexible thing, which can be bent or carried round corners, and tapped wherever desired. Moreover, it is motionless, and transmits large amounts of energy without being itself visibly in motion" (Fleming).

327. Induction without Mechanical Motion. A change in the strength of a magnetic field, however produced, will induce a current in a closed circuit placed in the field. Hence a current which is starting, or stopping, or changing in strength, induces currents in a closed circuit placed near it.

Suppose we place two coils of wire *A* and *B* (Fig. 274) side by side, join *A* in circuit with a battery, and *B* in circuit with a galvanometer. When the key *K* is pressed down, the deflection of the needle shows that for an instant a current circulates through *B*; this current is *opposite* in direction to that which flows through *A*, or is an *inverse* current. When the current in *A* is stopped, another brief current is induced in *B*; in this case the induced current is *direct*, or has the *same* direction as that which flowed through *A*.

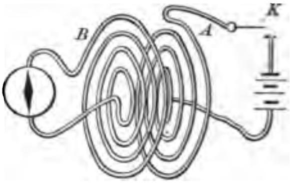


FIG. 274.

Experiment shows that every *increase* in the strength of a current flowing through *A* causes an *inverse* current in *B*, and every *decrease* of strength a *direct* current in *B*. *A* is called the *primary* coil, and *B* the *secondary* coil.

328. Self-Induction. The current in a coil of wire, when its strength is changing, acts inductively upon itself. This action is called *self-induction*. When the current is starting, self-induction prevents it from rising instantly to its full strength. When the current is stopping, self-induction tends to prolong the flow. In both cases the effect of self-induction is to oppose change in the strength of the current. In both cases the effect is merely momentary.

The momentary current on breaking the circuit is called the *extra current*. The extra current has a high E.M.F., and is the cause of the bright spark seen whenever a strong current is interrupted.

329. Induction Coil. This name is given to an apparatus consisting of a secondary coil placed around a primary coil, as sketched in Fig. 275. The secondary coil is composed of a great number (often many thousand) turns of fine wire. The primary coil contains comparatively few turns of coarser wire. Within the primary coil is an iron core composed of soft iron wires, and serving to increase the intensity of the magnetic field.

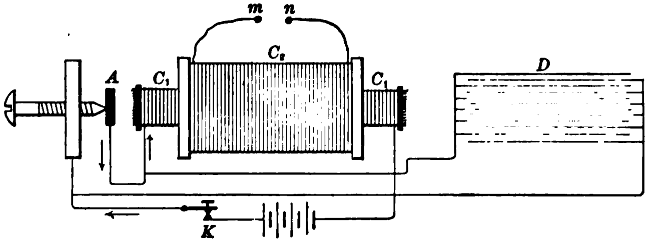


Fig. 275.

An *interrupter A* attached to a steel spring which forms part of the circuit makes and breaks the circuit automatically by vibrating backwards and forwards like the hammer of an electric bell (§ 285). At each interruption of the current a powerful E.M.F. is generated in the secondary coil; and if its terminals *m* and *n* are not too far apart, a brilliant spark will pass between them.

Ruhmkorff improved the induction coil by adding a *condenser D*, made of alternate layers of tinfoil and paraffined mica. The condenser prevents sparking at the interrupter, and adds greatly to the E.M.F. of the induced current.

From a large induction coil it is easy to obtain a spark 10 inches in length. To produce a spark of this length the potential difference between the terminals of the secondary coil must be more than 50,000 volts.

In an induction coil made for Mr. Spottiswoode, of London, the length of the secondary coil was about 280 miles, that of the primary being 1164 yards.

330. Principle of Transformation. The induction coil transforms a current of considerable magnitude but of weak E.M.F. (the primary current) into a small current of exceedingly high E.M.F. (the secondary current); in fact, the E.M.F. is increased nearly in the ratio of the number of turns in the two coils of wire.

This transformation takes place in strict accordance with the law of the conservation of energy. Electric power is the product of two factors, current strength and electromotive force. The induction coil enables us to increase one factor, electromotive force, at the expense of the other factor, current strength. But the change must be such that the product of the factors (disregarding small losses) remains the same.

Now, the induction coil is a reversible machine. This fact may be illustrated by referring to one of Faraday's early experiments on induction (Fig. 276).

Let two coils of wire *A* and *B* be wound on opposite sides of an iron ring, and suppose that *B* has 20 times as many turns as *A*. A current, sent through either coil, will magnetize the ring; and if it is interrupted or alternated in direction, it will induce currents in the other coil. But there is this important difference: If *A* is the primary coil, the effect of induction is to multiply the E.M.F. by 20 and to divide the strength by 20. But if *B* is the primary coil, the effect is to divide the E.M.F. by 20 and multiply the strength by 20.

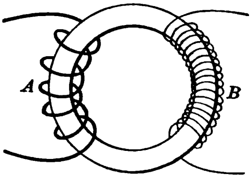


FIG. 276.

In general, a current of strength C_1 and voltage E_1 may be changed by induction to a current of strength C_2 and voltage E_2; but C_2 and E_2 must have such values that

$$C_2 E_2 = C_1 E_1.$$

331. Transformers. We see from the principle of transformation that it is possible to make any desired change in the strength or the voltage of a current with the loss of only a small fraction of the energy. Now, changes of this kind are actually required in every electric plant that generates power to be consumed at a distance. For, as we have seen (§ 326), the power cannot be economically carried through the line unless the current is small and its voltage high; while for electric lighting and most other uses a current of considerable strength at a low voltage is desired.

The requisite changes of strength and voltage are effected by machines called *transformers* or *secondary generators.* The essential parts of an alternating-current transformer are two coils of wire, one having more turns than the other, and both wound around a closed magnetic circuit; its mode of action has already been described (§ 330).

A transformer is called a *step-down* or a *step-up* transformer, according as it is used to *diminish* or to *increase* the voltage. Fig. 277 shows by a diagram how a high pressure of 2000 or 3000 volts in the main line is reduced by step-down transformers to a pressure of 50 to 100 volts, such as is suitable for feeding glow lamps placed in parallel circuit.

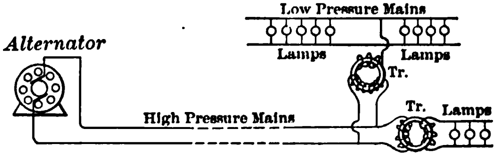


FIG. 277.

Continuous currents are transformed by a machine called a *motor-dynamo,* which is a combination of a motor and a generator. Continuous currents are converted into alternating currents or *vice versa* by machines called *rotary converters.*

332. The Telephone. A sectional view of the Bell telephone is given in Fig. 278. Originally it was used both as a transmitter and as a receiver. The speaker talks to a flexible disc of iron *A* placed just in front of one pole of a magnet *B*, round which is a coil of wire, the ends of the coil being joined to the line wires at *D*. The words of the speaker set the disc in vibration. The vibrations cause variations in the strength of the magnetic field. These variations induce currents in the coil which traverse the line wires, and act upon the magnet of a similar instrument at the other end of the line, so as to reproduce in its disc the original vibrations, and therefore the identical words uttered by the speaker.

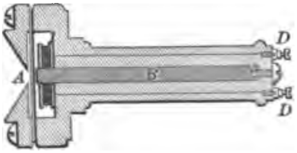


FIG. 278.

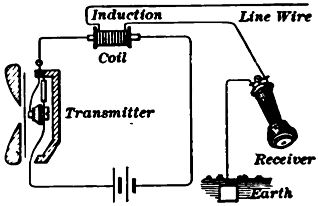


FIG. 279.

An instrument called the *Blake transmitter* is much used for transmitting messages, being found better adapted for that purpose than the original Bell telephone. Each end of a telephone line is provided with both instruments, one for transmitting, the other for receiving. A view of the connections at one end of the line is given in Fig. 279.

In Fig. 279 the ground is used instead of a return wire. The consequence is that, owing to earth currents and leakage from other circuits, the receiver emits an almost continuous frying noise that often seriously interferes with the speech. A complete metallic circuit is now almost always used for telephone lines in all our cities.

The telephone is wonderfully sensitive to inductive action; so much so that disturbances due to neighboring electric light wires often interfere seriously with the distinct transmission of speech.

REVIEW QUESTIONS ON CHAPTER VII.

1. Define the terms *declination* and *dip*, as applied to a magnetic needle. What are *isogonic* lines?

2. Explain the use of a *mariner's compass.*

3. Define a magnetic pole of *unit* strength.

4. Describe an experiment which illustrates magnetic induction.

5. What is the difference in the behavior of soft iron and steel as regards magnetism?

6. How would you impart magnetism to a bar of steel?

7. State briefly the theory of molecular magnets. What are some of the facts that support it?

8. Define a *line of magnetic force.* Describe (by a sketch or otherwise) the general arrangement of lines of force on the field of a bar magnet.

9. Describe an experiment which leads us to divide bodies into conductors and non-conductors of electricity.

10. How can a body be charged by induction?

11. Describe the *electrophorus* and its action.

12. How would you use a gold-leaf electroscope for the purpose of ascertaining what kind of a charge a body has?

13. Explain the action of a *plate electric machine.*

14. Define the *electrostatic unit* of electric quantity.

15. How would you prove that electricity is wholly confined to the outer surface of a charged insulated body?

16. Describe an electric *condenser* and its action.

17. How did Franklin prove that the charges of a Leyden jar reside on the surface of the glass, not on the tinfoil coatings?

18. What is meant by saying that two electrified bodies have *different potentials?* Point out the analogy between difference of potentials and difference of water levels or of temperatures.

19. What is the source of the energy of an electric current generated by a voltaic cell?

20. What is *polarization* in a voltaic cell, its cause, and its remedy?

21. Describe a Daniell cell and its action.

22. What was Oersted's discovery?

23. What two classes of galvanometers are there? Describe some one galvanometer, mentioning its merits and defects.

24. Describe the magnetic field which exists around an electric current.

25. What is a solenoid, and how does it behave?

26. What is the effect of putting an iron core into a solenoid?

27. Why are electromagnets usually made in the shape of a horseshoe and provided with an armature?

28. State Ohm's law. Apply it to two cells the zincs of which are joined by one wire and the coppers by another wire.

29. Define the C. G. S. units of current strength, quantity, electromotive force, and resistance.

30. Define the *ampere*, *coulomb*, *volt*, and *ohm*.

31. Prove that, when a current is shunted, the partial currents vary inversely as the resistances through which they pass.

32. Prove that the fall of potential along a conductor is directly proportional to the resistance passed over.

33. Describe a method of measuring the resistance of a wire.

34. When should voltaic cells be joined in series?

35. State Joule's law.

36. Define the *watt*, and show that the heat equivalent of 1 watt is 0.24 calories of heat per second.

37. What is a *thermopile*, and what is it used for?

38. Define the terms *electrolysis*, *anode*, *kathode*.

39. Describe the electrolysis of copper sulphate (1) if a platinum anode is used, (2) if a copper anode is used.

40. Explain why one Daniell cell is unable to decompose water, however large it may be, while two cells joined in series will decompose water, however small they may be.

41. Give an example of a *secondary* current. Describe the *Planté storage cell.*

42. State Faraday's laws of electrolysis.

43. Describe the process of electrotyping.

44. How are induced currents related to lines of force?

45. What are eddy currents?

46. Describe a *commutator*.

47. What are the essential parts of a *dynamo?*

48. The electric efficiency of a certain dynamo is 95%, and its commercial efficiency is 90%. Explain the meaning of these statements.

49. What is meant by saying that a dynamo is a reversible machine?

50. Why does the economic transmission of power by electricity require a high voltage?

51. Describe an *induction coil* and its action.

52. State and illustrate the principle of transformation.

53. Describe briefly the *telephone.*

CHAPTER VIII.

SOUND AND LIGHT.

Sound.

333. The Cause of Sound. The molecules of a sounding body are in a state of to-and-fro motion, or *vibration*. The sensation of sound is caused by this motion imparted to the organs of hearing. The word *sound* is used to denote the sensations, and also sometimes to denote the cause of the sensations. A body is made to vibrate by mechanical action.

We sound a tuning fork by striking the prongs on a table or rubbing them with a violin bow. The vibratory character of the motion is shown by bringing a cork ball, suspended by a thread, in contact with one prong, or by touching the prongs with the fingers or the lips.

The nature of the motion is also evident from the indistinct and enlarged outlines of the ends of the prongs when vibrating. A sounding piano string can be both seen and felt to be in motion.

Vibratory motion, in order to cause the sensation of sound, must be transmitted to the ear by some material medium. The usual medium is air; but other gases and most solids and liquids are capable of transmitting sound.

If an alarm clock is placed on felt under the receiver of an air pump, and the air removed, the sound of the bell is exceedingly faint. Sound cannot travel through a vacuum.

Vibratory motion is itself maintained by the force of *elasticity*. Musical sounds are usually produced by the vibrations of strings, steel wires, stretched membranes, and columns of air. Substances such as felt, sand, and clay are used to deaden vibrations and prevent the transmission of sound.

334. Vibratory Motion. Each particle of a vibrating body moves to and fro along the same path. The motion is said to be *periodic* if the particle returns to the same condition after equal intervals of time. The bob of a swinging pendulum has a periodic vibratory motion.

In the study of sound or acoustics we have to deal with the vibrations of *elastic* bodies. These vibrations may be *transverse* or *longitudinal*.

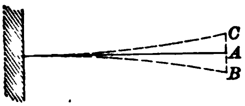


FIG. 280.

If one end of a steel rod, the other end of which is fixed (Fig. 280), is drawn aside from A to B and then released, the free end will vibrate for some time. The point A, for example, will describe repeatedly the path $B\ C$ such that the position of rest A bisects $B\ C$. The path of the vibrations is *perpendicular* to the direction of the rod; hence they are called *transverse* vibrations.

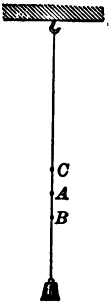


FIG. 281.

Suspend a small weight by a rubber cord (Fig. 281), pull the weight down gently, and then release it. The weight and the particles of the cord will execute vibrations *parallel* to the length of the cord. The particle A will oscillate between the extreme positions B and C equidistant from A. Vibrations of this kind are called *longitudinal* vibrations.

In acoustics, by *one vibration* of a particle is usually meant the motion of the particle from one extreme position to the other *and back again*. The time required to execute a vibration is called the *period* of the vibration. The distance from the position of equilibrium A to either extreme position B or C is called the *amplitude* of the vibration. The number of complete vibrations performed in one second is called the *vibration frequency*, or simply the *frequency*.

335. Simple Harmonic Motion. When the particles of an elastic body are made to vibrate by the action of some disturbing force, the value of which does not exceed the elastic limit of the substance, the force tending at any instant to restore the particle to its position of rest varies directly as the distance of the particle from the position of rest (Hooke's law). Vibration due to a force which obeys this law is called *simple harmonic motion.* The vibrations are periodic; and it can be proved that *the period is the same for all amplitudes.* Therefore the vibration frequency is the same for all amplitudes.

Simple harmonic vibration and uniform circular motion are very closely related. Suppose that a point *A* (Fig. 282), starting from the position *C*, describes with uniform velocity the circumference of a circle the center of which is at *O*. Consider what kind of motion the projection *B* of the moving point on the horizontal diameter *C D* will have. Evidently *B* will move in a straight line from *C* to *D* and then back from *D* to *C*. It is also clear that the motion of *B* will *not* be uniform, but most rapid at the center *O* of the circle, and slower and slower as we approach *C* or *D*. It can be proved that the motion of *B* is a simple harmonic motion.

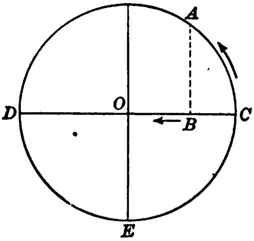


FIG. 282.

If *A* were a luminous point moving round the circle on a dark night, and if we suppose the eye of the observer to be placed in the line *O E* produced and at a great distance from the circle, then the motion of *A* would seem to coincide with that of *B*; *A* would appear to oscillate between *C* and *D* with a simple harmonic motion.

The motion of *B* is strictly periodic. The period is the interval of time between two successive passages of *B in the same direction* through any point of the horizontal diameter.

336. Wave Motion. When any portion of an elastic medium is thrown into vibration, the other portions very quickly acquire a similar motion. Consider a row of contiguous particles. When one is disturbed, the next is affected by reason of the mutual action between them, and also moves, only its motion is *a little later* in point of time. Thus the entire row is set in vibration; but at any given instant the particles are all in different stages of vibration, or, to use technical language, in different *phases* of vibration. This peculiar kind of motion is called *wave motion.* A wave consists of a series of particles all in different phases of vibration.

Waves, like vibrations, may be *transverse* or *longitudinal.*

337. Transverse Waves. A transverse wave is formed when one end of a long rubber cord is fastened to a wall, and the other end is moved up quickly from A to B, then down to C, and then back to A (Fig. 283). The more distant a point is from A, the longer is the interval of time before it begins to move. The portion of the cord between u and z contains particles in every phase of vibration; z is just beginning a vibration, y has performed $\frac{1}{4}$ of a vibration, x $\frac{2}{4}$, v $\frac{3}{4}$, and u a complete vibration. The distance from u to z is called a *wave length.* The portion x y z of the wave is the *crest*, and the portion u v x is the *hollow.*

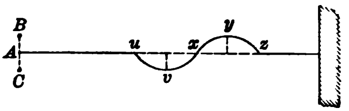


FIG. 283.

The horizontal motion of the *wave form* and the vertical motion of an individual particle such as u must be carefully distinguished. While the particle u makes one complete vibration in a vertical line, the wave motion advances a wave length from u to z.

Therefore, if l denotes a wave length, n the vibration frequency, and v the velocity of transmission of the wave motion, then in all cases of wave motion,

$$v = ln.$$

338. Longitudinal Waves. Vibrations are transmitted through air by *longitudinal* waves or waves of *compression and rarefaction.* A rough idea of their character is given in Fig. 284. When the bell plate vibrates, the adjacent layer of air is alternately compressed and rarefied. When under compression, its molecules act upon those of the next layer, setting them in motion, and causing this layer to be compressed; at the same time the molecules of the first layer rebound, and the layer passes into a state of rarefaction. The second layer acts similarly upon the third, and so on. Thus a *pulse,* or compressed state of air, travels from the bell outwards in all directions, and is followed immediately by a rarefied state; while the air molecules themselves merely move to and fro through very small distances.

When these air waves impinge upon the delicate organs of the ear the sensation of sound is perceived.

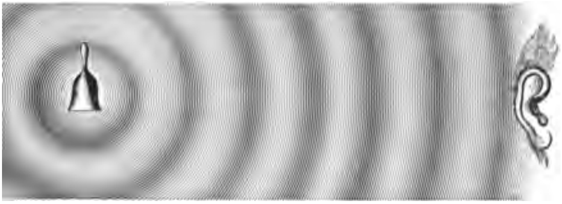


FIG. 284.

The waves produced by dropping a stone into still water visibly illustrate the propagation of air waves. The crests of the water waves correspond to the spherical shells in which the air is condensed, and the troughs to shells in which it is rarefied. "On the free surface of the water the mass when compressed can slip upwards, and so form ridges; but in the interior of the sea of air the mass must be condensed as there is no unoccupied spot for its escape" (Helmholtz).

The up-and-down motion of a bit of floating wood without advancing shows that the particles of water have only a motion of the same kind. What really advances as a wave is the altered form of the surface.

339. Velocity of Sound. The velocity with which air waves are transmitted has been repeatedly measured by observing the interval that elapses between seeing the flash of a cannon fired at night at a distance of several miles and hearing the report. The mean of the results is about 332 meters, or 1090 ft. per second, at 0° C.

The velocity of sound in air

(1) Increases with the temperature at the rate of about 55 cm. per second per degree C., or 1 ft. per second per degree F.

(2) Is independent of the barometric pressure.

(3) Is greater with the wind than against it.

(4) Is slightly greater for loud sounds than for faint ones.

(5) Is the same for both high and low musical sounds; as is proved by the fact that no discord is perceived when we listen to music played by a band at a distance.

The velocity of sound in water is about $4\frac{1}{2}$ times as great as in air, in copper 11 times as great, in steel 15 times as great.

Newton deduced mathematically a formula for the velocity of sound in any gas. The formula, as corrected by Laplace, is as follows:

$$V = \sqrt{1.41 \frac{P}{D}},$$

where V = the velocity in centimeters per second,
P = the pressure of the gas per qcm. in dynes,
D = the density of the gas in grams per ccm.,

and 1.41 is the ratio of the specific heat of a gas under constant pressure to its specific heat under constant volume (§ 228).

For air, $P = 1033 \times 981$, $D = 0.001293$; substituting these values in the formula, we obtain $V = 33{,}240$ cm. per second, nearly.

The formula shows that the velocity for different gases varies inversely as the square root of the density. For example, the density of hydrogen is about $\frac{1}{15}$ that of air. The square root of 15 is nearly 4. Therefore, sound will travel in hydrogen nearly 4 times as fast as in air.

340. Reflection of Sound. When sound waves meet the smooth fixed surface of a solid body, they are *reflected* from the surface. The amplitude of vibration is diminished by reflection, because a part of the energy of the vibrating molecules is communicated to the solid. The laws of reflection for sound and light are the same, and will be given when we come to the study of light.

The reflection of sound explains why it is easier to hear in a closed room than in the open air; the direct sound is strengthened by the waves that are reflected from all sides of the room. But in a very large room with bare walls the reflected waves will reach the ear later than the direct waves, and the increase in loudness is accompanied with a confused commingling of successive sounds.

In theaters, the curtains, the cushioned seats, the decorations, and the galleries tend to prevent disagreeable resonance; and the presence of the audience has the same effect.

The *speaking tube*, the *speaking trumpet*, and the *ear trumpet* are applications of the reflection of sound. In each case vibratory energy is concentrated into a small space by repeated reflection along the walls of the instrument.

341. Echoes. *Echoes* are caused by waves reflected from a surface so far away that they reach the ear entirely separate from the direct waves. If we take the velocity of sound as 1100 ft. per second, and suppose that 5 syllables are pronounced in 1 second, then to obtain a distinct echo the reflecting surface must be at least 550 ft. distant; if the distance were less, the last syllable would not be finished before the first reflected wave reached the ear. At less than 100 ft., therefore, a distinct echo of a single syllable is impossible.

When there is more than one reflecting surface, and the surfaces are suitably situated, *multiple* echoes are produced.

In *whispering* galleries the walls are so curved that sound waves coming from one place are nearly all reflected to another place; and a low whisper at one of these places can be distinctly heard at the other place.

342. Noise and Musical Sounds. *Musical tones* are produced by rapid periodic vibrations; *noise* is produced by non-periodic motion or vibration. Musical tones are agreeable to the ear; the sensation of mere noise is felt as a kind of shock or as a series of irregular shocks, and is often very disagreeable.

Musical sounds differ in *loudness*, *pitch*, and *quality*.

A series of noises succeeding each other at regular intervals may cause the sensation of a musical tone.

"Nothing can be imagined more purely a noise or less musical than the jolt of the rim of a cab wheel against a projecting stone; yet if a regularly repeated succession of such jolts takes place, the result is a soft, deep, musical sound that will well bear comparison with notes derived from more sentimental sources" (Haughton).

The report of a cork when drawn from a bottle is anything but musical. But "with a sufficient number of properly tuned bottles a skilful performer could, by merely withdrawing the corks, easily evoke a simple melody that every one would recognize" (Zahm).

343. Loudness. The loudness or intensity of a sound for the same ear under similar conditions depends upon the energy of the vibrating particles of air which impinge upon the ear. It follows that loudness increases with the amplitude of the vibrations.

The loudness of a sound diminishes as the distance of the ear from the source of the sound increases, and as the density of the air diminishes.

The decrease of loudness with increase of distance is due in part to the fact that air waves occupy more and more space as they advance, and in part to the transformation of the energy of the wave into heat by friction.

Why does the report of a gun on the top of a high mountain sound like the report of a firecracker?

Sound is strengthened by placing near the sounding body another body capable of vibrating with it. Thus, the sound of a tuning fork is greatly strengthened by merely bringing its base in contact with the top of a table. The use of sounding boards and resonators for the purpose of strengthening sound will be referred to later.

344. Pitch. The character of a musical sound as regards gravity or acuteness is called its *pitch.*

Pitch depends on vibration frequency; *the greater the number of vibrations per second, the higher is the pitch.*

Since period and frequency are reciprocals, and since in the formula $v = ln$ (§ 337) v is constant in value, therefore a rise in pitch implies a shorter period and a shorter wave length.

This truth is made evident by pressing a piece of cardboard against the teeth of a revolving wheel. If the wheel is turned very slowly, a distinct tap is heard every time a tooth strikes the cardboard. When the wheel revolves so fast that at least 16 taps occur each second the sounds coalesce into a deep, continuous, musical tone. As the velocity of the wheel is increased the tone becomes more and more acute. Finally, it ceases to be audible when the number of blows per second reaches a value varying from 8000 to 40,000 for different ears.

The *siren* is an instrument which enables us to measure the vibration frequencies of different notes. A metal disc (Fig. 285), perforated with holes at equal intervals near its circumference, is made to revolve horizontally. A current of air is forced through a tube the mouth of which is so placed that the holes in the disc pass directly underneath it as the disc revolves. Whenever a hole is below the mouth of the tube a puff of air is driven against the air below the disc. The regular succession of puffs produces musical tones which become more acute as the velocity of rotation is increased. This velocity is registered by the instrument itself. Multiplying the velocity by the number of holes in the disc, we obtain the number of vibrations of the air per second.

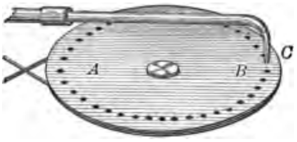


FIG. 285.

345. Harmonic Tones. A single musical sound of one definite pitch is called a *simple tone.* A series of simple tones such that their vibration frequencies are as the whole numbers 1, 2, 3, 4, 5, etc., is called a *harmonic series* of tones. Harmonic tones, when sounded together, have in general a very pleasing effect on the ear; hence they are of great importance in the theory of music and of musical instruments.

346. Musical Intervals. The number of simple tones is infinite. But the ear is so constituted that very few tones when heard together yield a *concord,* or impression that is pleasing to the ear. In all other cases *discord,* more or less unpleasant, is the result. Now, it is a very remarkable fact that, when tones are concordant, their relative vibration frequencies are always expressible by small whole numbers.

In music the ratio of the frequencies of two simple tones is called the *interval* between them. The most important interval is the *octave;* this is the interval between two simple tones one of which is produced by exactly *twice* as many vibrations per second as the other.

If we start from any tone we please and ascend an octave higher, then an octave above the last, and so on, we get a series of tones that make the most agreeable of concords when heard together. The same holds true if we descend in like manner from the tone selected as the starting point. Thus the whole scale of pitch may be regarded as a series of octaves referred to some one tone called the *keynote.*

The octave is subdivided into smaller intervals such that the intermediate tones, when heard in succession, sound agreeable to the ear.

The *diatonic* scale, or *gamut,* places six tones, commonly called *notes,* between those which begin two successive octaves, making in all *eight* tones; hence we see the origin of the term *octave.* Taking 24 as the frequency of the lowest note, the frequencies, intervals, and names of the series of notes in the diatonic scale are as follows:

Frequencies:	24,	27,	30,	32,	36,	40,	45,	48
Intervals:	1,	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2
Names:	C,	D,	E,	F,	G,	A,	B,	C_1

The effect of hearing C *at the same time* with D or B is a discord; but C combined with any one of the other tones yields a concord.

The extreme range of the human voice from the deepest bass tone to the highest soprano tone is confined between 50 and 1600 vibrations per second, or 5 octaves. The range from the deepest tone of a long organ pipe to the shrillest sound of a piccolo covers about 7 octaves.

347. Vibrations of a String. When a stretched string or wire, fastened at its ends, is plucked aside at the middle or rubbed with a violin bow, it vibrates as a whole, so that in the two extreme positions it has the form of half a wave (Fig. 286). If a rubber cord is used, the vibrations are distinctly visible.

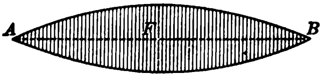


FIG. 286.

Experiment shows that the vibration frequency varies,

(1) *Inversely* as the length of the string.

(2) *Inversely* as the diameter of the string.

(3) *Directly* as the square root of the tension.

(4) *Inversely* as the square root of the density of the material of which the string or wire is made.

Taking into account the relation of pitch to frequency, we see that the pitch of a vibrating string or wire rises as its length is diminished, or its diameter diminished, or its tension increased.

These laws may be verified by means of the *sonometer* (Fig. 287). It consists of a string or wire fixed at one end and stretched over a sounding box by means of a weight. The fixed bridges are placed under the wire near the ends of the box. A movable bridge enables us to alter at pleasure the length of the vibrating portion. We vary the diameter by mounting two different wires of the same material side by side, the tension by using different weights, and the density by using two wires of the same size, but of different materials.

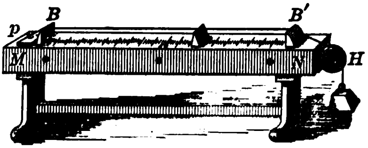


FIG. 287.

For comparing tones, either a siren, or two tuning forks that differ in pitch by exactly an octave, may be employed.

In stringed instruments of the violin class, the player varies the pitch of each string by pressing it with his finger so as to alter its length. The deeper tones are obtained by using thicker strings. The tuning is effected by turning the tuning pegs to which the strings are attached.

348. Harmonic Overtones. The tone emitted by a stretched string vibrating as a whole is the lowest which it is capable of emitting, and is called its *fundamental* tone.

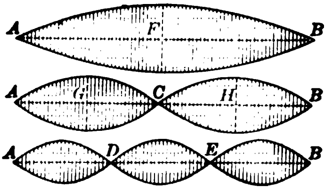


FIG. 288.

If the string is held by the fingers at its middle point *C* (Fig. 288), and either half is plucked or bowed, each half will vibrate as shown in the figure; and this state of vibration will continue after the bow and fingers have been removed. The tone emitted will be an octave above that emitted when the string vibrates as a whole; the frequency is doubled.

In like manner, by holding gently a long, tightly stretched string at ⅓, ¼, ⅕, etc., of its length, and plucking or bowing the smaller segment, the string will divide into 3, 4, 5, etc., equal segments, and emit tones whose frequencies are 3, 4, 5, etc., times that of the string when vibrating as a whole. These tones form a harmonic series; all above the fundamental tone are called *harmonic overtones*, or *upper partial tones.*

The peculiar kind of vibration just described is due to the meeting of waves reflected from the fixed ends of the string and moving in opposite directions. The waves thus produced are called *stationary* waves, because the wave form does not advance as in the case of sound waves in the open air. The points *C*, *D*, *E*, etc., which remain at rest are called *nodes.* The points *F*, *G*, etc., which have the greatest amplitude of vibration are called *antinodes.* The distance from a node to the nearest antinode is equal to a quarter of a wave length. The distance between two consecutive nodes or two consecutive antinodes is equal to half a wave length.

When a violin string is bowed, several upper partial tones are produced at the same time with the fundamental. The tone heard is the compound tone resulting from the coalescence of the separate simple tones.

349. Vibrations of Air Columns. The sounds of wind instruments are produced by vibrating columns of air inclosed in tubes called *pipes*. The organ pipe is a simple type of this class of instruments. One end of the pipe is always open; at this end the air column is set in vibration either by driving a current of air against a thin lip which partly covers a lateral opening in the tube (lip pipes) or by driving the current against a vibrating metal tongue (reed pipes).

The pipe is termed stopped or open according as the other end is closed or open (Fig. 289).

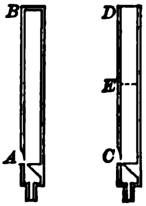


FIG. 289.

When a current of air is properly directed against the lip or reed, stationary air waves are produced in the pipe, with nodes and antinodes (§ 348). The frequency of the fundamental tone emitted by the pipe is determined by the following laws:

1. *The frequency varies inversely as the length of the pipe.*
2. *The frequency is twice as great for an open pipe as for a closed pipe of the same dimensions; therefore*

The pitch of an open pipe is an octave higher than that of a closed pipe of the same dimensions.

These laws are easily verified by experiment. They are necessary consequences of the conditions of vibration. In the stopped pipe there is always at the open lip end an *antinode*, or position of constant density, but of maximum changes in the velocity of the air molecules; at the closed end there is always a *node*, or position of *no motion*, but of maximum change of density. Therefore, the length of a stopped pipe is a *quarter* of the wave length of its fundamental note. In an open pipe there is an antinode at each end and a node in the middle, so that the length of the pipe is *half* of the wave length of the fundamental note. Hence the sound waves in an open pipe are *half* as long as those of a stopped pipe of the same length, and their frequency is *twice* as great.

When the pressure of the wind upon the lip of a pipe is gradually increased, the air column breaks up into segments and emits harmonic overtones along with the fundamental tone. An open pipe can emit any overtone; but a stopped pipe only the 2d, 4th, 6th, etc., overtones.

350. Composition of Vibrations. We know from daily experience that more than one sound can be heard at once. When we listen to the singing of a quartette, it is easy to follow the notes of any one performer, if we fix our attention exclusively upon them. When several persons are speaking at once, we may listen to one speaker in order to understand his words, but we hear at the same time the voices of the others, and often remember what they say.

Hence it follows that different systems of sound waves can be propagated at the same time through the same medium; and that the ear has the remarkable power of analyzing the resultant composite motion of the air into its constituent elements.

When several systems of waves of small amplitudes are excited simultaneously in any medium, the resultant effect is determined by a law identical in form with that of the composition of forces in Mechanics. The law may be thus stated:

Each separate cause of displacement acts independently of other causes; and the actual displacement of any particle at any instant is the resultant of the partial displacements, and can be obtained by applying the parallelogram law.

In Fig. 290 the heavy curves in I and II represent the resultant wave forms arising from the composition of the fundamental tone of a string with its first and third harmonic overtones respectively. The thinner waves in each case are the components. The displacement of each particle is the algebraic sum of the partial displacements. The resultant wave is periodic, and its period is the same as that of the fundamental component. In order that stationary waves may be formed, two wave systems (a direct system and a reflected system) must meet so that their crests coincide and their troughs also coincide. Hence at the antinodes the displacement is twice that due to either system alone.

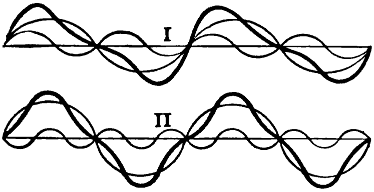


FIG. 290.

351. Sympathetic Vibrations. In certain cases a body may be made to emit sound by the mere presence in its neighborhood of another sounding body. Vibrations caused in this way are called *sympathetic vibrations.* The following are examples:

Place near each other two tuning forks, *A* and *B*, of the same frequency, and mounted on resonant boxes, and excite *A* with a violin bow. Very soon we hear the same tone from *B* also. If we stop the vibrations of *A* with the fingers, we continue to hear the sound of *B*. If *A* and *B* are at opposite ends of the room, the experiment will succeed nearly as well. If *A* and *B* differ in pitch, the effect is not produced.

Tuning forks and glass vases sometimes begin to sound when a musical instrument is played in the same room.

Stretch two wires on a sonometer so that they are in tune with each other. Set one of them in vibration. The other begins at once to quiver, and very soon it is emitting the same note as the first.

Sympathetic vibrations are due to *the cumulative effect of feeble impulses repeated many times at regular intervals.* The nature of the process is best seen in mechanical instances. The largest church bell can be set ringing by a boy who pulls the rope often enough just as it begins to descend. A suspended cannon ball can be made to swing through a large arc by attaching to it a cotton thread, and giving the thread many gentle pulls at those instants when the pulls assist the motion. If two pendulums, equal in length, are suspended side by side from the same support, and one is made to swing, the other will gradually acquire the same motion.

In the case of sound the cumulative effect requires that the two bodies concerned should have the same rate of vibration.

Suppose that the frequency of the tuning forks *A* and *B*, mentioned above, is 256. Then 256 gentle pulses of air every second are received by *B*, and are so directed that they all tend to increase the motion of *B*. A single pulse against the massive steel prong could in no way be detected, but many thousands of pulses produce the effect which we observe.

352. Resonance. The property of a sonorous body that enables it to absorb the vibrations of another sonorous body, and vibrate in unison with it, is called *resonance.* Bodies possessing this property in a high degree are called *resonators.* The best resonators are enclosed masses of air bounded by thin, flexible, elastic plates (resonant boxes), or thin, elastic boards of pine wood (sounding boards).

Resonators strengthen the sound of a vibrating body, because they intercept and absorb a part of the energy of vibration which would otherwise be dissipated into space, and then emit this energy in the form of vibrations of their own.

If the generator of sound and the resonator agree in vibration period we have sympathetic vibrations and the maximum strengthening effect. If the generator and resonator do not agree in vibration period, the vibrations of the resonator are said to be *forced*, and the strengthening effect is much less.

The resonant organ pipe, reinforcing the feeble sounds of the lip or reed; furnishes an example of sympathetic vibrations. The vibrations of the box of a violin in response to those of the strings are partly sympathetic and partly forced.

An instructive method of exhibiting resonance consists in holding a vibrating tuning fork, as shown in Fig. 291, over the mouth of a glass bottle, and then slowly pouring water into the bottle. When the water level reaches a certain point, the sound of the fork is greatly strengthened. If the level is raised higher, the sound becomes weaker. When the sound is loudest, the column of air in the bottle is vibrating in unison with the fork. While the prong of the fork executes half a vibration by moving from *a* to *b*, a pulse of compression travels down the tube and is reflected back just in time to meet the prong at *b* and reinforce its motion. The air wave is a stationary wave with a node at the bottom and an antinode at the top; and therefore the length of the vibrating column of air is *one quarter* of the wave length in air which corresponds to the frequency of the vibrating fork.

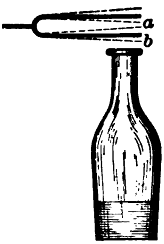


FIG. 291.

353. Interference of Sound. If we apply the law of the composition of vibrations to two simultaneous waves which agree in period and amplitude, but are *opposite* in phase, that is, differ in phase by just half a wave length, we find that the waves *completely destroy each other*, and that the medium will suffer no disturbance at all.

This is illustrated in Fig. 292. The displacements of a particle due to the separate waves are at every instant equal, but opposite in direction; hence their sum is zero.

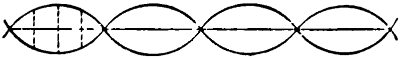


FIG. 292.

If the two waves are air waves, conveying sound, we should infer that their conjoint effect would be to yield no sound at all. Experience shows that this is the effect, and the phenomenon is called the *interference* of sound. The complete interference of sound seldom occurs, because exact equality in period and amplitude, and exact opposition in phase, seldom occur together; but partial interference is very common.

The simplest illustration of interference is afforded by a tuning fork. If a vibrating tuning fork is held to the ear and slowly rotated on its stem, during each revolution there are four positions in which the tone is clearly audible, and four others in which it cannot be heard at all. It is heard when the two prongs are parallel to the ear, and also when they are at right angles to the parallel position. Between these are four oblique positions, which are positions of silence. Partial interference always occurs in the sound of a tuning fork on account of the opposite motion of the two prongs; if a paper cylinder is passed over one prong without touching it, when the fork is vibrating, the sound is strengthened.

In stationary waves (§ 348) we have interference at the nodes and reinforcement at the antinodes. At half a wave length, or any whole number of half wave lengths, from the reflecting surface, the motions due to the direct and reflected waves are equal, but opposite; these points are the nodes. Halfway between the nodes are points where the motions are equal and in the same direction; these points are the antinodes.

354. Beats. If two tuning forks that vibrate in perfect unison are sounded together, their tones will combine and yield a single tone of uniform intensity. But if the vibrations of one fork are slightly retarded by sticking small pieces of wax on the ends of its prongs, the compound tone will become stronger and weaker at regular intervals. These periodic variations in the intensity of the tone are called *beats.* Any two sonorous bodies that differ slightly in pitch, if sounded together, will produce beats.

In general, *the number of beats per second is equal to the difference in the frequencies of the waves.* If, for example, the frequencies are 256 and 255, an exact second will elapse between two successive reinforcements or two successive interferences; for in this time one wave has fallen behind the other by just one wave length. We hear, therefore, in this case one beat every second.

355. Doppler's Principle. Most persons have observed that the sound of a locomotive whistle on a swiftly moving train suffers a sudden fall in pitch just as the train ceases to approach the ear and begins to recede from it.

This phenomenon is caused by a change in the number of sonorous vibrations which the ear receives in a given time. When the whistle is approaching the ear, more sound waves reach the ear in one second than when the whistle is at rest. On the other hand, when the whistle is moving away from the ear, fewer waves reach the ear in one second than when the whistle is at rest. The effect of the motion, therefore, is to raise the pitch of the whistle in the first case and to lower it in the second.

The motion of a vibrating body as a whole shortens the wave length, and increases the frequency of the waves that travel with the body, but has the reverse effect on the waves that travel in the opposite direction.

356. Compound Nature of Musical Sounds. The sensation of a simple tone is excited by a single system of vibrations which follow the law of simple harmonic motion (§ 335). A simple tone is heard when a tuning fork is set in vibration and mounted on a suitable resonance box, or when a wide, stopped organ pipe is gently blown.

But simple tones are seldom heard. As a rule, when a stretched string or a column of air is made to vibrate, several systems of simple vibrations, differing in period, are simultaneously produced; these systems combine together and form a single resultant system which is transmitted as a wave to the ear. Hence the sounds of musical instruments, with very few exceptions, are *compound* tones; in other words, *the sensation of a musical tone is compounded out of the sensations of several simple tones differing in pitch.*

The simple tones which enter into a compound tone are called *partial* tones. The partial tone lowest in pitch is generally the loudest, and hence it alone determines the pitch of the compound; it is called the *prime* or *fundamental* tone, and the others are called *overtones* or *upper partials.* If the upper partials are the harmonic overtones of the prime tone, a musical tone or note is the result; but if there are any inharmonic overtones, the compound sound is more or less discordant to the ear.

The unaided ear is able, to a certain extent, to resolve compound tones into their component partials. The inharmonic partials of bells are easily distinguished. When a tuning fork is struck sharply against a table we hear the tinkle of inharmonic upper partials along with the prime tone.

The note G_3 of a piano is the *third* harmonic partial of the note C_2. Strike gently G_3, and when its sound dies out strike C_2 strongly; the ear, already prepared for G_3, will hear it in the sound of C_2.

Raise the damper of G_3, and then give C_2 a quick, sharp stroke. After the wire of C_2 has come to rest, we hear faintly the sound of G_3. This could not happen unless the sound of G_3 was one of the component elements in the sound of C_2.

357. Analysis of Sound by Resonance. Helmholtz demonstrated the compound character of musical sounds by the aid of instruments which he called *resonators*. The resonator is a hollow brass ball with two openings at opposite ends of a diameter, as shown in Fig. 293. The large opening is presented to the source of sound, and the small funnel-shaped one is applied to the ear. Each resonator is so made that it resounds very powerfully to a single tone of definite pitch, but tends to damp all other tones. To use a resonator we close one ear and apply the nipple of the resonator to the other. Now let any musical note be sounded. If the tone of the resonator happens to occur among the partial tones of the note, the instrument will sing this tone into the ear of the listener with great emphasis. In this way an unpracticed ear is put in a position to pick out a simple tone, even when the tone is comparatively weak.

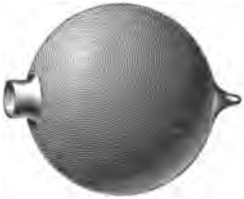


FIG. 293.

358. Quality. Besides loudness and pitch, there is another fundamental characteristic of sound called *quality*.

Quality is that difference which enables us to distinguish between tones of the same pitch and intensity when they are played on different musical instruments, or are sung by different voices. For example, a note sounded on a violin is instantly recognized by the ear as differing in quality from the same note when sounded on a piano. Even the tones of two violins often differ in quality, and this difference may be so marked as greatly to affect the value of the instruments. In the human voice the variety in quality is endless, and is one of the surest means of recognizing a person.

Helmholtz proved that *the quality of a sound is determined by the number, order, and relative intensity of the partial tones into which it can be decomposed.*

This truth may be established by the analysis and by the synthesis of musical sounds. Helmholtz employed both methods. He analyzed compound sounds with the aid of resonators (§ 357), and found that he could hear separately and identify the various partial tones. He confirmed the results of analysis by means of a series of tuning forks so arranged that he could obtain from each fork a simple tone of any desired intensity. By combining the simple tones of the forks he was able to imitate very closely the sounds characteristic of certain musical instruments.

When we analyze sound into wave motion, we find that loudness depends on the *amplitude* of the wave, and that pitch depends on the *frequency* of vibration. Quality depends on the *form* of the wave. The effect on the form of a wave when a simple tone coalesces with one of its upper partials is shown in Fig. 290. The number of different wave forms producible by the union of a prime tone with its upper partials, all variable in loudness, is infinite. If we take six partial tones, and allow to each only two degrees of loudness, more than 400 combinations may be made; if we allow four degrees of loudness, the number rises to over 8000. The endless variety of fine shades in quality which the ear is able to distinguish is thus accounted for.

Simple tones are soft, but weak, and dull at low pitches. The tones of tuning forks applied to resonant boxes and of wide, stopped organ pipes gently blown are examples.

Tones consisting of a predominant prime tone and the first five overtones are rich, full, and harmonious. Examples: the tones of the piano, the open organ pipe, and the French horn.

When overtones higher than the sixth or seventh are very distinct, the quality of tone is cutting and rough. A wire struck by a hard, sharp body yields such tones. Piano hammers are covered with a soft, elastic material in order to avoid high overtones, and to bring out a full, rich quality of tone. The higher overtones, if not very distinct, give character and expression to music. Instruments of the violin class furnish tones of this kind.

359. Harmony and Discord. Two or more musical sounds, produced simultaneously, constitute a *musical chord.* Comparatively few musical chords are able to give that smooth, pleasing effect to the ear which we call *harmony* or *concord.* The most perfect concord is that formed by two simple tones of the same pitch; next comes the harmony of two tones separated by the interval of an octave; then the concord of two partial tones in the same harmonic series; then the less perfect concords of certain pairs of notes in the diatonic scale (§ 346). In general, however, the simultaneous production of two or more simple tones causes a sensation of *dissonance* or *discord,* more or less jarring and disagreeable to the ear.

We have already mentioned those alternations of increase and decrease in loudness called *beats* (§ 354). Helmholtz proved that the roughness of tone characteristic of dissonance is caused by beats. Very slow regular beats in sacred music often produce a fine effect "by pealing through the lofty aisles like majestic waves, or by a gentle tremor giving the tone a character of enthusiasm and emotion." But when beats are more rapid, they produce a harsh grating effect on the ear, somewhat similar to the disagreeable impression produced on the eye by a flickering, unsteady light.

Helmholtz found that for two simple tones in the middle portions of the musical scale, any number of beats per second between 10 and 100 caused sensible dissonance, and that about 30 beats per second caused the maximum amount of dissonance.

If the two coexisting sounds are not simple tones but compound ones, each consisting of a well-developed series of partial tones, dissonance may arise between one overtone and another, or between a prime tone and one of the overtones. The amount of dissonance is also much affected by the order of the overtones which produce beats. The subject is too complex to be further considered here.

360. The Human Ear. The ear (Fig. 294) is divided into three parts: the *external* ear, the *drum* (tympanum), and the *labyrinth.* The drum is separated from the external ear by the *drumskin,* and contains three small bones, jointed together: the *hammer* H, the *anvil* A, and the *stirrup-bone* S. The stirrup-bone S is attached to a membrane stretched over a hole that opens into the labyrinth. The drum communicates with the atmosphere through a tube E called the *Eustachian tube,* and leading from the drum to the throat. The labyrinth consists of three parts: the *vestibule,* next to the drum, the *semicircular-canals* B, and a cavity C resembling in shape a snail's shell and called the *cochlea.* The labyrinth is filled with a fluid in which are spread out the delicate fibers of the auditory nerve.

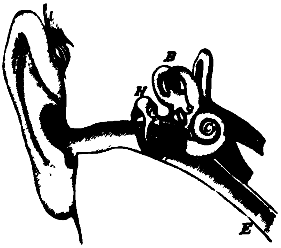


FIG. 294.

Suppose that pulses of air at the rate of 1000 per second impinge upon the ear. First, the drumskin, being elastic, is made to vibrate exactly 1000 times per second. These vibrations are taken up and transmitted through the little ear bones to the membrane that closes the labyrinth, and thence to the fluid that fills the labyrinth. The result is that the fibers of the auditory nerve are shaken exactly 1000 times every second. In some unknown way these tremors of the nerve so affect the brain that we have the sensation of sound.

In the cochlea there is a complicated apparatus consisting of several thousand fine hairs and rods (rods of Corti), all connected with filaments of the auditory nerve. According to Helmholtz the ear analyzes compound tones by means of the sympathetic vibrations of these hairs and rods. Corresponding to each simple tone there are certain rods which are set in strongest motion by that tone, but also set in vibration less strongly by tones of nearly the same pitch.

CLASS-ROOM EXERCISES.

1. The report of a cannon is heard 12 seconds after the flash is seen. How far is the observer from the cannon? Temperature 20° C.

2. The report of a cannon is heard 10 seconds after the flash is seen. The temperature of the air is 62° F. The distance of the cannon from the observer is 11,500 feet. Find the velocity of sound.

3. A string makes 256 vibrations per second. How many vibrations would it make if its length were doubled? If its tension were doubled?

4. Find the wave lengths of the fundamental tone of an open organ pipe 16 feet long, and of its first two overtones.

5. A man standing before a precipice shouts, and hears the echo in $4\frac{1}{2}$ seconds. How far away is the precipice?

6. What effect on the pitch of an organ pipe will be produced by filling it with hydrogen instead of air?

7. A glass jar containing water responds most loudly to a tuning fork when the length of the column of air is 17.5 cm. The temperature at the time is 20° C. What is the frequency of the fork?

8. Compute the velocity of sound in air if a column of air 33.5 cm. long reinforces most strongly a tuning fork of frequency 256.

9. Two sound waves, of the same amplitude, meet in the open air. What will happen if they meet in like phases? In opposite phases?

10. A sonometer string stretched by a 16-lb. weight gives a certain note. What weight will give a note an octave lower?

11. Compare the fundamental tones emitted by a stopped organ pipe 4 feet long and an open organ pipe 16 feet long.

12. Is the fundamental tone of an organ pipe the same in summer as in winter? Give reasons for your answer.

13. A certain wire 1 meter long, weighing 8 grams, and stretched by a weight of 10 kg. makes 64 vibrations per second. What is the frequency of a wire of the same material 50 cm. long, weighing 16 grams, and stretched by a weight of 20 kg.?

14. Why does the sound of a circular saw fall in pitch as the saw enters a log of wood?

15. How would you make a sonometer wire vibrate so as to yield upper partial tones? Why are piano wires struck near the end rather than at the middle?

16. If you have a string of catgut and wish to fit a sonometer with such lengths that they will under equal tension emit the successive tones of the diatonic scale, what lengths must be taken? Take for the lowest tone a length of 96 cm.

Transmission of Light.

361. First Notions. *Light* is the external cause of the sensation of sight. The study of light is called *Optics.*

A *luminous* body is one which of itself emits light; for example, the sun, a candle flame, the white-hot filament of carbon in a glow lamp. Most bodies are *non-luminous;* they become visible only when they receive light from a luminous body and reflect it to the eye. When we enter a dark room with a lamp, the objects in the room receive light from the lamp, and throw it off in all directions. Some of the light reflected from each object enters the eye, and the object is thus made visible; the object is said to be *illuminated* by the lamp.

A non-luminous body can be made to emit light by raising its temperature to a sufficient degree. A piece of iron, when heated, becomes first red-hot and finally white-hot. The light emitted by a candle is due to the presence in the flame of great numbers of particles of carbon raised to a white heat.

Light will pass through some substances, but not through others. A substance like air or glass which will allow light to pass through it is called *transparent.* A substance like wood or iron which will not transmit light is called *opaque.* Substances like ground glass and oiled paper which transmit light very imperfectly are said to be *translucent.* Very thin plates of all bodies, even the metals, are translucent.

The optical phenomena with which we are most familiar are necessary consequences of a very few fundamental facts, established by observation and experiment. They can be shown to be consequences of these facts without making any hypothesis as to the physical nature of light. The explanation of them in this sense is called Geometrical Optics. The attempt to explain optical phenomena by a theory about the physical nature of light is called Physical Optics. For the present we shall confine our attention to Geometrical Optics.

362. Rectilinear Motion of Light. *Light travels in straight lines* through any transparent medium of uniform density.

A line of light is called a *ray* of light, and a group of rays is called a *pencil* of light. If the rays all proceed from one point, the pencil is said to be *divergent*. If the rays are all made to proceed towards one point, the pencil is called *convergent*. If the point is so far away from the eye that the rays are sensibly parallel lines, the pencil is called a *beam* of light; a pencil of solar rays, or a sunbeam, is an example.

The fact that light travels in straight lines is evident from common phenomena. If an opaque body comes between the eye and any object, we can no longer see the object. When sunlight enters a darkened dusty room through a crack in the window shutter, the illuminated particles of dust form a straight line. On a foggy evening we see straight lines of light spreading from a street lamp in all directions.

Place three pieces of cardboard, in which small holes have been made, between a lamp and a screen, at different distances from the lamp. A spot of light can be seen on the screen only when the holes in the cardboard are in the same straight line.

363. Pinhole Camera. A small hole O (Fig. 295) is made in one side of a hollow box, and the opposite side is covered with a white screen. If we place a lighted candle directly in front of O, an inverted image $A'B'$ of the flame AB is seen on the screen. Rays of light proceed from the point A in all directions. A very small pencil passes through O, and falls on the screen at A'. Similarly, an image of every other point of the flame is formed.

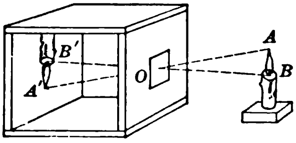


FIG. 295.

The outlines of the image are somewhat blurred; this is because the images of the different points of the flame are not points, but spots of light, and these spots overlap one another. The smaller the hole, the more distinct are the outlines. The images of distant objects are more distinct than those of very near ones.

364. Shadows. When light falls on an opaque body, it is excluded from the space behind the body; this space is called a *shadow.*

If the source of light is a point, the shadow is the frustum of a cone (Fig. 296). The section of the shadow made by a plane perpendicular to the axis of the cone is a circle, which increases in size as the distance of the plane from the opaque body increases.

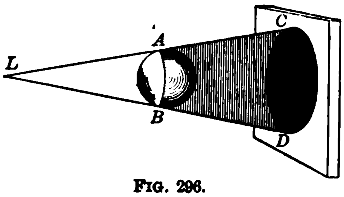


FIG. 296.

If the luminous body has a sensible magnitude, the shadow cast by the opaque body consists of two parts, the *umbra,* or space from which the light of the luminous body is *wholly* excluded, and the *penumbra,* or space from which the light is only *partially* excluded. The case of the sun shining on the earth furnishes a good illustration.

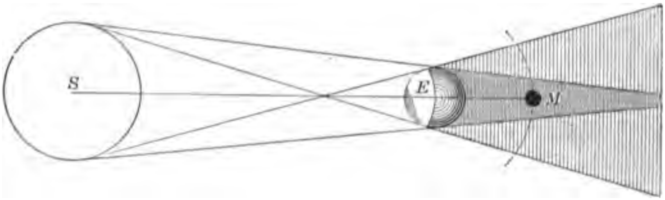


FIG. 297.

The umbra and the penumbra are bounded by tangents drawn to the circles S and E, which represent the sun and the earth, respectively. The dark shadow is the umbra; the lighter shadow, the penumbra. An eye situated in the umbra would not see the sun at all; the sun would be *totally eclipsed.* To an eye situated in the penumbra the earth would appear as a black body covering a part of the sun's disc; there would be a *partial* eclipse of the sun. When the moon M passes through the earth's umbra, it ceases to be illuminated by the sun; it suffers a total eclipse.

365. Velocity of Light. The velocity of light in a vacuum is about 186,000 miles, or 300,000 kilometers, per second. The velocity in air is slightly less, and is about a million times that of sound in air. This enormous velocity has been measured by no less than four different methods, and the results agree so closely that any serious error is out of the question. The methods may be distinguished by the names of the men who first employed them, as the methods of Roemer (1675), Bradley (1729), Fizeau (1849), and Foucault (1850).

Roemer's method. The planet Jupiter has four moons which revolve around him, and at regular intervals are eclipsed by passing through his shadow. These intervals can be found and the times of the eclipses predicted. Roemer noticed that when the earth was nearest to Jupiter, or at *A* (Fig. 298), the eclipses of all four moons occurred about 8¼ minutes *earlier* than the predicted times; and that when the earth was farthest from Jupiter, or at *B*, the eclipses occurred about 8¼ minutes *later* than the predicted times. He saw that these differences could be perfectly explained by supposing that light requires 16½ minutes to travel a distance equal to the diameter *AB* of the earth's orbit. Taking this distance as 184,000,000 miles, and dividing it by 990 (16½ × 60), we have as the velocity of light 185,856 miles per second.

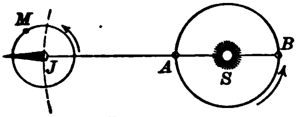


FIG. 298.

Bradley's method. When the telescope was applied to the observation of the fixed stars, it was found that each star underwent a series of small apparent variations in position, which repeated themselves with perfect regularity every year. This phenomenon is called the *aberration of light.* The cause was discovered by Bradley. Since light and the spectator on the earth are both in motion, the apparent direction of a star is determined by the composition of the two motions; just as the apparent direction of falling rain to a man running on the ground is determined by compounding the motion of the rain and the motion of the man (§ 164). The telescope is carried forward by the earth. Hence to see a star in the direction of the axis of the telescope, it must not be pointed in the true direction of the star, but tilted forward by a very small angle depending on the ratio of the velocity of the earth to that of light. From the known velocity of the earth and the observed value of the angle, the velocity of light can be found. The best observations by this method give as the result 186,000 miles per second very nearly.

Fizeau's method. A source of light *L* (Fig. 299) is reflected by a mirror *M* to a distant mirror *M′*. A toothed wheel *A* is so placed that the teeth of the wheel, as it revolves, pass directly between *M* and the distant mirror *M′*.

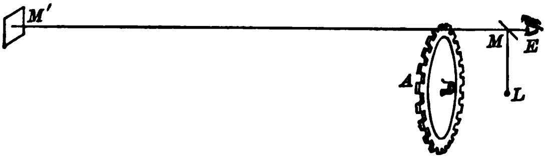


FIG. 299.

If the wheel is at rest with the space between two teeth in the line *MM′*, an observer looking along this line will see a bright spot by reflection in the mirror. But if the wheel is made to revolve fast enough, the light which passes through a space to the mirror will be reflected back just in time to meet a tooth and be stopped. From the velocity of the wheel, the number of teeth, and the distance of the mirror the velocity of light can be computed. The most trustworthy value found by this method is 186,600 miles per second.

Foucault's method. In this method the small angle through which a revolving mirror turns while a ray of light is reflected from that mirror to a fixed mirror and back again is observed. The best result obtained by this method is about 186,400 miles per second.

366. Intensity of Light. When light from a source small enough to be treated as a point falls upon a surface, the *intensity of light* at any point of the surface is measured by the quantity of light received by a unit of area at that point.

The intensity of light at any point of a spherical surface of unit radius with the source for its center is called the *illuminating power* of the source.

The intensity of light at any point of a surface

1. *Varies directly as the illuminating power of the source.*
2. *Varies inversely as the square of the distance from the point to the source.*
3. *Diminishes as the inclination of the surface to the rays of light increases.*

To see why law 2 holds true, let us imagine a series of spherical surfaces having a luminous point as their common center. Let the radii of the surfaces be to each other as the numbers 1, 2, 3, 4, etc. Then (by Geometry) the areas of the surfaces are as the *squares* of these numbers, or as 1, 4, 9, 16, etc. Now each surface, if those within it are removed, will receive the same amount of light, namely, all that is emitted by the source. Hence the intensity of light on the second surface will be only $\frac{1}{4}$ as great as that on the first surface; and the intensity on the third surface only $\frac{1}{9}$ as great, etc.

Law 3 is illustrated by holding a white pasteboard box so that its sides make different angles with the rays of light from a lamp.

Let P and P' denote the illuminating powers of two sources of light; and let I and I' denote the intensities of light which P and P' produce at the distances d and d'; then

$$I = \frac{P}{d^2}, \quad I' = \frac{P'}{d'^2}.$$

$$\text{If } I = I', \frac{P}{d^2} = \frac{P'}{d'^2}, \text{ or } P : P' = d^2 : d'^2.$$

The illuminating powers of two sources are proportional to the squares of their distances from a surface which they illuminate with equal intensity.

367. Photometry. *Photometry* is the measurement of the relative illuminating powers of different sources of light. The unit of illuminating power in common use is a sperm candle, weighing $\frac{1}{6}$ of a pound, and burning at the rate of 120 grains per hour. It is called the *standard candle.* The candle power of any light is the number of standard candles which will have the same illuminating power as the light.

Instruments for comparing illuminating powers are called *photometers;* one of the simplest arrangements for this purpose is Bunsen's photometer, shown in Fig. 300.

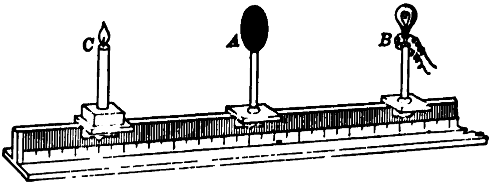


FIG. 300.

The photometer A is made by dropping a little melted paraffine upon the center of a sheet of clean, unsized paper, and then pressing a hot iron upon the paper till the paraffine is all melted and forms a nearly circular, translucent spot. If we hold the paper between the eye and a window, the spot appears brighter than the rest of the paper, because it allows more light to pass through than the paper does. If we hold the paper against a dark background, the spot appears darker than the rest of the paper, because it allows more light on the front side to pass through and reflects less to the eye. If, therefore, we find that the spot and the surrounding paper appear equally bright, we can be sure that the two sides are equally illuminated.

In Fig. 300 the photometer A, the light to be tested B, and the standard candle C are mounted so that they can be made to slide along a meter rod fixed in a horizontal position.

The distances of the candle and the light from A are adjusted till the translucent spot and the surrounding paper look alike when viewed from either side. The candle power of the light is then found by applying the principle stated at the end of § 366.

368. Division of Light at a Surface. When rays of light meet the surface of a body they may be

(1) Regularly reflected or thrown back from the surface in definite directions, according to a law soon to be given.

(2) Irregularly reflected or thrown back from the surface in all directions.

(3) Transmitted through the body.

(4) Absorbed by the body.

Usually the light is divided into two or more parts by the production of two or more of these effects.

LABORATORY EXERCISE.

1. Measure the candle power of some source of light by means of a Bunsen photometer.

CLASS-ROOM EXERCISES.

1. Under what conditions will the image of an object made by a pinhole camera be equal in size to the object? Larger than the object?

2. Draw a diagram to illustrate the formation of a shadow by an opaque body which is larger than the luminous body.

3. Under what conditions will a spectator on the earth see an eclipse of the sun? Illustrate by a diagram.

4. In one of Fizeau's experiments the distance of the mirror was 8663 meters, the number of teeth in the wheel 720, and 12.6 revolutions per second caused the image to disappear. Find the velocity of light.

5. Two lights, *A* and *B*, are 40 inches apart. The power of *A* is to that of *B* as 9 : 16. At what point between them must a screen be placed in order to be equally illuminated on both sides?

6. In measuring the illuminating power of a glow lamp by Bunsen's photometer the distances from the paraffine spot to the glow lamp and the standard candle were 60 cm. and 15 cm., respectively. What is the candle power of the lamp?

7. A gas jet, when burning 5 cubic feet of gas per hour, placed 50 inches from a screen illuminates it equally with a candle placed 15 inches from the screen and burning at the rate of 120 grains per hour. The gas costs $1 per 1000 cubic feet and the candles $0.25 per lb. Compare the cost of lighting a room with gas and candles, respectively.

Reflection of Light.

369. Regular Reflection. When a ray of light BA (Fig. 301) falls upon a smooth polished surface at A, the ray is reflected in a definite direction, AC. BA is called the *incident* ray, AC the *reflected* ray. Let AD be drawn perpendicular or *normal* to the surface. The angle BAD, formed by the incident ray and the normal, is called the *angle of incidence;* the angle CAD, formed by the reflected ray and the normal, is called the *angle of reflection.*

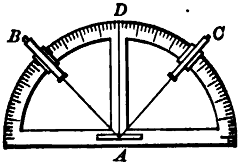


FIG. 301.

The law of regular reflection is as follows:

The angle of reflection is equal to the angle of incidence, and the two angles are in the same plane.

This law may be verified by simple experiments, such as are suggested by the arrangement shown in Fig. 301.

370. Irregular Reflection. If BA (Fig. 301) is a sunbeam, an image of the sun can be seen by looking in the direction CA. But if we place upon the polished surface at A a piece of unsized paper, the reflected beam AC and the image of the sun vanish, and a bright spot, visible from all sides, appears at A. The rough surface of the paper reflects the light in all directions. This kind of reflection is called *irregular* or *diffuse* reflection.

Non-luminous bodies are made visible by diffuse reflection. The moon shines by the diffuse reflection of light borrowed from the sun. At night a lamp enables us to see the walls and furniture in a room by diffuse reflection. What we call *daylight* is sunlight weakened and uniformly diffused by repeated reflections from the surfaces of trees, houses, the ground, dust, motes, etc.

371. Plane Mirror. A smooth surface used to reflect light is called a *mirror*. In the common plane mirror the reflecting surface is a metallic surface attached to the rear surface of a glass plate. The glass supports the reflecting surface and keeps it from being tarnished by the air.

Let *P* (Fig. 302) be a luminous point, *AB* a plane mirror, *PA* the ray of light normal to the mirror, *PB* any other ray. Draw *BD* perpendicular to *AB*. The ray *PB* is reflected in a direction *BC* such that the angles *PBD*, *CBD* are equal (§ 369).

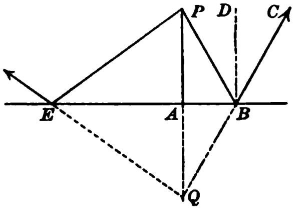


FIG. 302.

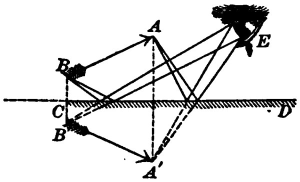


FIG. 303.

Produce the lines *PA* and *CB* behind the mirror till they meet at *Q*. The triangles *PAB*, *QAB* can be proved to be equal by Geometry; therefore, $AQ = AP$. Similarly, it follows that any other ray *PE* is reflected in a line which cuts *PA* produced at *Q*. Therefore, all the rays which diverge from *P* will be reflected as if they diverged from *Q*. An eye placed anywhere in front of the mirror will receive the rays as if they started from *Q* instead of *P*, and will therefore see an *image* of *P* apparently situated at the point *Q*.

The image of a luminous point placed before a plane mirror is situated on a normal from the point to the mirror, and as far behind the mirror as the point is before it.

If an object of sensible magnitude is placed before a mirror, the image of each point is formed according to this law. If the object is a straight line *AB* (Fig. 303), its image may be constructed by merely constructing the images *A'*, *B'* of any two points and drawing a straight line between them.

372. Symmetry of Object and Image. The nearer any point of an object is to a plane mirror, the nearer will be the image of the point. The object and its image are *symmetrical* with respect to the plane of the mirror; that is, the straight line joining any point of the object and the corresponding point of the image is normal to the plane of the mirror and bisected by it. Hence, although the image agrees with the object in size and in shape, there is *an inversion* in the order of the parts. This is well illustrated by the image of the face of a clock (Fig. 304); at 9 o'clock A.M. the position of the hands in the image indicates 3 o'clock P.M.

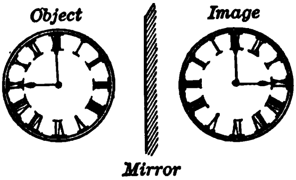


FIG. 304.

The *field of view* of an eye looking into a plane mirror is found by drawing straight lines from the eye to the boundary of the mirror, and producing them beyond the mirror (Fig. 305); the space behind the mirror enclosed by these lines is the field of view. It increases as the eye moves towards the mirror, and diminishes as the eye moves away from the mirror. In Fig. 305 the image of the cross is in the field of view of the eye *E*, but not in the field of view of the eye *E'*.

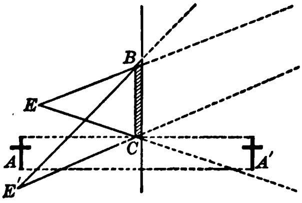


FIG. 305.

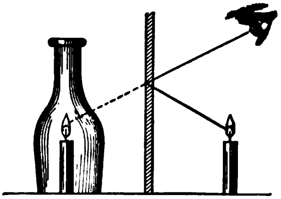


FIG. 306.

A polished glass plate will act as a mirror, and also permit objects behind it to be seen. If a lighted candle is placed in front of it and a bottle of water behind it, the illusion of a candle burning under water may be produced (Fig. 306).

373. Images of Images. When light, after reflection from a plane mirror, falls on another plane mirror, it will be reflected from the second mirror just as if it came from the image formed in the first mirror. In this way an *image of an image* will be formed in the second mirror. Two cases of this kind are illustrated in Figs. 307 and 308.

Parallel mirrors (Fig. 307). When a source of light is placed between two parallel mirrors, an observer looking into either mirror sees a row of images strung along a straight line. The images diminish in brightness, because at each reflection a portion of the light is lost by absorption.

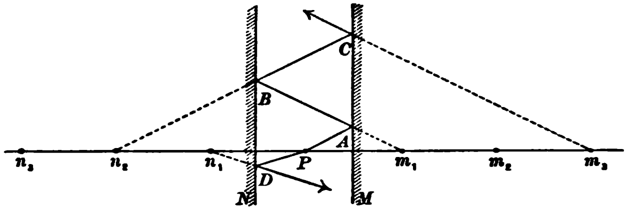


FIG. 307.

In Fig. 307 the course of a ray of light is traced until it has undergone three reflections. It is reflected first by the mirror *M*, as if it came from the image m_1; secondly, by the mirror *N*, as if it came from the image n_2; thirdly, by *M*, as if it came from the image m_3.

$$Am_1 = AP,\ Bn_2 = Bm_1,\ Am_3 = An_2.$$

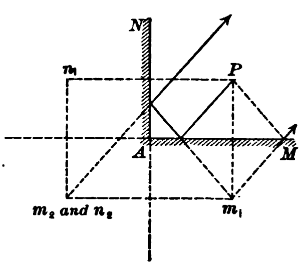


FIG. 308.

Perpendicular mirrors (Fig. 308). In this case, besides the direct images m_1 and n_1 of the object *P*, there is an image m_2 of n_1 formed in the mirror *M*, and an image n_2 of m_1 formed in the mirror *N*; m_2 and n_2, however, coincide in position, so that really there are only three images. The object *P* and the three images stand at the corners of a rectangle having its center at *A*.

374. Spherical Mirrors. A small portion of a spherical surface capable of reflecting light is called a *spherical mirror.* The mirror is termed *concave* or *convex,* according as the inner or the outer surface is the reflecting surface. Let *AB* (Fig. 309) be a section of a spherical mirror, made by a plane normal to the surface. The center *O* of the surface is called the *center* of the mirror, and the middle point *M* is called its *vertex.* The straight line *OM* is called the *principal axis,* and any other line through *O,* as *DB,* is called a *secondary axis.* An axis, whether principal or secondary, is normal to the surface of the mirror. The angle *AOB* is called the *aperture* of the mirror; in practice it never exceeds 10°.

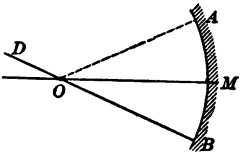


FIG. 309.

375. Principal Focus. A point from which rays of light diverge, or towards which they converge, is called a *focus.* When rays parallel to the principal axis fall on a concave mirror *of small aperture,* they are reflected very nearly to a point *halfway* between the center and the vertex of the mirror; this point *F* (Fig. 310) is called the *principal focus* of the mirror. The rays of the sun, for example, are reflected to the point *F* in such quantities that a match placed there is ignited, and gunpowder is exploded. Half the radius of the mirror is called its *focal length.*

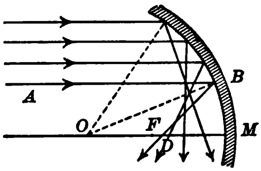


FIG. 310.

By the law of reflection the ray *AB* will be reflected along a path *BD* so that the angles *ABO* and *OBD* will be equal. Now, *ABO* = *BOM*. Therefore, *OBD* = *BOM*. Hence, *OD* = *BD*. If the aperture of the mirror is small, *DM* will be very nearly equal to *BD*, and therefore to *OD*. Therefore, *D* will be sensibly halfway between the center *O* of the mirror and its vertex *M*.

376. Real and Virtual Foci. When rays parallel to the principal axis fall on a convex mirror, they are reflected as if they diverged from the principal focus, which is now behind the mirror (Fig. 311). In a concave mirror the rays after reflection actually pass through the focus (Fig. 310); in a convex mirror they seem to diverge from the focus, but do not actually diverge from it. This distinction is expressed by calling the focus of the concave mirror a *real* focus, and the focus of the convex mirror a *virtual* focus.

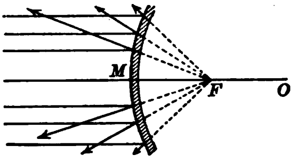


FIG. 311.

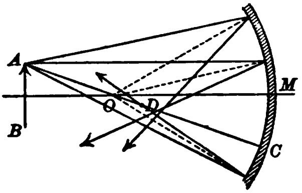


FIG. 312.

The same distinction is made between assemblages of foci or images. Real images can be thrown upon screens; virtual images can only be seen by looking into the mirror.

377. Spherical Aberration. When rays proceed from a point A in front of a concave mirror (Fig. 312), and fall on the mirror, they are reflected so that they cross the secondary axis AOC at points very near each other. This failure of the rays to meet accurately at a point is called *spherical aberration.* The consequence is that, although the mirror forms an image of an object AB placed in front of it, the image is indistinct, especially in the outer parts. The smaller the aperture of the mirror, the less marked will be the indistinctness of the image. In what follows we shall pay no attention to spherical aberration, but assume that all the rays from a point A are reflected to one definite point in the axis AOC. This point is called the *conjugate focus* of A.

378. The Mirror Formula. Let *P* (Fig. 313) be a luminous point on the principal axis of a concave mirror, and *PR* a ray from *P* meeting the mirror at *R*.

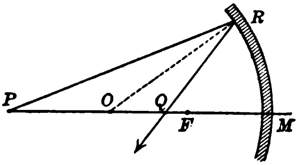


FIG. 313.

Join *OR* and draw *RQ*, making $ORQ = ORP$. Then the intersection *Q* of *RQ* and *MP* is the conjugate focus of *P*. Since the angle *PRQ* is bisected by *RO*,

$$PR : QR = PO : QO.$$

Let $MF = f$, $MP = u$, $MQ = v$. If the aperture is small, $PR = u$, and $QR = v$, very nearly. Also $PO = u - 2f$, $QO = 2f - v$. Substituting these values in the above proportion, we have

$$u : v = u - 2f : 2f - v,$$
$$2uf - uv = uv - 2vf,$$
$$vf + uf = uv.$$

Dividing by *ufv*, $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

By means of this formula any one of the three quantities *u*, *v*, and *f* can be found when the other two are given. If *u* and *v* have unlike signs, the meaning is that the object and the image are on opposite sides of the mirror.

It is evident, both from the formula and from Fig. 313, that if *Q* were the luminous point its image would be at *P*; for this reason *P* and *Q* are called *conjugate foci*. In other words, the *object and the image are interchangeable.*

We also see from the formula that when *u* diminishes, *v* must increase, and *vice versa*, since *f* remains constant; that is, *object and image move in opposite directions.*

The same fact is apparent from Fig. 313. As *P* moves towards the mirror the angle of incidence *PRO* of any ray *PR* diminishes. Therefore, the angle of reflection *ORQ* diminishes; therefore, *Q* moves away from the mirror.

379. Construction of Images. The chief characteristics of the images formed by spherical mirrors are most readily seen if the images are constructed for different positions of the object by a method which we proceed to explain.

We assume that all the rays proceeding from any point of the object and falling on the mirror are reflected to a definite point in the secondary axis which passes through that point. All we have to do, therefore, is to find where *two* reflected rays meet; for where they meet all the others also meet, and form the corresponding point of the image.

The two rays which can be most easily drawn after reflection are the ray along the secondary axis and the ray parallel to the principal axis. The former returns upon itself; the latter is reflected through the principal focus of the mirror.

The most important cases are the following:

Case 1. Concave mirror (Fig. 314). Object *AB* placed beyond the center of the mirror.

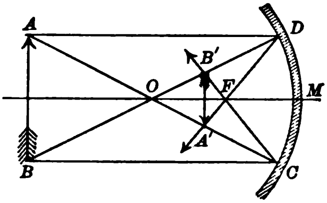


FIG. 314.

Draw the secondary axis *AOC*. Draw *AD* parallel to the principal axis *OM*. Draw a straight line through *D* and the principal focus *F*, meeting *AC* at *A′*. *A′* is the conjugate focus of *A*. *B′*, the conjugate focus of *B*, is found in the same way. Join *A′B′*. *A′B′* is the image of *AB*. The image is *inverted*, *real*, and *smaller* than the object.

If *A′B′* were the object, a precisely similar construction would give *AB* as its image; in this case the image would be inverted, real, and magnified.

In Fig. 314 the image *A′ B′* has been drawn *straight* like the object *AB*. Really *A′ B′* is curved with the convex side of the curve towards the mirror, as may be proved by experiment or by carefully constructing the images of a number of points.

Case 2. Concave mirror (Fig. 315). Object *AB* placed between the mirror and the principal focus.

In this case the ray *CO*, reflected along the secondary axis of *A*, and the ray *DF*, reflected through the focus *F*, do not meet in front of the mirror; but the straight lines *OC* and *FD*, if produced behind the mirror, meet at *A′*. This point is the conjugate focus of *A*. Similarly, *B′* is found. *A′B′* is the image of *AB*; it is erect, virtual, and magnified.

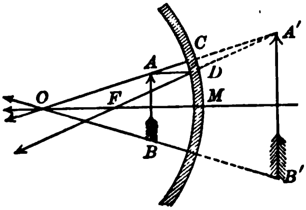


FIG. 315.

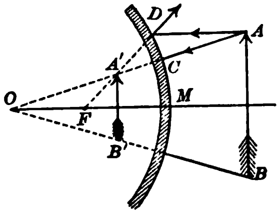


FIG. 316.

Case 3. Convex mirror (Fig. 316). Object *AB* placed anywhere on the principal axis.

The same method of construction gives an image *A′B′*, which is erect, virtual, and smaller than the object. The images in convex mirrors are always of this kind.

Magnifying power. Let *A′B′* (Fig. 317) be the magnified image of an object *AB* placed between the center and focus of a concave mirror. The ratio of *A′B′* to *AB* is the linear *magnifying power* of the mirror.

Since the triangles *A′OB′*, *AOB* are similar,

$$\frac{A'B'}{AB}=\frac{OA'}{OA}=\frac{\text{distance of image from } O}{\text{distance of object from } O}.$$

Join the center *M* of the mirror to *B*.

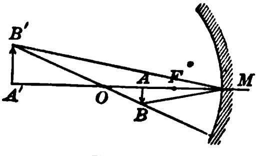


FIG. 317.

Since *B′* is the image of *B*, the incident ray *BM* is reflected along *MB′*. Therefore, the angles at *M* are equal, and the triangles *AMB*, *A′MB′* are similar. Therefore,

$$\frac{A'B'}{AB}=\frac{A'M}{AM}=\frac{\text{distance of image from mirror}}{\text{distance of object from mirror}}.$$

380. Applications. In the *ophthalmoscope*, a small concave mirror is placed so as to reflect the light of a lamp into the eye of the patient. The observer looks into the eye through a small hole made in the center of the mirror.

The *laryngoscope* consists of two mirrors. A concave mirror, strapped to the observer's forehead, reflects light from a lamp upon a small plane mirror held at the back of the patient's mouth. This light is reflected by the plane mirror so that it illuminates the throat. The observer sees the condition of the throat by light returned from the throat to the plane mirror, and thence to the observer's eye.

LABORATORY EXERCISES.

1. Verify the mirror formula by experiments with a concave mirror.

2. Verify the statements about images in § 379, as far as you can, by bringing your own face nearer and nearer to a concave mirror.

CLASS-ROOM EXERCISES.

1. Show by a figure how you would place a plane mirror so as to see a candle flame placed behind an opaque object.

2. A man 6 feet high sees his image in a plane mirror hung on a wall. The top of the mirror is 6 feet from the floor. Find its least length to enable the man to see his whole image in it.

3. Two parallel mirrors are 2 feet apart, and a luminous point is midway between them. Show by a diagram on a scale of $\frac{1}{12}$ the position of the first three images formed in each mirror.

4. A plane mirror is inclined to the floor of a room at an angle of 45°. Prove that the image of a man standing before it is horizontal.

5. An object is placed 15 inches from a concave mirror whose radius is 12 inches. Find the position and nature of the image.

6. An arrow 4 in. long is placed 4 in. from a concave mirror whose radius is 12 in. Find the position, nature, and magnitude of the image.

7. The image of a candle flame placed 10 inches from a concave mirror is most distinctly outlined on a screen 30 inches from the mirror. Find the focal length of the mirror.

8. Trace the changes in the position and nature of the image of a luminous point, as the point is moved from a great distance up to a concave mirror along its axis.

Refraction of Light.

381. Laws of Refraction. When rays of light pass from one medium to another, they are bent or *refracted* at the surface of separation. The change of direction is made visible by allowing a beam of sunlight to enter a darkened, dusty room, and fall on the surface of water containing a little milk. A ray *AO* (Fig. 318), in passing obliquely from air into water, is bent at the point *O* *towards* the normal *ROS* drawn to the surface of the water *MN*; in other words, the *angle of refraction* *BOS* which the refracted ray *OB* makes with the normal is smaller than the angle of incidence *AOR*. Conversely, a ray of light *BO*, which passes through the water to *O*, will take the course *OA* after it enters the air.

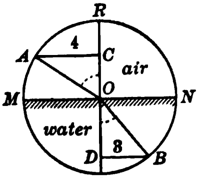


FIG. 318.

Describe a circle about *O* as center, meeting *OA* at *A* and *OB* at *B*. Draw *AC* and *BD* perpendicular to the normal *RS*. The ratio of *AC* to *BD* is found to have a constant value for all values of the angle of incidence *AOR*. This constant value is called the *index of refraction* from air into water. Conversely, the constant ratio of *BD* to *AC* is the index of refraction from water into air.

The laws of refraction, found by experiment, are the following:

1. *The angles of incidence and refraction lie in one plane.*
2. *The angle of refraction is smaller or larger than the angle of incidence, according as the light passes from a rarer to a denser medium, or the reverse.*
3. *The index of refraction has a constant value for the same two media.*

The index of refraction from air to water is $\frac{4}{3}$ (nearly); from air to crown glass $\frac{3}{2}$; from air to flint glass $\frac{8}{5}$; from water to air $\frac{3}{4}$.

Construction of a refracted ray. Let AO (Fig. 319) be a ray in air, meeting the surface of water at O. Through O draw the normal ROS. With center O and radius OA describe a circle. Upon AO produced take OD so that $\frac{OD}{OA} = \frac{3}{4}$, the index of refraction from water into air. Draw DB parallel to OS. Join OB; OB is the refracted ray.

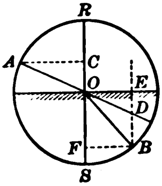


FIG. 319.

The learner should prove this construction to be correct by the help of the lines drawn in Fig. 319.

382. Some Effects of Refraction. Place a coin in a vessel with opaque sides. Move the vessel away from the eye till the coin ceases to be visible. Pour water into the vessel. The coin is again seen. The rays issuing from the point A (Fig. 320) are bent on leaving the water, so that some of them enter the eye as if they diverged from a point A' nearer to the surface. Similarly, the whole coin is apparently raised, and thus becomes visible. For the same reason the water in a pond appears shallower than it really is, and a stick partly immersed in water appears bent where it enters the water.

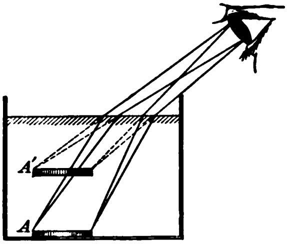


FIG. 320.

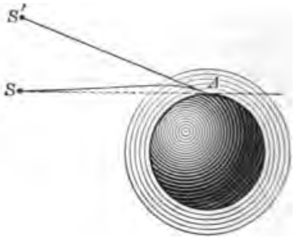


FIG. 321.

A star S (Fig. 321), when near the horizon, appears higher in the sky than it really is. Its light, before reaching the eye, is repeatedly refracted, because it has to pass through layers of air which differ in density. Consequently, the observer at A sees the star apparently raised to the position S'.

383. Total Reflection. If rays of light from every possible direction converge to a point *O* (Fig. 322) on the surface of water, they will all enter the water, and be condensed by refraction to a pencil *AOB*, the extreme rays of which, *OA* and *OB*, correspond to an angle of incidence equal to 90°. Conversely, if the rays of the pencil *AOB* proceed through the water to the point *O*, they will be scattered by refraction into the air, so that the extreme rays *OA* and *OB* will, after issuing into the air, just graze the surface of the water. What will happen to a ray *PO* which proceeds through the water from *P* to *O* at a greater angle of incidence than *AOS*? *It will not enter the air*, but be reflected back into the water to the point *Q*, such that the angles *QOS* and *POS* will be equal. The reflection is so perfect that the phenomenon is called the *total reflection* of light.

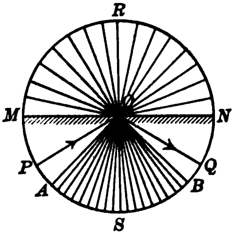


FIG. 322.

The angle *AOS*, which must not be exceeded if the ray is to pass out into the air, is called the *critical* angle; its value for water is about 49°, and for crown glass about 42°.

Examples. If a glass prism is properly held near a lighted lamp, one of its three faces will look like polished silver, and we can see in it a very bright image of the flame.

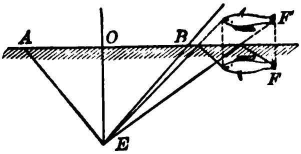


FIG. 323.

If a glass containing water and a spoon is held above the eye, a very bright image of the immersed part of the spoon is seen by total reflection.

To an eye *E* placed under water, all objects above the water will appear crowded into a cone whose vertex is *E* and whose vertical angle *AEB* (Fig. 323) is about 98°. Beyond the points *A* and *B* the water will act like a plane mirror, and images of objects in the water will be seen by total reflection. An image of the fish *F*, for example, will be seen at *F′*.

384. Refraction by a Glass Plate. When a ray of light *AB* (Fig. 324) passes through a glass plate with parallel faces (a window pane, for example) it is first refracted at the point *B* *towards* the normal at that point; and then, after traversing the glass, it is refracted at the point *C* *away from* the normal at *C* by the same angular amount, since the angle of incidence at *C* is equal to the angle of refraction at *B*. The consequence is that when we look at an object through such a plate we see it slightly displaced in position, but otherwise unchanged. The change in position for thin plates is too slight to attract attention.

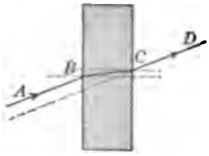


FIG. 324.

385. Refraction by a Prism. Let a ray of light *AB* fall obliquely on one of the faces of a triangular glass prism, as shown in Fig. 325. On entering the glass at *B* it is refracted towards the normal; on emerging into the air it is refracted away from the normal. The total change of direction is the sum of the two partial changes, and is equal to the angle *DOE*. This angle is called the *angle of deviation* of the ray. The angle *MRN* is called the *refracting angle* of the prism; usually it is either 60° or 90°.

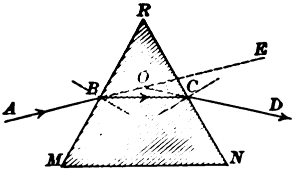


FIG. 325.

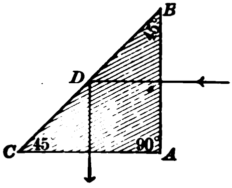


FIG. 326.

When light is allowed to enter the face *AB* of a right isosceles prism, as shown in Fig. 326, it suffers total reflection at the face *BC* (why ?), and issues from the face *AC* *perpendicular* to its original direction. This is the best way to change the direction of light by a right angle.

386. Refraction by a Lens. A *lens* is a small portion of a transparent substance, usually glass, with spherical surfaces. A straight line drawn through the centers of the two spherical surfaces is called the *principal axis* of the lens. Lenses are divided into two classes : *converging* and *diverging*. Converging lenses are thickest at the axis ; the *double convex* lens (Fig. 327) is a type of this class. Diverging lenses are thickest at the edges ; the *double concave* lens (Fig. 328) is a type of this class. When a ray of light passes through a lens, it is always bent by refraction towards the thickest part of the lens.

If a converging lens is held so that the rays of the sun fall on it parallel to the principal axis, they are refracted very nearly to a point *F* on the axis (Fig. 327). A bright image of the sun is seen when a piece of paper is held at *F*, and the paper is quickly charred or burned.

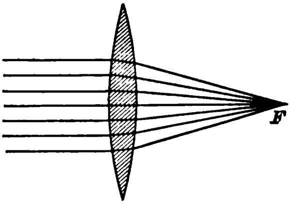


FIG. 327.

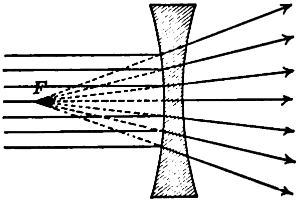


FIG. 328.

If we allow sunlight to fall on a diverging lens, the rays after refraction will diverge as if they issued from a point *F* on the same side of the lens as the sun (Fig. 328).

The focus *F* of rays parallel to the principal axis is called the *principal focus* of the lens ; it is real in Fig. 327, and virtual in Fig. 328. The distance from *F* to the lens is called the *focal length* of the lens.

An incident ray and the corresponding refracted ray are *reversible ;* rays coming from *F* in Fig. 327, and going towards *F* in Fig. 328, are made parallel by refraction.

387. Conjugate Foci. Let P (Fig. 329) be a luminous point on the principal axis of a converging lens beyond the principal focus F. The rays from P which meet the lens are brought by refraction nearly to a point Q of the axis beyond the principal focus on the other side of the lens. Conversely, if Q were the source of light, its focus would be the point P; hence, P and Q are called *conjugate foci.*

Whatever be the position of P on the axis, the angle PAQ formed by any ray before and after refraction remains nearly constant in value. If, therefore, P moves towards the right, Q will also move towards the right. If P moves up to F, Q will move away so far that the refracted rays will be sensibly parallel. If P is between F and the lens, the rays from P will diverge after refraction from a *virtual* focus Q on the same side of the lens as the source P itself (Fig. 330).

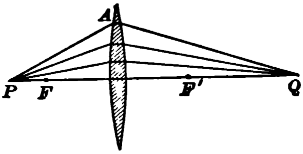


FIG. 329.

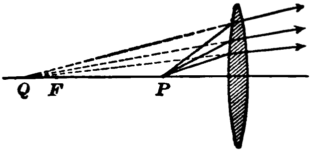


FIG. 330.

In lenses of every kind the rule is that *object and image move in the same direction.* If the lens is a diverging lens, and the source of light P is on the axis at a great distance from the lens, the conjugate focus Q will be sensibly at the principal focus F on the same side of the lens (Fig. 328). As P moves up to the lens, Q will move much more slowly from F up to the lens. Whatever be the position of P, the conjugate focus Q is a virtual focus (Fig. 331).

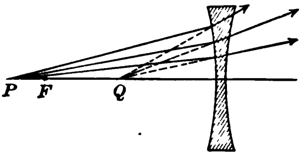


FIG. 331.

388. Optical Center. There is a point O (Fig. 332) on the principal axis of a lens, either at or near its center of volume, so situated that rays of light which pass through this point and the lens *suffer no change of direction.* This point is called the *optical center* of the lens. If the surfaces of the lens are equal in curvature, the optical center lies midway between them.

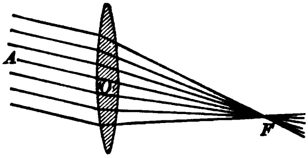


FIG. 332.

Any straight line AOF through the optical center, other than the principal axis, is called a *secondary axis.*

Rays parallel to a secondary axis have for their focus a point on that axis; and the conjugate focus Q of a luminous point P situated near the principal axis is on the secondary axis drawn through P.

389. The Lens Formula. It can be proved that the formula for mirrors given in § 378,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

may be applied also to a *converging* lens of small thickness and curvature; u denoting the distance of the object from the lens, v the distance of its image, f the focal length of the lens.

If the image is virtual, v is negative; and if in computing the value of v from given values of u and f we obtain a negative value, the image is virtual and on the same side of the lens as the object.

For a *diverging* lens the image is always virtual, v is always less than u, and the formula should be written

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

390. Images Formed by a Lens. The images formed by lenses may be constructed in the same way as those formed by spherical mirrors (§ 379). To find the conjugate focus of any point of the object, we draw the secondary axis on which the point is situated, and then find where it is cut after refraction either by the ray parallel to the principal axis or by the ray which passes through the principal focus.

The chief cases are given in Figs. 333–335, and should be carefully studied. The thickness of the lens is disregarded.

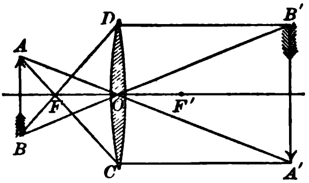


FIG. 333.

In Fig. 333 the object is the arrow AB placed beyond the principal focus of a convex lens. The image $A'B'$ is inverted, real, and in this case larger than the object.

In Fig. 334 the arrow AB is placed between the focus F and the lens. The image $A'B'$ is erect, virtual, and magnified; in order to see it we must look through the lens.

In Fig. 335, the arrow AB is placed in front of a concave lens. The image $A'B'$ is erect, virtual, and diminished.

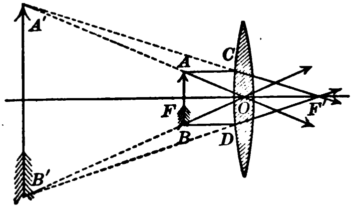


FIG. 334.

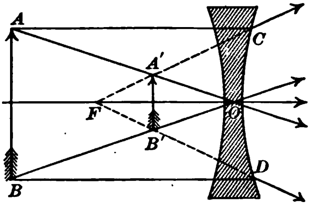


FIG. 335.

In every case the following proportion holds true:

$$\frac{\textit{size of image}}{\textit{size of object}} = \frac{\textit{distance of image from lens}}{\textit{distance of object from lens}}.$$

391. Spherical Aberration of Lenses. In the constructions of § 390 it is assumed that all the rays proceeding from a luminous point to a lens are accurately refracted to one point. But this assumption is not true. The rays that fall on the outer portions of a lens are refracted more strongly than the rays that fall near the optical center, and therefore after refraction they cross the principal axis at points nearer to the lens. This want of exact concurrence is called *spherical aberration.*

The effects of spherical aberration are:

(1) To make all images more or less indistinct.

(2) To make them differ in shape from the object.

Thus, in Fig. 333 the image $A'B'$ of the arrow AB is not really straight, as shown in the figure, but curved so as to appear concave as seen from the lens. In Fig. 334 the image $A'B'$ is curved so as to appear convex as seen from the lens.

The distinctness of an image may be improved by cutting off the outer rays from the lens by means of a *diaphragm* or *stop;* but at the same time the brightness is diminished.

In large telescopic lenses the outer portions are ground so that their curvature, and therefore their refracting power, is diminished by the proper amount to insure distinct images.

LABORATORY EXERCISES.

1. Allow a beam of sunlight to fall on a glass prism so as to have total reflection from one of its faces. Then draw carefully a diagram representing the path of the rays through the prism.

2. Find the focal length of a convex lens (1) by throwing an image of the sun or some distant object on a screen; (2) by the "pin" method.

3. Verify the formula given for a convex lens in § 389.

4. Construct by points the real image of a straight arrow formed by a convex lens.

5. Construct by points the virtual image of a straight arrow formed by a convex lens.

CLASS-ROOM EXERCISES.

1. Construct to scale the path of a ray of light through a thick plate of crown glass. Index of refraction, $\frac{3}{2}$.

2. Show by a diagram how a straight stick held obliquely and immersed partly in water appears to a person in the air, and trace a pencil of rays from the lower end of the stick to the eye.

3. The angles of a glass prism are 90°, 70°, and 20°. A ray of light enters the prism normally at the face bounded by the angles 90° and 70°. The critical angle for the glass is 41°. Prove that the ray will suffer two internal reflections before it leaves the prism.

4. The focal length of a convex lens is 20 cm. Find the positions of the images of a small object if the distances of the object from the lens are 20 meters, 40 cm., 15 cm., and 5 cm., respectively. Are the images real or virtual?

5. If the distance of an object from a convex lens is twice the focal length of the lens, prove that the image is at the same distance on the other side of the lens.

6. An object is 60 cm. from a lens on one side and the image is 15 cm. on the other side. What is the focal length?

7. An object is 12 cm. from a lens, and its image is 72 cm. from the lens on the same side. Is the lens convex or concave? What is its focal length? Illustrate by a diagram.

8. An object is 36 cm. from a lens, and its image is 3 cm. from the lens on the same side. Is the lens convex or concave? What is its focal length? Illustrate by a diagram.

9. A small object moves along the axis of a concave lens towards the lens. Trace the changes in the position and size of the image. Is the image real or virtual?

10. A candle is 8 feet from a wall. When a lens is held 2 feet from the candle a distinct image is thrown on the wall. Find the focal length of the lens. Compare the image and the object in respect to size.

11. At what distance from a convex lens must an object be placed in order that the image may be half as large as the object? Focal length of lens = 30 cm.

12. A convex lens of 6 inches focal length is used to read the graduations on a scale. How far must it be held from the scale to magnify them three times?

13. An arrow 5 cm. long placed 10 cm. from a lens at right angles to the axis has a virtual image 30 cm. from the lens. If the arrow were placed 30 cm. from the lens, how far from the lens would the image be?

Dispersion of Light.

392. Analysis of Light. We have thus far assumed that a ray of white light, when refracted, suffers merely a change of direction. But the ray is also separated into an indefinite number of rays, all differing in refrangibility and also in color. This phenomenon is called the *dispersion* of light, and it was first investigated with success by Newton (1672).

Newton's fundamental experiment is illustrated in Fig. 336. He allowed sunlight to pass through a small hole *A* into a dark room and form an image *B* of the hole on a screen. Then he held a glass prism in the path of the rays as shown in the figure. He found that the image *B* was not only raised by refraction to a higher part of the screen, but expanded to a colored band *RV* of light about five times as long as it was broad. This colored band is called the *solar spectrum.* Newton distinguished in it *seven* primary colors: red, orange, yellow, green, blue, indigo, and violet. The red rays are the least refracted; the violet rays are the most refracted. The colors change by insensible gradations from the extreme red to the extreme violet.

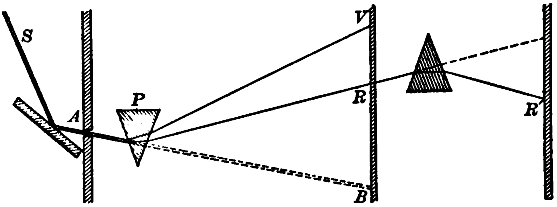


FIG. 336.

If a red ray is allowed to pass through a small hole in the screen and traverse another prism *P*, it will be refracted, but will suffer no further dispersion. By turning the prism *P*, this can be shown to be true for all the colored rays.

393. Synthesis of Light. If we interpose in the path of rays of white light two exactly similar prisms reversed in position, as shown in Fig. 337, and hold a screen in the proper position, we obtain on the screen simply a spot of white light. The second prism cancels the dispersion produced by the first prism, by producing an equal amount of dispersion in the opposite direction. The two prisms together have the same effect as a thick plate of glass with parallel faces.

If the spectrum produced by a prism be allowed to fall on a large convex lens, the rays which suffer dispersion at the point *P* of the prism will be brought together again at *Q*, the conjugate focus of *P* (Fig. 338). If a screen be placed at *Q*, we see a spot of white light. If we allow only a portion of the spectrum to fall on the lens, the image at *Q* will assume various shades of color, due to the union of the differently colored rays that pass through the lens.

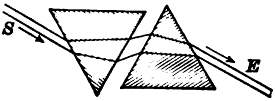


FIG. 337.

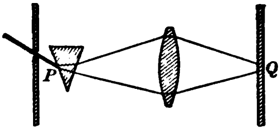


FIG. 338.

394. Composite Nature of White Light. The experiments mentioned in the last two sections prove that *white light is composed of innumerable constituent parts, each part having a distinctive color and a definite degree of refrangibility.*

A transparent prism decomposes white light by refraction into its component parts. The red components are refracted the least, the orange components are refracted more than the red ones, the yellow ones still more, and finally the violet components are refracted most of all. By combining together all these different components white light is again produced.

395. The Rainbow. In the *rainbow* Nature shows us a solar spectrum on a most magnificent scale. In order to see a rainbow we must look at falling rain with the sun behind us, and within about 42° of the horizon. Very often two bows are seen: the *primary* bow and the *secondary* bow. The primary bow is red on the outside and violet on the inside. The secondary bow is larger, but fainter, and the order of colors is reversed. Both bows are arcs of circles having a common center on the line which passes through the sun and the eye of the observer. The inner radius of the primary bow is about 40°, and its width is 2° 17′. The inner radius of the secondary bow is about 50° 40′, and its width 3° 40′.

A general idea of the formation of the rainbow may be obtained from Fig. 339. Sunlight will enter a spherical drop of rain at all angles of incidence, and will suffer refraction and also dispersion both on entering and on emerging from the drop. Most of the colored rays on emerging are so divergent in direction that they fail to produce any impression of color on the retina of the eye. But for certain angles of incidence the rays of each component color will emerge in nearly parallel directions; in this case they act upon the eye with force enough to cause the sensation of color. Let *SA* (Fig. 339) represent incident rays of the kind just mentioned. They are twice refracted by the drop *A*, and suffer total reflection at the back surface of the drop. The *red* components emerge so that they enter the eye at *E*; but the *violet* components pass above the eye. Every drop situated at the same angular distance as *A* from the line *EO* will have precisely the same effect; hence, the eye *E* sees a red circular arc in the sky. Below *A* there are drops such as *B* that send violet rays to *E*, and between *A* and *B* the other colors will be seen.

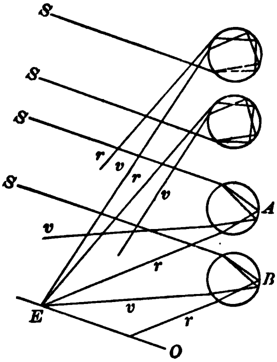


FIG. 339.

The secondary bow is produced by light which has suffered *two* internal reflections as well as two refractions.

396. Chromatic Aberration. If we allow sunlight to pass through a small round hole into a dark room, and project an image of the hole upon a screen by means of a convex lens, we observe that the image is surrounded by a colored fringe. This phenomenon is called *chromatic aberration.*

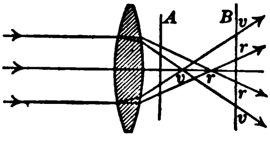


FIG. 340.

The violet rays, being more refrangible than the red rays, are brought to a focus nearer to the lens (Fig. 340). The consequence is that the lens really forms a series of images, beginning with a violet image and ending with a red one. When we hold a screen in this series of images a composite blurred image is seen on the screen, white in the central parts and colored around the border. If the screen is held at *A*, the border is red; if it is held at *B*, the border is blue or violet.

Chromatic aberration interferes very seriously with the formation of good images by lenses, and for a long time was thought to be an incurable defect. But Dolland (1757) discovered a very effectual remedy. He took advantage of the fact that the dispersive power of flint glass is about twice as great as that of crown glass, while their refracting powers are nearly the same. He combined a convex lens of crown glass with a concave lens of flint glass, having such a shape that its dispersive power was equal to that of the convex lens, but exerted in the opposite direction (Fig. 341). In this way chromatic aberration is mostly prevented, while the refracting power of the convex lens is merely weakened to a certain extent by the presence of the concave lens. Such a combination is called an *achromatic* lens. The object glasses of telescopes and microscopes are always achromatic lenses. The eye pieces consist of two lenses of the same kind of glass so arranged as to counteract both spherical and chromatic aberration.

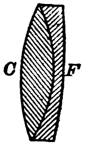


FIG. 341.

397. Pure Spectrum. A spectrum obtained as described in § 392 is composed of overlapping images of different colors. A spectrum in which this overlapping of images is prevented, and each color obtained free from all admixture with the others, is called a *pure* spectrum. To obtain a pure spectrum, we must use a *fine line* for the object, and take care to have all the rays well focused on the screen.

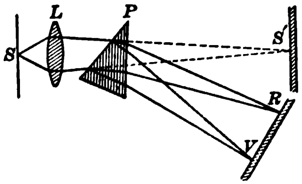


FIG. 342.

One method is shown in Fig. 342. The light from a narrow slit *S* passes first through a convex lens *L*, so placed that it will throw a sharp image of *S* upon a screen at *S'*. A glass prism *P* is then held between the lens and the screen, and turned till the deviation of the rays is a minimum. If we now hold the screen normal to the refracted rays and at the same distance from the lens as before, a pure spectrum *RV* will be seen on the screen.

398. The Spectroscope. For the purpose of studying spectra an instrument called a *spectroscope* is employed. Its chief parts are outlined in Fig. 343. The rays of light from the source are made parallel by passing through a convex lens in the *collimator*, whence they pass through the *prism* into the *telescope*, through which each color in succession can be distinctly seen by slowly rotating the prism. A *scale-tube*, containing a lens and a graduated scale illuminated by a lamp, is so placed that parallel rays from the scale are reflected from the face of the prism into the tube of the telescope. Thus the image of the scale and the image of the spectrum are seen at the same time by the observer.

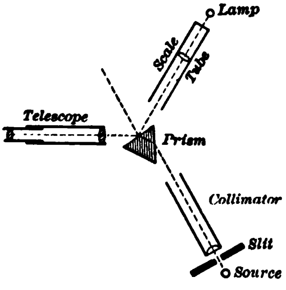


FIG. 343.

399. Classes of Spectra. The examination of different spectra has led to their division into three classes.

(1) *Continuous spectra.* In these there is present light of every degree of refrangibility and every shade of color, from the extreme red at one end to the extreme violet at the other end. Continuous spectra are obtained from the electric light, the lime light, white-hot platinum, and, in general, from *all white-hot solids or liquids.* Even gases under great pressure yield continuous spectra. The flames of lamps and candles give continuous spectra, because they owe their brightness to particles of carbon floating in them.

(2) *Discontinuous, or bright-line spectra.* These consist of a number of bright lines on a dark background. The spectra of *incandescent gases and vapors* at all ordinary pressures are of this kind.

The spectrum of sodium vapor may be obtained by holding in a Bunsen flame before the slit of the spectroscope a platinum wire previously dipped in a solution of sodium carbonate. It consists of a pair of *bright yellow* lines called the *D* lines, so near together that they are often treated as one line. Lithium gives a splendid red line; hydrogen gives three lines, one red, another greenish blue, and the other dark blue.

(3) *Absorption, or dark-line spectra.* In these the continuity of the spectrum is interrupted by a number of fine dark lines.

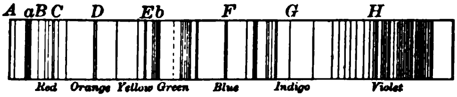


FIG. 344.

The most important example is the solar spectrum. It contains many dark lines fixed in position, called the *Fraunhofer lines.* The most important of these lines are designated by letters, as shown in Fig. 344. The spectra of the moon, the planets, and the fixed stars also contain dark lines.

400. Principle of Reversal. If we place a flame, colored with sodium vapor, before the slit of the spectroscope, and examine the spectrum, we see the bright lines *D* (Fig. 345, I). Let now an electric or lime light be so placed that its rays can traverse the sodium vapor and then enter the spectroscope. Instantly the bright sodium line vanishes, and in its exact position there appear the *dark* lines *D* (Fig. 345, II), flanked on either side by a continuous spectrum of color. In the same way the bright lines characteristic of other incandescent vapors may be changed to dark lines or *reversed.* These phenomena are perfectly explained by the principle of reversal, which may be thus stated:

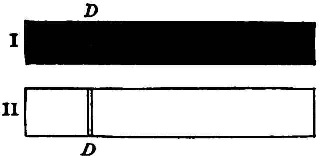


FIG. 345.

A gas or vapor absorbs exactly those kinds of rays which it emits when incandescent, and allows all other rays to pass through it with undiminished intensity.

The sodium vapor, when incandescent, emits only those rays which by refraction are brought to the position *D* in the spectrum. When the powerful lime light, containing rays of all degrees of refrangibility, passes through the sodium vapor, most of the rays which would be refracted to the position *D* are absorbed, while all the other rays are freely transmitted. Hence dark lines are seen at *D.*

401. Explanation of the Fraunhofer Lines. Many of the dark Fraunhofer lines in the solar spectrum coincide exactly in position with the bright lines of certain incandescent vapors. Thus, the dark lines *D* have the same position as the bright yellow lines of sodium vapor. The spectrum of iron vapor contains more than 400 bright lines, and every one has its dark counterpart in the solar spectrum. The following explanation, first given by Kirchhoff, is based on the principle of reversal.

The glowing white-hot surface of the sun or *photosphere*, whether the mass of the sun be a solid, a liquid, or a very dense gas, emits white light, and would by itself give a continuous spectrum. Outside the photosphere is an atmosphere of glowing vapors, called the *chromosphere*, lower in temperature than the photosphere, but still hot enough to maintain metals in the state of vapor. The light of the photosphere, before it can reach the earth, must traverse the chromosphere and be subjected to the absorbing action of the cooler vapors contained therein. To this action the Fraunhofer lines owe their origin. The sodium rays, for example, are mostly absorbed in passing through the chromosphere; hence the dark line *D* appears in the solar spectrum.

This explanation is universally accepted as the true one, and as demonstrating the existence in the sun of sodium, hydrogen, iron, and numerous other elements.

402. Spectrum Analysis. The position of the bright lines in the spectrum of a vapor or gas depends on the nature of the gas, and not on its temperature; moreover, no two vapors or gases give the same set of bright lines. On these facts is based a new method of chemical analysis known as *spectrum analysis*. This method far excels any other in point of delicacy. A millionth part of a milligram of sodium carbonate, a speck of matter so small that it cannot be seen without the aid of a powerful microscope, is yet capable of giving a bright yellow line in the spectroscope.

With the aid of the spectroscope several new elements have been discovered, and the presence of certain elements in the fixed stars has been revealed by the appearance of their absorption spectra.

The practical applications of the spectroscope are numerous and important. Adulterations in food, for example, can be easily found by examining their absorption spectra, and traces of poisoning matter in the blood so minute as to elude other tests can be surely detected.

Optical Instruments.

403. The Human Eye. The *eye* (Fig. 346) is divided into two chambers by the *crystalline lens*, *l*, a double convex lens composed of layers increasing in density inwards. The front chamber is filled with a watery liquid called the *aqueous humor*, and the back chamber is filled with a transparent, jelly-like substance, called the *vitreous humor*. In front of the crystalline lens is the *iris*, *i*, an annular diaphragm which allows light to reach the crystalline lens by passing through the *pupil*, *p*, and which has the power to contract and expand automatically, according to the brightness of the light. Over the surface of the rear chamber is spread the *retina*, *r*, a membrane containing a highly complex system of nerve filaments, and connected with the *optic nerve*, *n*, the nerve that leads to the brain. Back of the retina is the *choroid coat*, *ch*, so intensely black that it absorbs all light reflected internally. The eyeball is covered by the opaque *sclerotica*, *s*, on the back and sides, and the transparent *cornea*, *c*, in front.

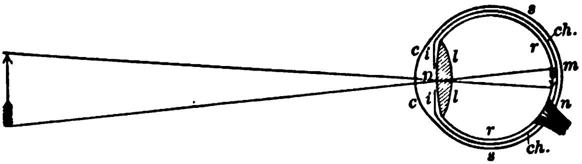


FIG. 346.

The crystalline lens and the two humors act together like a single converging lens, and form a real, inverted, diminished image of any object to which the eye is directed. If this image falls on the retina, the nerve filaments are set in vibration, and the disturbance is conveyed by the optic nerve to the brain, where, in some unknown way, it causes the sensation of sight. The object, if sufficiently luminous, is seen.

404. Accommodation. The distance of an image from a lens varies with the distance of the object. And yet we can see distinctly objects at very different distances from the eye. Certain muscles attached to the crystalline lens act so as to increase the curvature and the refracting power of the lens for near objects, and diminish them for distant objects. This is called the *power of accommodation.*

For most eyes there is no upper limit of distinct vision. For the normal eye in youth the lower limit is 5 to 6 inches (12½ to 15 cm.); but before this limit is reached the exercise of the power of accommodation is attended with a sense of effort or strain. An eye which can see the details of an object most distinctly, without sense of effort, at a distance of 8 to 10 inches (20 to 25 cm.), is considered to be a *normal* eye.

405. Near-sighted and Far-sighted Eyes. A *near-sighted* eye is one which cannot see objects distinctly unless they are at a less distance than 10 inches from the eye. In all other cases the image is formed *in front* of the retina. A *far-sighted* eye is one which cannot see objects distinctly unless their distance from the eye is more than 10 inches. In all other cases the image is formed *behind* the retina, even when the person exerts his power of accommodation as much as he can. Far sight is very common among old people, because the power of the crystalline lens to refract light, as well as the power of accommodation, diminishes as old age approaches.

These defects of vision are remedied by the use of *spectacles.* The glass required for the near-sighted eye is a *concave* lens of suitable power, because such a lens tends to neutralize the refracting power of the crystalline lens, and thus to increase its focal length. The glass required for a far-sighted eye is a *convex* lens of the proper strength. Such a lens reinforces the weak refracting power of the crystalline lens, and thus shortens its focal length.

406. Visual Angle. The angle which a straight line subtends at the optical center of the eye is called the *visual angle* under which it is seen. The greater the visual angle the greater is the *apparent* length of the line. If the angle is small, the *real* length of the line varies directly as its distance from the eye.

A very small object near the eye subtends a larger visual angle than a much larger object very far away; thus a nickel may be held before the eye so that it will completely hide from view the full moon.

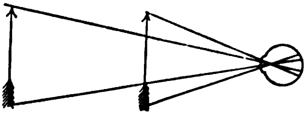


FIG. 347.

If an object moves towards the eye, its visual angle increases, and it appears to be larger than before, although its real size remains the same. This is illustrated in Fig. 347.

407. Field of Distinct Vision. Only a very small space on the retina near the optic axis is capable of giving distinct vision. Hence the field of distinct vision for an eye at rest is exceedingly small. When we wish to examine a small object carefully we point the optic axis straight at it; other objects are indeed visible, but we see them indistinctly. If the reader will look steadily at a word in the middle of some line on this page, he will find it impossible to read the words at the end of the line, or those which stand directly above or below on the third line from the word looked at.

But the eyeball is capable of turning freely in its socket in every possible direction, and by means of this motion we can bring a great number of objects in rapid succession into the field of distinct vision. If our eyes were firmly fixed in our head, we should be obliged to turn the head every time we transferred our gaze from one object to another. By moving the eyeball, and also turning the head, we can enlarge our field of view to more than 180° in every direction.

408. Binocular Vision. Vision with *one* eye is imperfect, since it affords no means of estimating distance, except the sense of strain when we try to accommodate the eye to near objects. If we attempt to estimate distance by one eye only we are liable to curious illusions.

If we lie on our back looking at the sky, and a small insect poises itself just above one eye so that it cannot be seen by the other eye, we imagine that we see a great bird in the sky.

Suspend a ring by a thread so that the thin edge alone is visible to you. Then close one eye and try to pass a pencil through the ring. In all probability several trials will be necessary before you succeed. If both eyes are open, a single trial will be enough.

Vision with *two* eyes greatly increases our power to estimate distance. We are obliged to make the two optic axes converge to the point which we wish to see most distinctly, and the sense of muscular effort required for this purpose is a valuable aid in estimating distances. Moreover, we get two slightly different views of the object, and from these we gain a clear conception of its depth or third dimension in space.

An image of the object is formed on the retina of each eye, but the brain usually combines the two images into one, so that we see the object single. The necessary condition for single vision with two eyes is that the image of each point of the object should fall on *corresponding* points of the retina. Corresponding points are the centers a, a' (Fig 348) of the two retinae, and any pair of points, one on each retina, equidistant from the center, and in the same direction from the center. Hence the point A, directly looked at, is always seen single. The point B will be seen single if $ab = a'b'$. This condition will be satisfied if B lies on a circle passing through A, a, and a'. If B has any other position in the plane of the figure except on the circumference of this circle, it will really be seen double, but we seldom notice the fact because our attention is fixed upon the point A.

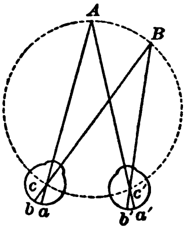


FIG. 348.

409. After-Images. The impression of light on the retina lasts an appreciable time after the light is withdrawn, and the more intense the light is, the longer the impression lasts. The consequence is that bright objects continue to be seen for a very short time after they cease to send light to the eye. The image seen for this brief period is called a *positive after-image.*

If a stick burning at one end is moved rapidly to and fro in the dark, we see a fiery line. The image on the retina produced in any one position of the flame lasts till it is renewed by the flame again coming to the same position. Similarly, a rising rocket leaves behind it a train of light.

If a disc, painted with alternate sectors of white and black (Fig. 349), is made to rotate rapidly, it will appear to have a uniform gray color, due to the superposition of the black and white colors. If the disc is rotated at great speed in the dark, and a flash of lightning occurs, the disc will appear to be at rest, and the black and white sectors will be distinctly seen. The illumination is so brief that there is no time for the superposition of the images.

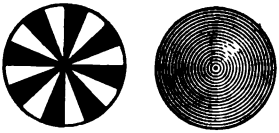


FIG. 349.

If we look directly at the sun and then turn our eyes to the ground or to a dark cloud, we see a round red sun lasting sometimes for several minutes. In this case the light is so intense that the retina is liable to be permanently injured.

There is another kind of after-image, called *negative,* in which the light and dark colors of the object are exactly reversed; what was light will be dark, and what was dark will be light. They are caused by the retina becoming fatigued at the spots where the light has acted strongly.

Look steadily for a minute through a window at the bright sky and then turn the eyes quickly towards a white wall. A dark window will now be seen with a bright frame.

Fix the eyes for some time upon a large ink blot on strongly illuminated, white paper, and then turn them upon a white wall; the wall will appear gray with a white patch on it.

410. Optical Lantern. This instrument (Fig. 350) is used for the purpose of projecting upon a screen a bright enlarged image of a picture or photograph made on glass.

The source of light *A*, with the assistance of a concave mirror *M*, throws a large quantity of light upon the convex lens *L*, called the *condenser*, which concentrates the rays upon the picture or photograph *P*. The picture *P* is placed at a distance from the combination lens *L′* greater than its focal length, but less than twice the focal length. A real, inverted, magnified image is formed upon the screen *S*.

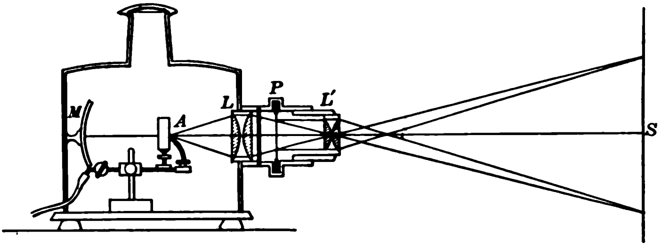


FIG. 350.

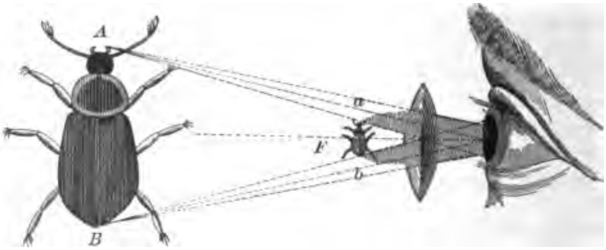


FIG. 351.

411. Simple Microscope. The action of a simple *magnifier* is illustrated in Fig. 351. The object is placed between the lens and its focus *F*. The position of the image is determined as explained in § 390.

412. Compound Microscope. The essential parts of a compound microscope (Fig. 352) are the object glass or *objective*, the *eyepiece*, and the *illuminating apparatus*. The object glass C is an achromatic lens of short focal length; this produces a real inverted, magnified image AB of the object ab; this image is then observed through the eyepiece O, which acts precisely like a simple magnifying glass, so that the observer sees an enlarged virtual image $A'B'$ of the image AB. The object ab is fixed in a glass slide, and illuminated by the light of the sun or a lamp thrown in the proper direction for this purpose by a concave mirror M.

The compound microscope, therefore, is an instrument which produces at the distance of most distinct vision a bright, distinct, and highly magnified image of a very small object.

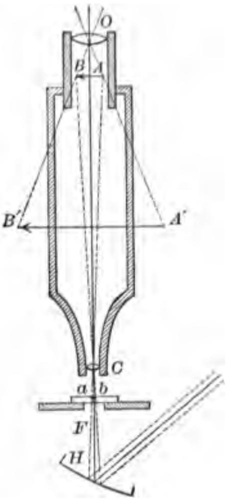


FIG. 352.

The internal reflection of light is prevented by blackening the walls of the tube, and spherical aberration is mostly avoided by the use of diaphragms placed in suitable positions. There are also special arrangements by means of which the observer is enabled to measure the dimensions of the object under examination.

413. Telescope. The *telescope* enables us to see distant objects more clearly and distinctly than with the naked eye. It increases the visual angle under which the object is seen, and concentrates a great quantity of light from the object into the pupil of the eye. It resembles the microscope in having an object glass which produces a real image of the object, and an eyepiece which produces a virtual image. But the objective of a telescope is very large, has a great focal length, and produces a real image much smaller than the object.

414. Astronomical Telescope. In this telescope (Fig. 353) an achromatic object glass *C* produces a real, inverted, and diminished image of the distant object, and this image is then viewed through an eyepiece. The circumstance that the image is inverted is not a very serious disadvantage in the case of stars. In telescopes used for seeing terrestrial objects, or *spyglasses*, a system of two lenses is interposed between the object glass and the eyepiece. These produce an erect image of the image formed by the object glass. Therefore, the image finally seen is erect.

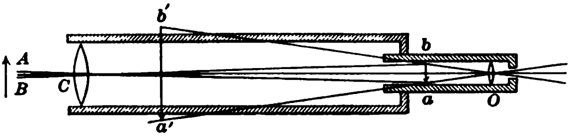


FIG. 353.

415. The Opera Glass. In the *opera glass* the eyepiece is a concave lens (Fig. 354). The object glass tends to make a real image *ab* of the object *AB* beyond the eyepiece; but before the rays meet they are made by the eyepiece to diverge as if they proceeded from points *a'b'* at the distance of most distinct vision. Thus, the observer sees an erect magnified image *a'b'* of the object *AB*.

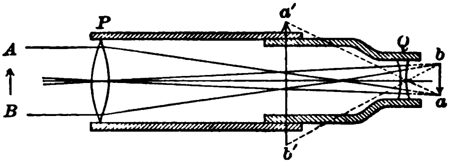


FIG. 354.

The magnifying power of an opera glass is small compared with that of a telescope, but its construction is such that only a short tube instead of a long one is required.

416. Photographer's Camera. A sketch of this instrument is shown in Fig. 355. The tube *AD* contains an achromatic lens. A glass plate coated with a film containing the proper chemicals is inserted in the slide *C*. The slide is connected to the frame supporting the tube *AD* by means of the bellows *B*, so that it can be moved to and from the lens, and thus be properly focused for an object placed in front of the lens.

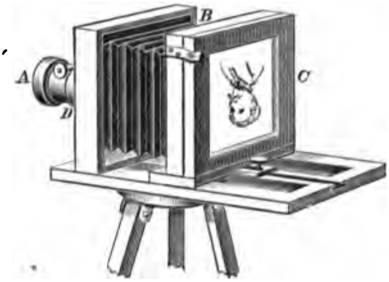


FIG. 355.

The basis of the art of photography is the power of light to blacken certain salts of silver, especially silver chloride and silver iodide. The blackening is the result of a chemical decomposition which takes place. The production of a photograph involves two distinct sets of operations. We begin by covering a glass plate with a collodion film containing silver iodide. The plate is then placed in the slide of the camera, the chamber is darkened, and the plate exposed to light from the object for a very short period of time. By the action of the light an image is formed. This image is then *developed* by the aid of a substance which continues the action begun by the light. Finally, the image is *fixed* by washing it with a solution of sodium hyposulphite, which removes from it all matter sensitive to light.

But this image is *negative;* that is to say, the spots that should be light are dark, and *vice versa.* To obtain a positive picture, the negative is placed upon a sheet of paper covered with a film containing chloride of silver, so that the image is in contact with the paper. The two are then exposed to a good light for some time. The dark parts of the negative shut off the light from the parts of the paper which they cover, so that at these places the paper suffers no change; in all other places the paper is blackened. The result is that a *positive* picture is made on the paper, in which the contrasts of light and shade of the object are faithfully reproduced.

Physical Nature of Light and Color.

417. Theories about Light. According to the *emission* theory, held by Newton, luminous bodies throw off in all directions exceedingly minute, elastic particles, which are capable of traversing transparent bodies, and which produce the sensation of light by impact on the retina of the eye.

According to the *wave* theory, propounded by Huyghens, light is caused by vibratory motion of the *ether*, a perfectly elastic medium of exceedingly small density, which is assumed to fill all space, even the densest solid bodies.

The two theories will explain equally well the law of distances and the laws of reflection and refraction. But there are optical phenomena for which the emission theory has no reasonable explanation to offer. On the other hand, the wave theory not only gives a rational explanation of all known optical phenomena, but also connects them with thermal and electric phenomena, and with the doctrine of energy, in a way which adds greatly to the probability of its truth.

The wave theory holds that *color* depends on vibration frequency, or, what comes to the same thing, on wave length. Corresponding to every frequency, or every wave length, there is a definite shade of color. White light results from the composition of waves of all frequencies from extreme red to extreme violet. The wave frequencies have been measured by experimental methods; that of extreme red is about 400 billions per second and that of extreme violet is about 760 billions per second. The wave length for extreme red is about 0.00077 mm., and the wave length for extreme violet is about 0.00040 mm.

By dispersion white light is decomposed into its component parts. Each component ray is bent; and the shorter the wave length of the ray, the more it is bent.

418. Interference of Light. Every one who has seen soap bubbles has observed their brilliant colors and the rapid changes which they undergo. A small bubble is colorless, and we see in it by reflected light a distorted image of the window. As the bubble grows larger, faint red and green tints begin to appear, and move uneasily about. As the soapy film becomes thinner, other colors of great brilliancy successively make their appearance, and please the eye by their constant motion and change of hue. Finally, pale white and gray tints are seen, a sure sign that the bubble is about to burst.

The emission theory of light is unable to explain these phenomena, but the wave theory shows us that they are simply the effects of the *interference of waves* differing in phase. Just as the interference of sound waves alters or destroys sound, so the interference of luminous waves alters or destroys light.

To simplify matters, suppose that light of a single wave length, sodium light for example, falls on a very thin film (Fig. 356). That portion of the ray *AB* which enters the film is reflected from the rear surface, and emerges in the direction *DE*. A portion of another ray *CD* is directly reflected along *DE*. The two rays meet at *D*, but one has traveled farther than the other by about twice the thickness of the film. According as this thickness is equal to the wave length, or to half the wave length, the rays will reinforce or destroy each other.

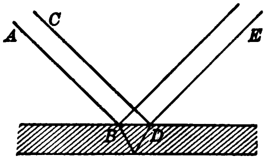


FIG. 356.

In the case of the soap bubble white light loses by interference one or more of its components, and is thereby changed into colored light.

Many beautiful color effects are caused by the interference of luminous waves reflected from very thin films. Examples: the colors of oil films on the surface of water; the colors of thin films of oxide on metals; the colors of pearls and opals; the brilliant metallic colors exhibited by beetles and many species of flies; the gorgeous colors of the peacock's feather, changing in hue as we turn the feather in the hand.

419. Diffraction. The supporters of the emission theory argued that, if the wave theory were true, light would pass round corners, and after going through an aperture would be diffused in all directions, as sound is. But this difference in the behavior of light and sound can be perfectly accounted for on the wave theory, as Huyghens himself pointed out.

When a wave, either of light or sound, passes through an aperture, secondary systems of waves are started at the aperture and travel outwards from it. The laws of wave motion show that if the size of the aperture is very small compared with the length of the waves, then the secondary waves travel on without mutual interference to any extent. This is the case with sound waves. But if the size of the aperture is very large compared with the length of the waves, then the secondary waves are completely destroyed by interference. This is the case with luminous ether waves. Hence, light darts through any ordinary orifice in straight rays, while sound is diffused in all directions.

If, however, the aperture through which light passes is extremely small, the image of the hole or slit thrown on a screen is found to be bordered by colored fringes, blurring the sharpness of the edges. This phenomenon is called *diffraction*.

The emission theory cannot explain diffraction at all.

420. Confirmation of the Wave Theory. The phenomena of *polarization* and *double refraction*, which cannot here be described, furnish additional evidence in favor of the wave theory, and prove that ether vibrations are *transverse*, and not longitudinal as in the case of the air waves of sound.

The crucial test was applied to the two theories by Foucault. The emission theory asserts that light travels faster in water than in air. The wave theory asserts that light travels slower in water than in air. Foucault proved by direct experiment that light travels *slower* in water than in air.

421. Invisible Rays of the Spectrum. We receive heat from the sun as well as light. Examination of the solar spectrum shows that its heating effect extends far beyond the visible red end, and has, in fact, its maximum value beyond the red end. The invisible heat rays have less refrangibility and smaller vibrative frequency than the extreme red rays. They constitute what is called the *infra-red* or *dark heat* spectrum.

Light also produces chemical changes. The fading of colored cloths and the blackening of silver chloride are examples; and the decomposition of carbon dioxide by plants under the action of the sun's rays is the most conspicuous example of all.

It has been found that the chemically active rays of the solar spectrum begin in the yellow part, show their greatest power in the violet part, and extend far beyond the visible violet end of the spectrum. The invisible rays beyond the extreme violet are called the *ultra-violet* or *actinic* spectrum.

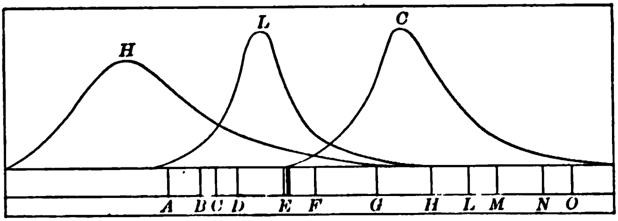


FIG. 357.

The curve *H* (Fig. 357) indicates roughly the calorific power of the spectrum at different points, the curve *L* the luminous power, and the curve *C* the chemical or actinic or photographic power.

The physical difference between dark heat rays and luminous rays, or between luminous rays and actinic rays, is simply a difference in refrangibility and in vibration frequency. The invisibility of certain rays results not from any peculiarity in the rays themselves, but from the nature of the eye.

422. Absorption of Light. When luminous rays are extinguished on meeting the surface of a body, they are said to be *absorbed.* What really takes place is a transfer of energy from the vibrating ether to the molecules of the body. Radiant heat is thereby converted into sensible heat. The body tends to rise in temperature, and a rise will occur unless other conditions prevent it.

Absorption is in almost all cases *selective,* that is, the body absorbs only rays of certain definite wave lengths or frequencies. Sodium vapor absorbs only rays whose wave lengths correspond to the lines *D* of the spectrum. The rays which a given body will absorb must be ascertained by experiment.

The rays which a body does not absorb are either reflected or transmitted. The rays reflected diffusely render the body visible. If the body transmits rays, we can, by looking through it, see the source from which the transmitted rays proceed.

423. Color. Color depends on wave length; it bears the same relation to light that pitch bears to sound. In a pure spectrum each ray has a definite wave length, and causes a definite sensation of color. The colors of a pure spectrum are called *pure, simple,* or *homogeneous.*

Two or more simple colors, when superposed, fuse together and yield a *compound* or *mixed* color.

Two colors, simple or compound, which by fusion yield white light are called *complementary* colors.

Examples of mixed colors: red + green = yellowish white; red + yellow = orange; green + indigo = blue; red + orange + green = yellow.

Complementary colors: red and blue-green, yellow and blue-indigo, orange and light blue, green-yellow and violet. The fusion of red and violet gives *purple,* which is complementary to green.

The ear can analyze a compound tone into simple tones, but the eye cannot analyze a color wave into its components. We cannot tell by the eye a blue-yellow white from a seven-color white.

424. Colors of Bodies. The colors of bodies are mostly due to the absorption of light, and result from the fusion of the rays which *are not absorbed*, but reflected diffusively to the eye. Thus, in white light a red ribbon appears red, because it reflects the red rays, but absorbs all other rays; a blue ribbon appears blue, because it reflects the blue rays, but absorbs all the others.

The color of a body, however, is never pure; there is always an admixture of colors other than that which gives the predominant color tone. In most cases there is also an admixture of white light reflected directly from the surface of the body without absorption. A color which is undiluted with white light is said to be *saturated*.

An opaque substance that reflects all kinds of rays in equal proportions is bright *white*, if it reflects them in great numbers, and *gray*, if it reflects them in smaller numbers. A substance which absorbs practically all the light that falls upon it is *black*; lampblack is a good instance.

A piece of blue glass allows only those rays to pass through it that produce in the eye the sensation of blue; hence, objects viewed through a blue glass appear blue. A combination of a green glass and a red glass is almost opaque, because the first absorbs almost all rays except the green, the second almost all except the red, and therefore both absorb practically all the rays.

If we pulverize a yellow crayon and a blue crayon, and mix the two powders thoroughly with the addition of a little water, we obtain a *green* paste. But if we combine together on a screen the yellow and blue rays of the spectrum the result is a *white* spot. This difference is explained as follows:

The yellow crayon reflects green light as well as yellow, but absorbs all other rays; examination of its spectrum proves this. The blue crayon reflects green light as well as blue, but absorbs all other rays. On mixing, all rays except green are absorbed; hence, the paste appears green. On the other hand, the yellow and the blue of the spectrum are pure colors, and produce white by simple fusion.

When we mix pure colors a simple *addition* of the colors occurs; when we mix colored materials (pigments), a *subtraction* of certain elements occurs, and only the common element remains.

425. Influence of Incident Light on Color. Bodies cannot reflect or transmit rays that do not reach them. Blue paper will not appear blue unless illuminated by light containing blue rays. Daylight contains such rays. But if held at the red end of a spectrum blue paper will appear black, and if illuminated with the yellow light of sodium vapor it will also appear black.

In homogeneous light, differences of color are no longer perceptible; only variations of light and shade are visible. If a room were illuminated with incandescent sodium vapor, all objects in the room would appear either yellow or black. "It requires the white light of the sun in which innumerable colors are blended to disclose to our eyes the variegated tints of natural objects" (Lommel).

426. Theory of Color Sensations. Color is not a property of bodies, but a *sensation*. According to the theory propounded by Dr. Young and developed by Helmholtz, there are *three* primary color sensations, namely, red, green, and violet.

For each of these sensations there is provided on the retina of the eye a set of nerves especially well fitted to produce it. But each set is also excited in a weaker degree by rays of various other wave lengths. When all three sets of nerves are equally excited we have the sensation of white. When the red nerves, for example, are more strongly excited than the others we have the sensation of red, or orange, or yellow.

This theory is supported by the fact that all known tints of color may be produced by blending red, green, and violet in the proper proportions. It explains *color blindness* as due to the absence either of the red nerves or the green nerves. It also explains the cause of *subjective colors*. For example, if a square of bright green paper be placed on white paper, gazed at steadily for some time, and then suddenly withdrawn, the spot it occupied has a reddish hue. The green nerves have become fatigued and fail to respond to the green element in the white light at the spot in question.

REVIEW QUESTIONS ON CHAPTER VIII.

1. What is the cause of sound? What property of matter makes sound possible? Give reasons for your answers.

2. How would you show that sound cannot travel through a vacuum?

3. Describe *transverse* and *longitudinal* vibrations, illustrating by diagrams. Define *period* and *frequency.*

4. What law of force will cause simple harmonic vibrations? What relation exists between such vibrations and uniform circular motion?

5. Explain the meaning of the formula $v = ln$.

6. What is the velocity of sound in air? How is it affected by temperature?

7. Explain echoes.

8. What is the distinction between noise and musical sound?

9. On what does the loudness of a sound depend?

10. On what does the pitch of a sound depend?

11. Define a musical interval and an octave.

12. What are the laws of the vibration of strings?

13. Define, by taking the case of a vibrating string, *harmonic overtones*, *nodes*, *antinodes*, and *stationary waves.*

14. Compare the vibrations of open and closed organ pipes.

15. What is the law of the composition of vibrations?

16. What are *sympathetic* vibrations? Give examples.

17. Define and illustrate *resonance.*

18. Give an example of the interference of sound.

19. What are *beats?* How is their number in any case determined?

20. Describe Helmholtz's resonators and how he used them.

21. Illustrate what is meant by the *quality* of a musical sound. How are different qualities of sound explained?

22. What is the physical explanation of harmony and of discord?

23. Explain by a diagram the formation of a shadow, and distinguish between the umbra and the penumbra.

24. What is the velocity of light? Describe one method by which it has been determined.

25. Prove that the illuminating powers of two sources of light are proportional to the squares of their distances from a surface which they illuminate with equal intensity.

26. Prove that the image of a luminous point in front of a plane mirror is on the normal from the point to the mirror, and as far behind the mirror as the point is before it.

27. An object is placed midway between two parallel mirrors. Show that a series of images arranged on a straight line will be formed. Illustrate by a diagram how the series of images are formed.

28. Define the terms *principal focus* and *focal length.*

29. Prove the mirror formula. What are *conjugate* foci?

30. Construct the image of an object placed before a concave mirror beyond the principal focus.

31. Construct the image of an object placed between a concave mirror and the principal focus.

32. What are the laws of refraction? Define the index of refraction.

33. Explain why a stick appears bent where it enters the surface of water. Illustrate by a diagram.

34. Explain, with a diagram, the total reflection of light. Define the critical angle. What is its value for water? For glass?

35. Explain by a diagram the passage of light through a window pane.

36. Draw a diagram showing the passage of light through a prism. What is the best way to change the direction of rays of light by 90°?

37. How do convex and concave lenses differ in their action on light?

38. Define the *optical center* of a lens.

39. Construct the image of an object placed between a convex lens and the principal focus.

40. What is *spherical aberration?* What are its effects? How is spherical aberration prevented?

41. How did Newton analyze white light?

42. How can white light be produced by synthesis?

43. Give a general explanation of the rainbow.

44. What is *chromatic aberration?* How is it prevented?

45. Describe briefly the spectroscope and its uses.

46. Explain the Fraunhofer lines in the solar spectrum.

47. Why can we see distinctly objects at very different distances?

48. What are the advantages of vision with *two* eyes?

49. Give an example of an *after-image.* How do you account for it?

50. Describe, with diagram, the compound microscope.

51. Describe, with diagram, the astronomical telescope.

52. Give a brief account of the two theories about light, and reasons for preferring the wave theory.

53. How are the colors of soap bubbles explained?

54. What is the physical cause of color? Give examples of *pure* colors, *mixed* colors, and *complementary* colors.

55. In a room illuminated with sodium light all objects would appear either yellow or black. Explain.

APPENDIX.

Useful Propositions in Plane Geometry.

1. The sum of the angles of a triangle = 180°.

2. Two triangles are equal, if they have their three sides equal, each to each, or two sides and the included angle equal each to each, or one side and two angles equal each to each.

3. Two right triangles are equal, if they have a side and hypotenuse of one equal, respectively, to a side and hypotenuse of the other.

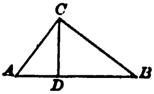
4. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

5. Two triangles equiangular with respect to each other are similar.

6. The sides opposite equal angles of similar triangles are proportional.

7. The line bisecting an angle of a triangle divides the opposite side into parts proportional to the adjacent sides.

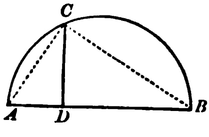
In the figure, $\angle ACD = \angle BCD$, and $\frac{AD}{BD} = \frac{AC}{BC}$.



8. The perpendicular from any point in the circumference of a circle to the diameter is a mean proportional between the segments of the diameter; and a chord drawn from the point to either end of the diameter is a mean proportional between the diameter and the adjacent segment.

In the figure,

$$\overline{DC}^2 = AD \times DB.$$
$$\overline{AC}^2 = AB \times AD.$$
$$\overline{BC}^2 = AB \times BD.$$



Useful Formulas in Plane Geometry.

Area of a triangle $= \frac{1}{2}$ base × altitude.

Area of a parallelogram = base × altitude.

Area of a trapezoid $= \frac{1}{2}$ sum of bases × altitude.

Area of a regular polygon $= \frac{1}{2}$ perimeter × radius of inscribed circle.

Circumference of a circle $= 2\pi r$.

Area of a circle $= \pi r^2$.

π = ratio of circumference to diameter $= \frac{22}{7}$ very nearly.
r = radius of the circle.

Useful Formulas in Solid Geometry.

Volume of a rectangular solid $\}$ = length × width × height.

Area of lateral surface of right prism or cylinder $\}$ = perimeter of base × height.

Volume of right prism or cylinder $\}$ = area of base × height.

Area of lateral surface of right pyramid or cone $\}$ $= \frac{1}{2}$ perimeter of base × slant height.

Volume of right pyramid or cone $\}$ $= \frac{1}{3}$ area of base × height.

Area of the surface of a sphere (r = radius of sphere) $\}$ $= 4\pi r^2$.

Volume of a sphere (r = radius of sphere) $\}$ $= \frac{4}{3}\pi r^3$.

INDEX.

The numbers refer to pages.

Aberration, chromatic, 406.
  of light, 378.
  spherical, 388, 401.
Absolute temperature, 103.
Absorption, of gases, 148.
  of heat, 133.
  of light, 424.
  selective, 134.
  spectra, 408.
Acceleration, 180.
  of gravity, 183.
Accommodation, power of, 412.
Achromatic lens, 406.
Actinic spectrum, 423.
Action, 15.
Adhesion, 157.
After-Images, 415.
Agonic line, 263.
Air, composition of, 168.
Air pump, 86.
  Sprengel's, 91.
Alternators, 339.
Ammeter, 296.
Amorphous, 147.
Ampere, 297, 299.
Amperes defined, 307.
Amplitude, 205.
Annealing, 154.
Anode, 327.
Antinodes, 362.
Archimedes, 37; law of, 62.
Arc lamp, 323.
Armature, 300, 336.
  drum, 337.
  ring, 338.
Artesian wells, 58.
Astatic galvanometer, 296.
Atmospheric pressure, 72.
Atom, 4, 134.
Atomic theory, 167.
Atomizer, 91.
Attraction, 202.
  of gravitation, 214.

Balance, 19, 48.
Balance wheel of a watch, 98.
Balanced forces, 14.
Balloon, 82.
Barometer, 74.
Beats, 368, 372.
Bending stress, 20.
Bicycle, 232.
Binocular vision, 414.
Blake transmitter, 348.
Body, 1.
Boiling, 109.
Boyle, law of, 78.
Bradley, 378.
Bright line spectra, 408.
Brittleness, 154.

Buoyant force, 62, 63.
Burning, 169.

Caisson, 81.
Calorie, 115.
Calorific power, 249.
Calorimeter, 121.
Capacity, inductive, 276.
Capillary action, 158.
Capstan, 227.
Carnot's principle, 250.
Center, of buoyancy, 62.
  of gravity, 44.
  of oscillation, 207.
  of percussion, 207.
Centrifugal force, 211, 212.
Centripetal force, 210.
Change of momentum, 199.
  of state, 106.
  of volume on melting, 107.
Charles' law, 102.
Chemical changes, 3, 161.
Chemical combination, 162.
  law of, 166.
  and heat, 169.
Chemical effects of an electric current, 327.
Chemical exchange, 165.
  separation, 163.
  symbols, 167.
Chemistry, 4.
Chromatic aberration, 406.
Circuit, divided, 309.
Clouds, 138.
Coefficient, of expansion, 101.
  of friction, 26.
Cohesion, 153.
  in liquids, 155.
Cold, from evaporation, 119.
Colloids, 150.
Color, cause of, 420, 424.
Color, theory of, 426.
Colors of bodies, 425.
Combustion, 169.
Commutator, 312, 337.
Compass needle, 263.
Complementary colors, 424.
Component, 30.
Composition of motion, 174.
Compound machines, 231.
Compound pendulum, 207.
Compounds, 164.
Compressibility, 1.
Compression pump, 87.
Concurrent forces, 29.
Condensation, 109.
Condenser, air, 87.
  electrostatic, 281.
Condensing engine, 252.
Conduction, of electricity, 272.
  of heat, 128.
Conservation of energy, 256.
Constant force, its effect, 195.
Convection of heat, 126.
Conversion of heat to mechanical energy, 244.
  of mechanical energy to heat, 242.
Coulomb, 25.
Coulomb, defined, 307.
Critical angle, 395.
Cryophorus, 119.
Crystalline structure, 147.
Crystallization, 145.
  uses of, 146.
Crystalloids, 150.
Currents, chemical effects of, 327.
  eddy, 336.
  electric, 286.
  extra, 344.
  induced, 334.
  magnetic effects of, 294.

Currents, mutual action of, 299.
  strength of, 289.
  thermal effects of, 318.
Cyclone, 75, 140.

Daniell cell, 291.
D'Arsonval's galvanometer, 296.
Davy, 129, 245.
Declination of needle, 269.
Density, 9.
Dew, 138.
Dew point, 139.
Dialysis, 150.
Diathermacy, 133.
Diatonic scale, 360.
Dielectric substance, 276.
Differential air thermometer, 132.
Diffraction, 422.
Diffusion, of gases, 151.
  of liquids, 149.
Discontinuous spectra, 408.
Dispersion of light, 403.
Dissipation of energy, 260.
Distillation, 113.
Distribution of electric charge, 279.
Divided circuits, 309.
Diving bell, 81.
Doppler's principle, 368.
Double Weighing, 48.
Draft of chimney, 126.
Ductility, 22, 154.
Dynamical, measure of force, 192.
  units, 220.
Dynamics, 16.
Dynamo, 336.
  efficiency of, 340.
  series and shunt, 338.
Dyne, 193.

Ear, 373.
Ebullition, 110.
Ebullition, laws of, 111.
Echoes, 337.
Eddy currents, 336.
Efficiency, 225, 253, 340, 342.
Elasticity, 18.
  elastic limit, 18.
  modulus of, 20.
Electric attraction, 273.
  bell, 301.
  density, 280.
  energy, 320.
  firing of explosives, 324.
  machine, 278.
  motor, 341.
  efficiency of a motor, 342.
  potential, 287.
  pressure, 290.
  repulsion, 273.
  spark, 276.
  telegraph, 302.
  transmission of power, 343.
Electric current, first notions of, 286.
  strength of, 289.
Electricity, in the air, 283.
  statically measured, 279.
Electrification by contact, 271.
  two kinds of, 273.
Electro-chemical equivalent, 331.
Electrodes, 327.
Electrolysis, 327.
  laws of, 331.
  of water, 163, 329.
  uses, 332.
Electromagnet, 300.
Electromagnetic, induction, 334, 344.
  units defined, 306.
Electromotive force, 290.
  counter, 329.
  how measured, 329.
Electrophorus, 275.

Electroplating, 332.
Electroscope, 277.
Electrostatic induction, 274.
  unit of quantity, 279.
Electrotype, 327.
Electrotyping, 332.
Elements, 164.
  of a force,16.
Emission theory of light, 420.
Endosmose, 150.
Energy, and velocity, 236.
  conservation of, 256.
  dissipation of, 260.
  in a machine, 237.
  kinetic, 234.
  of a steam engine, 253.
  of the sun, 258, 259.
  potential, 237.
  sources of, 257.
  transformation of, 254.
Equilibrium, kinds of, 46.
  stability of, 47.
Erg, 220.
Ether, 420.
Evaporation, 109, 119.
Exchanges, theory of, 134.
Exosmose, 150.
Expansion, coefficient of, 101.
  of gases, 96.
  of liquids, 96.
  of solids, 97.
Extra currents, 344.
Eye, 411.

Falling bodies, laws of, 180.
Fall of potential, 310.
Farad, 281.
Faraday, 112, 268, 280, 334, 336, 346.
  laws of, 331.
Far-sighted eye, 412.
Fire engine, 89.
  syringe, 243.
First law of motion, 188.
Fixed pulley, 27.
Fizeau, 378.
Floating docks, 65.
Flotation, law of, 64.
Focal length of lens, 397.
Focus, conjugate foci, 389, 398.
  principal, 387, 397.
  real and virtual, 388.
Fogs, 138.
Foot-pound, 217.
Foot-poundal, 220.
Force, buoyant, 62.
  composition of, 30, 39.
  defined, 13.
  effect of constant, 195.
  elements of a, 16.
  impulse of, 196.
  in circular motion, 211.
  internal, 203.
  measure of, 192.
  molecular, 153.
  parallelogram of, 30.
  physical independence of, 196.
  resolution of, 32.
Forced vibrations, 366.
Foucault, 336, 379, 422.
Franklin, 111, 282, 283.
Fraunhofer lines, 408, 409.
Freezing mixtures, 118.
Frequency of vibration, 352.
Friction, 25; its laws, 25.
Fusion, 106.

Galvani, 288.
Galvanometer, 295.
Galvanoscope, 295.
Gamut, 360.
Gases compared with liquids, 71.

Glow lamp, 322.
Governor, 252.
Gravitation, 214.
  measure of force, 16.
Grove cell, 292.
Gunpowder, 161.

Hail, 139.
Hardness, 154.
Harmonic overtones, 362.
  tones, 359.
Harmony and discord, 372.
Heat, absorption of, 133.
  change of volume due to, 96, 107.
  conduction of, 128.
  considered as a quantity, 115.
  convection of, 125.
  converted into mechanical energy, 244.
  disappearance of, 117, 118.
  due to compression, 243.
  due to electric currents, 318.
  due to friction and collision, 242.
  effects of, 120.
  in the atmosphere, 136.
  latent, 117, 118.
  law of radiant, 132.
  mechanical equivalent of, 246.
  mechanical theory of, 247.
  of chemical combination, 249.
  radiant, 131.
  specific, 116.
  spectrum, 423.
  true nature of, 245.
Helmholtz, 259, 370, 371, 372, 373, 426.
Hoar frost, 138.
Hooke's law, 19.
Hope's experiment, 100.
Horse power, 220.
Humidity, 137.
Huyghens, 207, 208, 420, 422.
Hydraulic press, 53.
Hydrodynamics, 16.
Hydrostatic balance, 62.
Hydrostatics, 16.

Ice, contracts on melting, 107.
Illuminating power, 380.
Images, by small apertures, 387.
  of images, 386.
  in lenses, 400.
  in plane mirrors, 384.
  in spherical mirrors, 390.
Impact, 200.
Impulse of a force, 196.
Impulsive forces, 196.
Incandescent light, 322.
Inclination of needle, 263.
Inclined plane, 32.
  friction on, 33.
Indestructibility of matter, 4.
Index of refraction, 393.
Induced currents, 335, 344.
  of air, 91.
Induction, coil, 345.
  electromagnetic, 334.
  electrostatic, 274.
  magnetic, 265.
Inductive capacity, 276.
Inertia, 188.
  applications of, 189.
  of rotation, 190.
Insulators, 272.
Intensity of light, 380.
Interference, of light, 421.
  of sound, 367.
Intermittent springs, 90.
Internal forces, 203.
Isobaric lines, 75.
Isogonic lines, 263.

Joule, 246, 301.
  defined, 320.
Joule's equivalent, 246.
  law, 319.

Kater, 208.
Kathode, 327.
Key-note, 360.
Kinetic energy, 234.
  theory of gases, 248.
Kirchhoff, 410.

Land breezes, 140.
Lantern, optical, 416.
Laryngoscope, 392.
Latent heat, of evaporation, 118.
  of fusion, 117.
  of steam, 118.
Lawn sprinkler, 56.
Leclanché cell, 292.
Lenses, 397.
  lens formula, 399.
Lentz's law, 335.
Lever, 36.
Leyden jar, 282.
Light, absorption of, 424.
  analysis of, 403.
  composite nature of, 404.
  intensity of, 380.
  rectilinear motion of, 376.
  reflection of, 383.
  refraction of, 393.
  synthesis of, 404.
  velocity of, 378.
Lighters, 65.
Lightning rod, 283.
Lines of magnetic force, 268, 335.
Liquefaction, 109, 112.
Local action, 290.
Locomotive, 252.
Lodestone, 262.
Loudness of sound, 358.
Luminous body, 375.

Machines, compound, 231.
  simple, 223, 226.
Magdeburg hemispheres, 75.
Magnetic action of the earth, 263.
  attraction, 262, 264.
  effects of a current, 294.
  field of force, 267, 297.
  induction, 265.
  meridian, 163.
  needle, 263.
  poles, 262, 263.
Magnetization by contact, 266.
Magnifying power, 391, 400.
Malleability, 22, 154.
Manometer, 79.
Mariner's compass, 263.
Mariotte, law of, 78.
Mass, 7.
  proportional to weight, 194.
Maximum density of water, 100.
Mayer, 246, 259.
Measure of force, 192.
Mechanical advantage, 38, 225.
  equivalent of heat, 246.
  mixture, 161.
  theory of heat, 247.
Mechanics defined, 16.
Microscope, compound, 417.
  simple, 416.
Mirror galvanometer, 311.
Mirrors, parallel, 386.
  perpendicular, 386.
  plane, 384.
  spherical, 387.
Mists, 138.
Mixtures, method of, 116.
Molecular forces, 153.
  phenomena, 143.

Molecular theory of magnets, 266.
  theory of matter, 266.
Molecule, 4, 143.
Moment of a force, 38.
Momentum, 192.
Monsoons, 140.
Motion, accelerated, 180.
  composition of, 174.
  in a circle, 210.
  Newton's laws of, 188,192,199.
  of a projectile, 184.
  on inclined plane, 183.
  relative, 176.
  resolution of, 175.
  uniform, 173.
Motor dynamo, 347.
  electric, 341.
Music and noise, 358.
Musical sounds, 358.
  compound nature of, 369.
  intervals, 369.
Mutual action of currents, 299.

Near-sighted eye, 412.
Neutral axis, 21.
Newton, 188, 192, 199, 214, 215, 403, 420.
Newton's law of cooling, 132.
Nodes, 362.
Noise, 358.

Octave, 360.
Oersted's discovery, 294.
Ohm defined, 307.
Ohm's law, 304.
  application of, 305.
Opaque body, 375.
Opera glass, 418.
Ophthalmoscope, 392.
Optical center, 399.
  lantern, 416.
Organ pipes, 363.
Oscillation, 205.
  center of, 207.
Osmosis, 150.
Overtones, 362, 369.
Oxidation, 169.

Parabola, 185.
Parachute, 83.
Parallel forces, 39.
Parallelogram law, 30.
Partial tones, 369.
Pascal's law, 52.
Pencils of rays, 376.
Pendulum, compensation, 104.
  compound, 207.
  formula of, 206.
  laws of, 206.
  simple, 205.
  uses of, 208.
Permeability, magnetic, 269.
Photographers' camera, 419.
Photometry, 381.
Physical change, 3.
  independence of forces, 196.
Physics defined, 4.
Pigments, mixing of, 425.
Pile driver, 238.
Pin-hole camera, 376.
Pitch, 359.
Plasticity, 22, 154.
Pneumatic inkstand, 94.
Pneumatics, 16.
Polarization, 291.
Poles of a magnet, 262, 263, 264.
Porosity, 2.
Potential, electric, 287.
  energy, 238, 240.
Poundal, 193.
Power, 220, electric, 321.
Pressure, 15, 199.

Pressure gauge, 79.
  of the atmosphere, 72.
  of a liquid, 54, 56, 57.
Principle of work, 224.
Projectiles, 184.
Pulley, 27, 227.
Pump, air, 86.
  force, 89.
  steam, 89.
  suction, 88.

Quality of sound, 370.

Radiation, 131.
  laws of, 132.
Rain, 139, 258.
Rainbow, 405.
Rate of working, 220.
Reaction, 15.
Receiver, 86.
Reduction, 171.
Reflection, of light, 383.
  of sound, 357.
  total, 395.
Refraction, by glass plate, 396.
  by lens, 397.
  by prism, 396.
  effects of, 394.
  laws of, 393.
Regelation, 108.
Relative motion, 176.
Relay, 302.
Repulsion, 202.
Residual magnetism, 301.
Resistance, electric, 308, 310, 312.
Resolution, of force, 32.
  of motion, 175.
Resonance, 366.
Resonators, 366, 370.
Respiration, 170.
Resultant, 30.
Retentive power, 265.
Rheostat, 312.
Rigidity, 154.
Rods of Corti, 373.
Roemer, 378.
Rotary converter, 347.
Rotation, motion of, 177.
Rumford, 245.

Safety valve, 80.
Saturated air, 137.
Screw, 230.
Sea breezes, 140.
Secondary action, 328.
Secondary currents, 330.
Second law of motion, 192.
Selective absorption, 134.
Self-induction, 344.
Shadows, 377.
Shearing stress, 22.
Simple harmonic motion, 353.
  machines, 223, 226.
  pendulum, 205.
Siphon, 90.
Snow, 139.
Soap bubbles, 156, 160, 421.
Solar spectrum, 403.
Solenoid, 298.
Solution, 144.
Sonometer, 361.
Sound, 351.
  cause of, 351.
  interference of, 367.
  loudness of, 358.
  pitch of, 359.
  quality of, 370.
  reflection of, 357.
  velocity of, 356.
Sounder, 303.
Sources of energy, 257.

Specific gravity, 66.
  methods of finding, 67, 68.
  heat, 116.
  method of finding, 116, 121.
Spectacles, 412.
Spectroscope, 407.
Spectrum, absorption, 408.
  analysis, 410.
  continuous, 408.
  discontinuous, 408.
  invisible rays of, 423.
  reversed, 409.
Spherical aberration, 401.
Sprengel's air pump, 91.
Spring balance, 19.
Statics, 16.
Steam engine, 251, 255.
  pump, 89.
Steel and iron, magnetic properties of, 265.
Storage cell, 330.
Strain, 15.
Stress, 15, 199.
Subjective colors, 426.
Sublimation, 109.
Substitution, method of, 312.
Sucker, 94.
Summary of heat effects, 120.
Surface tension, 156.
Sympathetic vibrations, 365.

Tangent galvanometer, 294.
Tantalus, vase of, 90.
Telephone, 348.
Telescope, 417.
  astronomical, 418.
Temperature, 95.
  measurement of, 98.
  of the air, 136.
Tempering, 154.
Tensile stress, 19.
Tension, 15, 199.
Theories about light, 420.
Thermal effects of a current, 318.
Thermodynamics, 247.
Thermo-electricity, 325.
Thermometer, 98.
Thermopile, 324.
Third law of motion, 199.
Torricelli's experiment, 72.
Torsion, 22.
Total reflection of light, 395.
Toughness, 154.
Trade winds, 140.
Transformation, of energy, 254.
  principle of, 346.
Transformers, 347.
Translation, motion of, 177.
Translucent body, 375.
Transparent body, 375.
Tyndall, 108, 130, 134, 243, 259.

Uniform motion, 173.
Units, electromagnetic, 306.
  of force, 16, 193.
  of heat, 115.
  of work, 217, 220.
  tables of fundamental, viii.
Universal gravitation, 214.

Value of *g*, 183.
Vapor, 112.
Vaporization, 109.
Variation of compass, 263.
Velocity, of light, 378.
  of sound, 356.
  uniform, 173.
  variable, 179.
Ventilation, 127.
Vibrating spiral, 299.
Vibration, 205.
  composition of, 363.

Vibration, of air columns, 363.
  of a string, 361.
  sympathetic, 365.
Vibratory motion, 352.
Vice, 14, 230.
Viscosity, 159.
Vision, binocular, 414.
  distinct, 413.
Visual angle, 413.
Volatilization, 109.
Volt defined, 307.
Volta, 288.
Voltaic arc, 323.
  battery, 293.
  cell, 288.
  cells, how arranged, 314.
  cells in common use, 291, 292.

Watch spring, 239.
Water, maximum density of, 100.
  wheel, 238.
Watt defined, 320.
Wave defined, 354.
  longitudinal, 355.
  stationary, 362.
  theory of light, 420, 422.
  transverse, 354.
Wedge, 229.
Weight, 7, 43.
Wheatstone's bridge, 310.
Wheel and axle, 226.
  differential, 228.
Windlass, 226.
Winds, 139, 258.
Work, 217.
  against gravity, 217.
  by gravity, 218.
  of oblique forces, 219.
  principle of, 224.
  waste, 225.

Young, C. A., 258.
  Thomas, 20, 426.

$$a = \frac{V_1 - V_0}{t}$$

Formulas for acceleration.

$$s = \left(\frac{V_1 + V_0}{2}\right) t$$

$$2as = V_1^2 - V_0^2$$

$$s = tV_0 + \tfrac{1}{2} at^2$$

Laurence I. Neale
Fairfax 28
Harvard